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A STUDY ON THE COMPLEMENTS OF THE ELEMENTS OF Z-SOFT COVERING BASED ROUGH LATTICE AND ITS APPLICATION

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Abstract. In this paper, we discuss information system $I = (\Omega, B)$ and related Z-soft covering based rough lattices $(T_S, \vee, \wedge)$ in which $\vee$ denotes join and $\wedge$ denotes meet. We prove the existence of maximal and minimal elements for Z-soft covering based rough lattice and define the complement of elements of the set T_S. The proposed concepts are explained through examples.

Keywords: Soft set; Rough set; Lattice; Boolean algebra.

AMS Subject Classification: 03E72, 03G10, 90B50.

1. Introduction

Zadeh [24] investigated the general theory of uncertainty. In this theory, information is represented as general constraints derived from fuzzy set theory and fuzzy logic, and uncertainty is linked to information through the idea of granular structures. In 1982, Pawlak [16] initiated a rough set (RS). This formal technique was developed in information systems to handle incomplete data. RS is used in a variety of fields, including artificial intelligence, such as pattern recognition, intelligent systems, expert systems, knowledge discovery and others [1, 5, 6, 10, 11, 14]. Extensions of rough sets are covering rough

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sets (CRS), which have recently become a significant area of research. Based on CRS, researchers can examine uncertainty and roughness in a broader context. In 1999, Molodtsov presented a soft set, another mathematical approach to dealing with uncertainty. Many other information representations and computational operations are made possible by soft sets. Ali et al. [7] introduced different operations such as restricted intersection, union and difference on soft sets. In 2021, Al-Shami introduced a new types of soft compactness on finite spaces and different kinds of soft separation axioms in [2] and [3], respectively. Likewise, soft somewhat open sets and their behaviours are studied in [4] through some specific topologies. Ali [8] defined fuzzy soft set with the help of soft set. Roy and Maji [21] proposed a decision-making model by creating a comparison table using fuzzy soft sets. In [9], semiring structures of soft sets are discussed. Feng et al. initiated soft P-rough set in [12]. Shabir et al. [22] developed a modified soft rough set (MSR) using Feng's soft set theory. MSR performs better in terms of accuracy than other existing models. Compared to the Shabir-soft rough set, the development of the soft P-rough set requires extra criteria. Feng et al. [13] created a Multiple Attribute Group Decision Making (MAGDM) using soft rough set. Yüksel et al. [23] established soft covering based rough sets (SCRS) to form a decision making algorithm.

Zhan et al. [25] proposed five new kinds of SCRS. They proved that the third type of SCRS provides a exact representation of sets than other types of SCRS. Praba et al. [17] defined two new operations praba ∇ and praba Δ to prove that the collection of all rough set (T) is lattice. Praba et al. [19] developed a lattice structure for minimal soft rough sets and provided a new decision-making technique based on it. Then, the existence of a maximum and a minimum element of this lattice is proved, and the complement of elements in T is defined in [18]. Pavithra and Manimaran [20] defined two new operations join $\vee$ and meet $\wedge$ on the collection of all Z-soft covering based rough set (T_S). Using these two operations, it is demonstrated that every pair of components has a least upper bound (lub) and a greatest lower bound (glb), and as a result, T_S is a lattice. Inspired by these ideas, a lattice structure is constructed for Z-soft covering based rough set and developed a decision-making algorithm.

The structure of this paper is outlined below: Definitions required for understanding the following sections are provided in Section 2. Section 3 defines the complement of the elements of T_S. Section 4 presents a decision making algorithm developed using a Z-soft covering based rough sets. The conclusion is discussed in Section 5.

2. Preliminaries

The basic definitions necessary to comprehend the topics that follow are covered in this section. Ω represents the finite universe throughout this article.

Definition 2.1. [16] *Let R to be an equivalence relation and (Ω, R) be an approximation space. For any $M \subseteq \Omega$, the lower and upper approximation of M with respect to R are given by $\underline{R}(M) = \{v \in \Omega : [v]_R \subseteq M\}$ and $\overline{R}(M) = \{v \in \Omega : [v]_R \cap M \neq \emptyset\}$, respectively and the corresponding rough set is defined as $RS(M) = \left(\underline{R}(M), \overline{R}(M)\right)$.*

Definition 2.2. [15] *Let E be the set of all parameters and $B \subseteq E$. A pair $K = (N, B)$ is known as a soft set over Ω, if N is defined by $N : B \to P(\Omega)$ where $P(\Omega)$ indicates the power set of Ω.*

Definition 2.3. [12] *A soft set $K = (N, B)$ is called a full soft set over Ω, if $\bigcup\limits_{b \in B} N(b) = \Omega$.*

Definition 2.4. [12] *A full soft set $K = (N, B)$ over Ω is called a covering soft set denoted as C_K, if $N(b) \neq \emptyset$, $\forall\ b \in B$.*

Definition 2.5. [23] *Let $K = (N, B)$ be a covering soft set over Ω. A pair $S = (\Omega, C_K)$ represents a soft covering approximation space (SCA).*

Definition 2.6. [25] *Let $S = (\Omega, C_K)$ be a SCA. The soft adhesion of v is defined by $SA(v) = \{u \in \Omega : \forall\, b \in B(v \in N(b) \leftrightarrow u \in N(b))\}$, for each $v \in \Omega$.*

Definition 2.7. [25] *Let $S = (\Omega, C_K)$ be a SCA. The soft covering lower approximation (SCLA) and upper approximation (SCUA) are respectively defined as $\underline{SC}(M) = \{v \in \Omega : SA(v) \subseteq M\}$ and $\overline{SC}(M) = \{v \in \Omega : SA(v) \cap M \neq \emptyset\}$, for each $M \subseteq \Omega$. If $\underline{SC}(M) \neq \overline{SC}(M)$, then M is called Z-soft covering based rough set. It is denoted as $SCRS(M)$ and defined by $SCRS(M) = (\underline{SC}(M), \overline{SC}(M))$.*

Example 2.1. *Let $\Omega = \{v_1, v_2, v_3, v_4\}$ be the universe set and $B = \{b_1, b_2, b_3\}$ be the collection of parameters. Then the soft set over Ω is given by TABLE 1 where $N(b_1) = \{v_1, v_2, v_3, v_4\}$, $N(b_2) = \{v_2, v_4\}$ and $N(b_3) = \{v_1, v_2, v_3\}$.*
Then, $SA(v_1) = \{v_1, v_3\}$, $SA(v_2) = \{v_2\}$, $SA(v_3) = \{v_1, v_3\}$, $SA(v_4) = \{v_4\}$.

Table 1. Tabular representation of the soft set

	b_1	b_2	b_3
v_1	1	0	1
v_2	1	1	1
v_3	1	0	1
v_4	1	1	0

(i) Let $M = \{v_1, v_4\} \subseteq \Omega$; then $\underline{SC}(M) = \{v_4\}$ and $\overline{SC}(M) = \{v_1, v_3, v_4\}$.
Hence, $SCRS(M) = (\{v_4\}, \{v_1, v_3, v_4\})$.
(ii) Let $M = \{v_2, v_3, v_4\} \subseteq \Omega$; then $\underline{SC}(M) = \{v_2, v_4\}$ and $\overline{SC}(M) = \Omega$.
Hence, $SCRS(M) = (\{v_2, v_4\}, \Omega)$.
The equivalence classes formed by soft adhesion are $[v_1] = \{v_1, v_3\}$, $[v_2] = \{v_2\}$, $[v_4] = \{v_4\}$.

Definition 2.8. [20] *Let $T_S = \{SCRS(M) : M \subseteq \Omega\}$ and define a relation R_S on T_S by $R_S = \{(SCRS(M), SCRS(O)) : SCRS(M) \subseteq SCRS(O)\}$.*

Lemma 2.1. [20] *R_S is a poset on T_S.*

Definition 2.9. [20] *For each two subsets M and O of Ω. $SAW(M) = \{SA(v) : SA(v) \subseteq M\}$. Define the set $M \vee O$ as follows:*
(1) $M \vee O = M \cup O$, if $|SAW(M \cup O)| = |SAW(M)| + |SAW(O)| - |SAW(M \cap O)|$.
(2) If $|SAW(M \cup O)| > |SAW(M)| + |SAW(O)| - |SAW(M \cap O)|$ then there exists $v \in \Omega$ such that $SA(v) \subseteq SAW(M \cup O)$, $SA(v) \nsubseteq M$ and $SA(v) \nsubseteq O$.
(3) Remove v from M (or O).
(4) Name the newly formed set as M (or O).
(5) Redo Step 1 if there is no v such that $SA(v) \nsubseteq M$ and $SA(v) \nsubseteq O$ is found, then $M \vee O = M \cup O$.

Definition 2.10. [20] *For each subset M and O of Ω, any element $v \in \Omega$ is called pivot element and $\widehat{P}_{M \cap O} = \{v \in \Omega : SA(v) \cap M \neq \emptyset, SA(v) \cap O \neq \emptyset, SA(v) \nsubseteq M \cap O\}$ is the pivot set for Z-soft covering based rough set.*

Definition 2.11. [20] *For each subset M and O of Ω. The meet of M and O is defined by*

$M \wedge O = \{v \in \Omega : SA(v) \subseteq M \cap O\} \cup \widehat{P}_{M \cap O}$.

Theorem 2.1. [20] *If M and O are any two subsets of Ω then $SCRS(M \vee O)$ is the lub of $SCRS(M)$ and $SCRS(O)$.*

Theorem 2.2. [20] *If M and O are any two subsets of Ω then $SCRS(M \wedge O)$ is the glb of $SCRS(M)$ and $SCRS(O)$.*

Theorem 2.3. [20] *Let $K = (N, B)$ be a soft set over Ω, then $(T_S, \subseteq)$ is a lattice. $(T_S, \subseteq)$ is known as Z-soft covering based rough lattice.*

3. Complement of elements of Z-soft covering based rough lattice

In this section, the complement of the elements of Z-soft covering based rough lattice are defined and a lattice structure for Z-soft covering based rough sublattice is proposed.

Definition 3.1. *If $SCRS(O) \subseteq SCRS(M)$, for any $SCRS(O) \in T_S$, then the element $SCRS(M) \in T_S$ is said to be the maximal element.*

Definition 3.2. *If $SCRS(M) \subseteq SCRS(O)$, for any $SCRS(M) \in T_S$, then the element $SCRS(O) \in T_S$ is said to be the minimal element.*

Theorem 3.1. *If $(T_S, \vee, \wedge)$ is the Z-soft covering based rough lattice then $SCRS(\Omega)$ is the maximal element and $SCRS(\emptyset)$ is the minimal element.*

Proof. Let $SCRS(M) \in T_S$ then $SCRS(M) = \big(\underline{SC}(M), \overline{SC}(M)\big)$ where $\underline{SC}(M) \subseteq \underline{SC}(\Omega)$ and $\overline{SC}(M) \subseteq \overline{SC}(\Omega)$, therefore $SCRS(M) \subseteq SCRS(\Omega)$. Hence, $SCRS(\Omega)$ is the maximal element. Now, $SCRS(\emptyset) = (\emptyset, \emptyset)$ implies $\underline{SC}(\emptyset) \subseteq \underline{SC}(M)$ and $\overline{SC}(\emptyset) \subseteq \overline{SC}(M)$, therefore $SCRS(\emptyset) \subseteq SCRS(M)$. Hence, $SCRS(\emptyset)$ is the minimal element. □

Theorem 3.2. *A Z-soft covering based rough lattice $(T_S, \vee, \wedge)$ is a bounded lattice.*

Proof. The proof is trivial from the statement of Theorem 3.1. □

Definition 3.3. *If $SCRS(M) \vee SCRS(O) = SCRS(\Omega)$ and $SCRS(M) \wedge SCRS(O) = SCRS(\emptyset)$, then the complement of $SCRS(M) \in T_S$ is $SCRS(O) \in T_S$.*

Theorem 3.3. *Let $I = (\Omega, B)$ be an information system, $T_S = \{SCRS(M) : M \subseteq \Omega\}$ and let $(T_S, \vee, \wedge)$ be the Z-soft covering based rough lattice. If M is the union of one or more equivalence classes formed by soft adhesion, then $SCRS(M)$ has a complement in T_S.*

Proof. Let $M \subseteq \Omega$ and $\overline{M} \subseteq \Omega$ be the union of one or more equivalence classes formed by soft adhesion then $SCRS(M) \vee SCRS(\overline{M}) = SCRS(\Omega)$ and $SCRS(M) \wedge SCRS(\overline{M}) = SCRS(\emptyset)$. Hence, $SCRS(\overline{M})$ is the complement of $SCRS(M)$. □

Theorem 3.4. *Let $(T_S, \vee, \wedge)$ be Z-soft covering based rough lattice. Let $J = \{M_1, M_2, ..., M_k\}$ be the collection of all equivalence classes formed by soft adhesion then $(T_P, \vee, \wedge)$ is a Boolean algebra where $T_P = \{SCRS(M) : M \in P(J)\}$ where $P(J)$ is the power set of J.*

Proof. It is clear that T_P is a sublattice of T_S, that is for each $SCRS(M)$ and $SCRS(O) \in T_P$ such that $SCRS(M \vee O)$ and $SCRS(M \wedge O) \in T_P$. T_P is bounded because $SCRS(\emptyset)$ and $SCRS(\Omega) \in T_P$.

$(T_P, \vee, \wedge)$ is distributive since $(T_S, \vee, \wedge)$ is distributive.

Using Theorem 3.1, Every element of T_P has a complement in T_P.
Therefore, $(T_P, \vee, \wedge)$ is a Boolean algebra. $\square$

Definition 3.4. *If there is at least one equivalence class induced by soft adhesion M_i such that $M_i \nsubseteq M$ and $M_i \cap M \neq \emptyset$ then M is called Z-soft covering based roughly weak.*

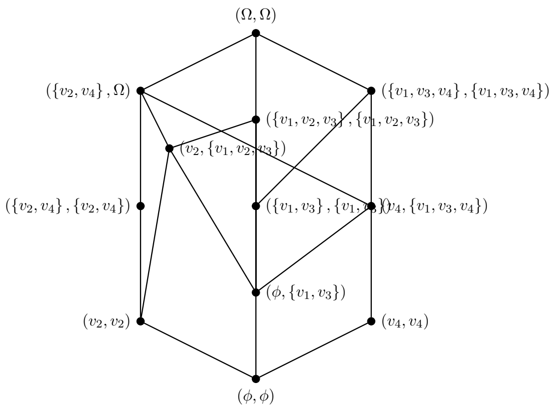
Theorem 3.5. *Let $(T_S, \vee, \wedge)$ be Z-soft covering based rough lattice. If M is a Z-soft covering based roughly weak then $SCRS(M)$ does not have a complement in T_S.*

Proof. Let us consider that, $SCRS(O)$ is the complement of $SCRS(M)$ then by Definition 3.3 we get, $SCRS(M) \vee SCRS(O) = SCRS(\Omega)$. Let $\underline{SC}(M \vee O) = \Omega$, then all $M_i \subseteq M \vee O$, but from the Definition 2.9, we know that $M_i \nsubseteq M$ and $M_i \cap M \neq \emptyset$ and by using the same definition of $\vee$, O cannot contain the remaining elements of M_i. Therefore, $\underline{SC}(M \vee O) \neq \Omega$. Hence, $SCRS(O)$ cannot be the complement of $SCRS(M)$. $\square$

Example 3.1. *Let $\Omega = \{v_1, v_2, v_3, v_4\}$ be a universal set and $B = \{b_1, b_2, b_3\}$ be the set of parameters. Then the soft set over Ω is given by TABLE 1 where*
$N(b_1) = \{v_1, v_2, v_3, v_4\}$, $N(b_2) = \{v_2, v_4\}$ and $N(b_3) = \{v_1, v_2, v_3\}$.
Then, the equivalence classes formed by soft adhesion are $[v_1] = \{v_1, v_3\}$, $[v_2] = \{v_2\}$, $[v_4] = \{v_4\}$ and from [20]
$T_S = \{$ $SCRS(\emptyset)$, $SCRS(v_1)$, $SCRS(v_2)$, $SCRS(v_4)$, $SCRS(\{v_1, v_2\})$, $SCRS(\{v_1, v_3\})$, $SCRS(\{v_1, v_4\})$, $SCRS(\{v_2, v_4\})$, $SCRS(\{v_1, v_2, v_3\})$, $SCRS(\{v_1, v_2, v_4\})$, $SCRS(\{v_1, v_3, v_4\})$, $SCRS(\Omega)$ $\}$, where $(T_S, \vee, \wedge)$ is a Z-soft covering based rough lattice.

The Hasse diagram of Z-soft covering based rough lattice on T_S is shown in FIGURE 1.

FIGURE 1. Lattice structure for Z-soft covering based rough set



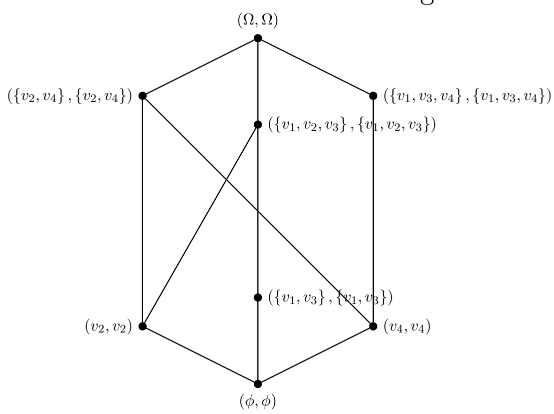
Example 3.2. *Let $M_1 = \{v_1, v_3\}$, $M_2 = \{v_2\}$ and $M_3 = \{v_4\}$ are the equivalence classes formed by soft adhesion and let $M = M_1 \cup M_2$, $\overline{M} = M_3$ then $SCRS(M) \vee SCRS(\overline{M}) = SCRS(M \vee \overline{M}) = SCRS(M_1 \cup M_2 \cup M_3) = SCRS(\Omega)$ and $SCRS(M \wedge \overline{M}) = SCRS(\Omega)$. Therefore, $SCRS(\overline{M})$ is the complement of $SCRS(M)$.*

Example 3.3. *Let $M = \{v_1, v_2\}$ and $\overline{M} = \{v_3, v_4\}$, then*
$SCRS(M) = (\{v_2\}, \{v_1, v_2, v_3\})$ and $SCRS(\overline{M}) = (\{v_4\}, \{v_1, v_3, v_4\})$ also $M \vee \overline{M} = M \cup \overline{M} = \{v_2, v_4\}$, therefore $SCRS(M \vee \overline{M}) = (\{v_2, v_4\}, \{v_2, v_4\}) \neq SCRS(\Omega)$. Hence, $SCRS(\overline{M})$ is not a complement of $SCRS(M)$.

Remark 3.1. *(1) The complement of $SCRS(\Omega)$ is $SCRS(\emptyset)$.*
(2) The complement of $SCRS(\emptyset)$ is $SCRS(\Omega)$.
(3) If M is the union of one or more equivalence classes formed by soft adhesion then the complement exists in T_S.
(4) If $M_i \not\subset M$ for any i then the complement of $SCRS(M)$ doesnot exist in T_S.
(5) T_S is a Z-soft covering based rough lattice and T_S is not a boolean algebra because all the elements doesnot have complements.

Example 3.4. *Let $\Omega = \{v_1, v_2, v_3, v_4\}$ be a universal set and $B = \{b_1, b_2, b_3\}$ be the set of parameters. Then the soft set over Ω is given by TABLE 1.*
Then, the equivalence classes formed by soft adhesion are $[v_1] = \{v_1, v_3\}$, $[v_2] = \{v_2\}$, $[v_4] = \{v_4\}$ and from [20]
$T_S = \{$ $SCRS(\emptyset)$, $SCRS(v_1)$, $SCRS(v_2)$, $SCRS(v_4)$, $SCRS(\{v_1, v_2\})$, $SCRS(\{v_1, v_3\})$, $SCRS(\{v_1, v_4\})$, $SCRS(\{v_2, v_4\})$, $SCRS(\{v_1, v_2, v_3\})$, $SCRS(\{v_1, v_2, v_4\})$, $SCRS(\{v_1, v_3, v_4\})$, $SCRS(\Omega)$ $\}$, where $(T_S, \vee, \wedge)$ is a Z-soft covering based rough lattice then the complements of elements of Z-soft covering based rough lattice form a sub lattice and it is denoted by T_P where
$T_P = \{$ $SCRS(\emptyset)$, $SCRS(M_1)$, $SCRS(M_2)$, $SCRS(M_3)$, $SCRS(M_1 \cup M_2)$, $SCRS(M_1 \cup M_3)$, $SCRS(M_2 \cup M_3)$, $SCRS(\Omega)$ $\}$. The Hasse diagram of Z-soft covering based rough sublattice on T_P is shown in FIGURE 2 which is a Z-soft covering based rough boolean algebra.

Figure 2. Lattice structure for Z-soft covering based rough sublattice



Remark 3.2. $(T_P, \vee, \wedge, SCRS(\emptyset), SCRS(\Omega))$ *is a Z-soft covering based rough boolean algebra and there are only three equivalence classes induced by soft adhesion. Therefore $|T_P| = 2^3 = 8$.*
In general, if there are n equivalence classes formed by soft adhesion then $|T_P| = 2^n$.

4. A new MAGDM approach using Z-SCRS

A unique decision-making process is developed in this section to select the best alternatives from the list of possible Ω objects.

4.1. **Description and process.** Let $\Omega = \{v_1, v_2, ..., v_j\}$ be j alternatives and let B be the parameter set. Consider that we have an expert group $E = \{E_1, E_2, ..., E_k\}$ made up of k experts to assess each alternative in Ω. All alternatives in Ω must be examined by a panel of experts, and only after doing so are they allowed to suggest the best alternative.

As a consequence, each specialist's main evaluation outcome is a subset of Ω. We believe that the assessments of these experts in E are similarly significant. The main assessment result of expert group E is described as the assessment soft set $H_1 = (L, E)$ over Ω, where $L : I \to P(\Omega)$ is given by $L(E_k) = M_k$. We derive the original assessment dataset from the soft set $H_1 = (L, E)$. However, soft rough approximation enables us to gather more pertinent data. We consider a soft rough approximation of the main assessment result M_k of the expert over the soft approximation space. According to the expert group E_k, the soft covering lower approximation $\underline{L}(E_k)$ can be considered the set of alternatives that are the most promising options. Similar to this, the soft covering upper approximation $\overline{L}(E_k)$ can be seen as a collection of items that, according to experts, are the best prospects. We eventually identify two additional soft sets $\underline{H_1} = (\underline{L}, E)$ and $\underline{H_1} = (\underline{L}, E)$ using soft rough approximations over Ω where,

$\underline{L} : E \to P(\Omega)$,
$\underline{L}(E_k) = \underline{SC}(L(E_k))$, k = 1,2,...,m.
$\overline{L} : E \to P(\Omega)$,
$\overline{L}(E_k) = \overline{SC}(L(E_k)$, k = 1,2,...,m.

Find the cardinality of $\underline{SC}(L(E_k))$, $\overline{SC}(L(E_k))$ and $L(E_k)$ where $L(E_k) = \overline{SC}(L(E_k)) - \underline{SC}(L(E_k))$. Calculate the maximal value of soft covering lower and upper approximation of assessment results of expert group E. Find the minimal value of assessment results of expert group E. If the cardinality of soft covering lower approximation of $L(E_k)$ is the maximal value then the cardinality of $E_k^{'} = 1$, otherwise it is Zero. If the cardinality of soft covering upper approximation of $L(E_k)$ is the maximal value then the cardinality of $E_k^{''} = 1$, otherwise it is Zero. If the assessment result of expert group $L(E_k)$ is the minimal value then the cardinality of $E_k^{''} = 1$, otherwise it is Zero. Find the value of $|E_k|$ using the formula, $|E_k| = \left|E_k^{'}\right| + \left|E_k^{''}\right| + \left|E_k^{'''}\right|$.

Find the $|E_k|$ which has the maximal value then the E_k is the optimal set.

The decision-making method is summarized as follows:

Step 1: Consider the original soft set $K = (N, B)$.

Step 2: Formulate the soft set $H_1 = (L, E)$ by using the first assessment results of the specialist group I.

Step 3: Calculate SCLA and SCUA and get the soft set $\underline{H}_1 = (\underline{L}, E)$ and $\overline{H}_1 = (\overline{L}, E)$.

Step 4: Calculate $|\underline{SC}(L(E_k))|$, $\left|\overline{SC}(L(E_k))\right|$ and
$|L(E_k)| = \left|\overline{SC}(L(E_k)) - \underline{SC}(L(E_k))\right|$.

Step 5: Compute the maximal values $|\underline{SC}(L(E_k))|$, $\left|\overline{SC}(L(E_k))\right|$ and the minimal value $|L(E_k)|$.

Step 6: If $|\underline{SC}(L(E_k))|$ is the maximal value then $\left|E_k^{'}\right| = 1$; otherwise, the value is Zero. If $\left|\overline{SC}(L(E_k))\right|$ is the maximal value, then $\left|E_k^{''}\right| = 1$; otherwise, the value is Zero. If $|L(E_k)|$ is the minimal value, then $\left|E_k^{'''}\right| = 1$; otherwise, the value is Zero.

Step 7: Find $|E_k| = \left|E_k^{'}\right| + \left|E_k^{''}\right| + \left|E_k^{'''}\right|$.

Step 8: Obtain the maximal value $|E_k|$ and the final result is E_k.

4.2. **Illustrative example.** In this work, we use soft adhesion to find SCLA and SCUA. A software company needs to select a team for an upcoming project related to networks. Experts conduct an evaluation to select the best candidates to form a team.

Step 1: Candidates attending the interview form a set $\Omega = \{v_1, v_2, ..., v_7\}$ and the parameter set includes their essential features such as Knowledge on 2G, 3G, 4G and 5G (b_1), Knowledge on fiber networks (b_2), Cisco Certified Network Associate (CCNN) knowledge (b_3), Communication skill (b_4) and Experience (b_5). We construct a soft set $K = (N, B)$ which is mainly on parameters over Ω given in TABLE 2. Let $S = (\Omega, C_K)$ be the SCA.

TABLE 2. Tabular representation of $K = (N, B)$

	b_1	b_2	b_3	b_4	b_5
v_1	1	1	1	0	1
v_2	0	1	1	1	1
v_3	1	1	0	0	1
v_4	1	1	1	0	1
v_5	1	1	0	0	1
v_6	1	1	1	0	1
v_7	0	1	1	1	1

Then the soft adhesion is given by
$SA(v_1) = \{v_1, v_4, v_6\}$, $SA(v_2) = \{v_2, v_7\}$, $SA(v_3) = \{v_3, v_5\}$, $SA(v_4) = \{v_1, v_4, v_6\}$, $SA(v_5) = \{v_3, v_5\}$, $SA(v_6) = \{v_1, v_4, v_6\}$ and $SA(v_7) = \{v_2, v_7\}$.

Step 2: With the aid of parameters, the experts $E = \{E_1, E_2, E_3, E_4\}$ will evaluate the candidates. Using the initial evaluation values of expert group, we produce a soft set $H_1 = (L, E)$ over Ω. Each expert evaluates every individuals in Ω and then identifies the best alternatives as the conclusion of their evaluation. As a result, the major assessment values of each expert are subsets of Ω. We equally value the opinions of these experts.
$L(E_1) = \{v_1, v_4, v_6, v_7\}$, $L(E_2) = \{v_2, v_3, v_7\}$, $L(E_3) = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ and $L(E_4) = \{v_1, v_3, v_4, v_6\}$.

Step 3: Now we use SCLA and SCUA in this decision making problem. Let $S = (\Omega, C_K)$ be a SCA. By using this, we get two soft sets $\underline{H}_1 = (\underline{L}, E)$ and $\overline{H}_1 = (\overline{L}, E)$ over Ω where,

$\underline{L} : E \to P(\Omega)$,
$\underline{L}(E_k) = \underline{SC}(H_1(E_k))$, $K = 1, 2, 3, 4$.
$\overline{L} : E \to P(\Omega)$,
$\overline{L}(E_k) = \overline{SC}(H_1(E_k)$, $K = 1, 2, 3, 4$.

The soft sets $\overline{H}_1$ and $\underline{H}_1$ are the assessment values of the experts group E. We get the SCLA and SCUA of first assessment values of experts to obtain the soft sets $\underline{H}_1$ and $\overline{H}_1$.
Consider,

$$\underline{SC}(L(E_1)) = \{v_1, v_4, v_6\},$$
$$\underline{SC}(L(E_2) = \{v_2, v_7\},$$
$$\underline{SC}(L(E_3) = \{v_1, v_3, v_4, v_5, v_6\},$$
$$\underline{SC}(L(E_4)) = \{v_1, v_4, v_6\}.$$
$$\overline{SC}(L(E_1)) = \{v_1, v_2, v_4, v_6, v_7\},$$
$$\overline{SC}(L(E_2)) = \{v_2, v_3, v_5, v_7\},$$
$$\overline{SC}(L(E_3)) = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\},$$
$$\overline{SC}(L(E_4)) = \{v_1, v_3, v_4, v_5, v_6\}.$$

Step 4: The cardinality of $L(E_k)$, SCLA and SCUA are given by
$|L(E_1)| = 4$, $|L(E_2)| = 3$, $|L(E_3)| = 6$, $|L(E_4)| = 4$.
$|\underline{SC}(L(E_1))| = 3$, $|\underline{SC}(L(E_2))| = 2$, $|\underline{SC}(L(E_3))| = 5$ and $|\underline{SC}(L(E_4))| = 3$.

Table 3. Table for ranking outcomes

Different models	Obtain a decision
Zhan's model 1 [25]	$E_3 = E_2 \geq E_1 = E_4$
Zhan's model 2 [25]	$E_3 \geq E_2 = E_1 = E_4$
Zhan's model 3 [25]	$E_3 = E_2 \geq E_1 = E_4$
Our model	$E_3 \geq E_2 \geq E_1 = E_4$

$\left|\overline{SC}(L(E_1))\right| = 5$, $\left|\overline{SC}(L(E_2))\right| = 4$, $\left|\overline{SC}(L(E_3))\right| = 7$ and $\left|\overline{SC}(L(E_4))\right| = 5$.
Step 5: The maximal values of $|\underline{SC}(L(E_k))| = 5$ and $\left|\overline{SC}(L(E_k))\right| = 7$.
The minimal value of $|L(E_k)| = 3$, where $k = 1, 2, 3, 4$.
Step 6: $\left|E_1^{'}\right| = 0$, $\left|E_1^{''}\right| = 0$, $\left|E_1^{'''}\right| = 0$.
$\left|E_2^{'}\right| = 0$, $\left|E_2^{''}\right| = 0$, $\left|E_2^{'''}\right| = 1$.
$\left|E_3^{'}\right| = 1$, $\left|E_3^{''}\right| = 1$, $\left|E_3^{'''}\right| = 0$.
$\left|E_4^{'}\right| = 0$, $\left|E_4^{''}\right| = 0$, $\left|E_4^{'''}\right| = 0$.
Step 7: $|E_1| = 0$, $|E_2| = 1$, $|E_3| = 2$ and $|E_4| = 0$.
Step 8: Since E_3 takes the maximal value, we choose the elements in set E_3, that is, v_1, v_2, v_3, v_4, v_5, v_6 as optimal solutions.

4.3. **Comparison with other existing techniques.** The literature contains a wide range of methods that can be used to solve various decision making problems. Each of these decision making strategies has its own advantages and disadvantages. The effectiveness of each technique is determined by the chosen problem. Here, we compare the proposed decision making strategies with some existing decision making techniques. In this subsection, we compare our model with Zhan's three different types of models proposed in [25] to illustrate the significance of our model in the decision-making process. As shown in TABLE 3, the newly proposed model gives more precise and accurate results compared to other existing models, which shows the effectiveness and importance of our model.

5. Conclusion

In this paper, the complement of the elements of Z-soft covering based rough lattice is found. A lattice structure is developed for Z-soft covering based rough sublattice. We showed that a Z-soft covering based rough sublattice is a Z-soft covering based rough boolean algebra. Finally, we proposed a novel MAGDM model to select a team for a software company project. As shown in TABLE 3, the proposed model provides accurate results when compared with other existing models, which helps in finding the optimal results in the decision making process. In future work, we focus on determining the prerequisites that must be met for the Z-soft covering based rough lattice to qualify as a boolean algebra.

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