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"YARMAN-36 MAKAM TONE-SYSTEM" FOR TURKISH ART MUSIC

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Abstract. This study offers a mathematically rigorous, yet straightforward, fixed-pitch tuning strategy to the problem of adequate sounding and notating of essential Turkish *makam* genera, in contradistinction to the praxis-mismatched music theory cast in effect known as *Arel-Ezgi-Uzdilek* (AEU). It comprises 36 tones locatable just by ear, via counting exact 0, 1 and 2 beats per second when listening to given octave, fifth and third intervals, starting from an algebraically attained reference frequency for A at 438.41 Hertz, very near the international standard A=440 Hz. The so-named *Yarman-36 makam tone-system* proposed in this paper accounts for hitherto omitted pitches in *Uşşak*, *Saba*, *Hüzzam, etc...* at popular transpositions, each corresponding to a habitually used *Ahenk* (concert pitch level specified by a chosen *Ney* reed), by virtue of being based on a twelve-by-twelve triplex structure of exclusively tailored Modified Meantone Baroque Temperaments. It thus also features pleasant shades of key-colors supporting polyphonic endeavours in line with Western common-practice music.

Keywords: Arel-Ezgi-Uzdilek, makam (maqam), tone-system, tuning, temperament.

AMS Subject Classification: 00A65

1. Problems with the established 24-tone makam theories

Makam in Turkey, and homonymously elsewhere across the Middle East from Morocco to Uyghur autonomous region of China, designates a musical mode, or a family of kindred modes, consisting of a set of “more or less” fluid pitches (called *perdeler*), with distinctly embedded intonation (*baskı*) and inflexion (*kaydırma*, *oynaklık*) attributes, the entirety of which remains dependent on the classical rules of thematic flow (called *seyir*). [16,20,24,38,48,49,52,53,54,70]

Due to said ambiguous traits of *Makam* music, it is exceedingly difficult to pinpoint – especially within the context of live performance – the precise microtones typically used by a genus or scale. In addition, the reference frequency for any particular ensemble can be selected from among no less than a dozen options, each bearing such habitual names

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as *Bolahenk*, *Süpürde*, *Mansur*, *etc...*, any of which indicates a specific *Ney Ahenk* [i] that corresponds to a chosen size of reed (**Fig. 1**).

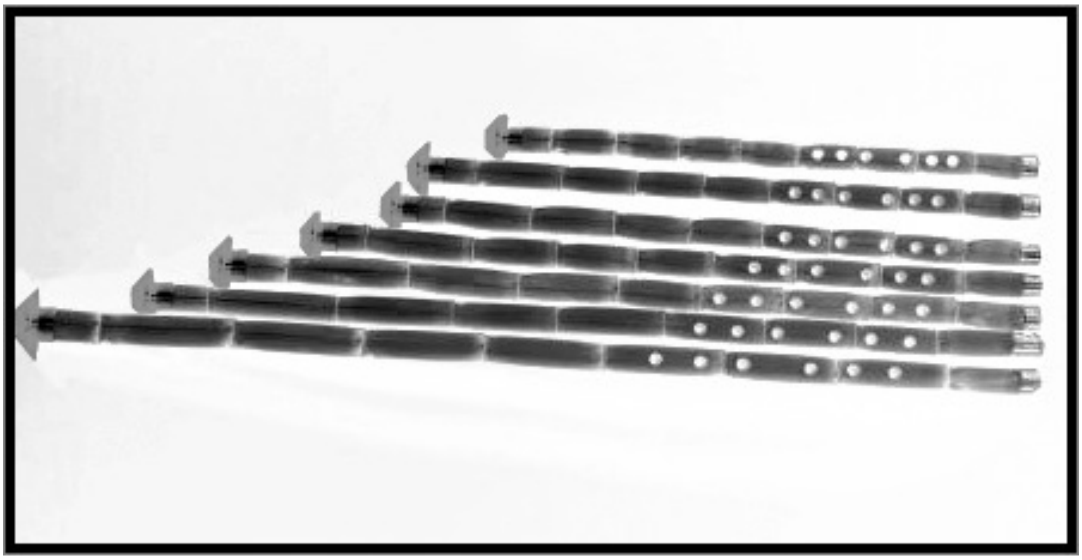


FIGURE 1. Different sizes of *Ney reeds* corresponding to different *Ahenks* (Fingering is preserved despite the change of dimensions, which results in a key transposed instrument serving as the ensemble's reference pitch).

Understandably, and owing also to the adverse influence of a bicentennial flurry of Westernization in Turkey and beyond, there has been considerable efforts to accurately identify or determine the total amount of *perdeler* that make up the master tuning grid of Makam music. All the way from 17 through 24, 29, 36, 41, 48, 51, 53, 60, 65, 72 up to 79 unequal or equal divisions of the octave have thus far been variously endorsed and/or applied in the literature. [1,4,19,21,22,23,28,32,33,36-38,43,50,53,55,56,59-64,66-69]

Howcome so many arithmetically interesting, yet basically irreconcilable, tuning schemes contend side by side to explain *makamlar* today should not astound the reader. The latecomer majority of the above-mentioned tone-systems aspire to reinstate what the historically adopted Turkish/Arabic/Persian 24-tone music theory templates fundamentally neglect or evade: *i.e.*, "unruly microtones" that significantly overshoot or undershoot 12-tone Equal Temperament pitches at customary musical registers, transpositions, and modulations; which are pertinaciously executed by performers on their instruments, despite having remained non-systematized heretofore to the agreement of the majority.

As it so happens, investigations have firmly ascertained of late that, the official tone-system of Turkish Classical/Art music known as *Arel-Ezgi-Uzdilek* (AEU) does not at all reliably reflect practice. It is demonstrably on account of the failure of this 24-tone

[i] *Bolahenk* with *perde rast* (second partial blown from all fingerholes of the *Ney* closed) at D; *Davud* with *rast* at E; *Şah* with *rast* at F; *Mansur* with *rast* at G; *Kız* with *rast* at A; *Müstahsen* with *rast* at B; and *Süpürde* with *rast* at C. Observe, that *perde rast* can be made to correspond to any tone of Western common-practice music, including all the half-tones in-between the naturals.

Pythagorean tuning [ii] (**cf. Formula 1.1**) [25,46,58] at embodying or expressing via staff notation the multifarious neutral or middle second flavors [iii] [5,60] measured in audio recordings [2,8,10-15,26,30,31,34,35,44,45,49], and thus, inextricably peculiar to the genre.

In contrast, manifestly amiss in the quarter-tonal setup of Arab and Persian *maqam /dastgah* treatises [19,23,50,51,53] is a mathematically complete (*i.e.* transposition-wise fully navigable) model incorporating minute "commatic alterations" inherently found in AEU, which are otherwise at the disposal of executants of native free-pitched instruments.

The situation can be just as perplexing in the Arabic and Persian world of traditional music-making as it is in Turkey, due to each faction prioritizing an idiosyncratic body of tuning criteria at the outset. *The issue is markedly as complex as the methodical and clever blending of the commatic soundscape with the quarter-tonal.*

Let's provide a direct analogy to shed more light on the complexity of the matter: Whereas the utilization of 12-tone Equal Temperament or a sibling cyclic tuning [iv] [7,9,29, 47,57] on keyboard and fretted instruments in Western Classical/Contemporary music is not in the least unacceptable – *viz.* one can interchangably perform (and even re-arrange) a piece written for Trombone or Violin on a Piano or Guitar without grossly misrepresenting or distorting the intended music (*i.e.*, within the recognized boundaries of instrumentalism), fretting the *Tanbur* or affixing *mandallar* [v] on a *Kanun* strictly according to AEU will be disastrous for Turkish Art music performance.

[ii] AEU is generated from an initial relative frequency (1/1) – dubbed *perde kaba çargah* and notated as a ledger lined C (Do) below the first line of a G-cleffed staff () – by going up eleven pure fifths (3/2) therefrom, then again up twelve pure fourths (4/3) once more therefrom, and bringing all resultant ratios within the range of a single octave (2/1) via the required octave transpositions. "Going up" here signifies multiplication of either the initial 3/2 or 4/3 ratio by itself to arrive at the next ratio, which is again multiplied by same to yield a further ratio, *etc...* Transposing by the octave means that a fraction's even number numerator should be divided by 2 or even number denominator multiplied by 2 (should the fractional value be greater than 2) until the ratio comes to reside between 1 and 2. The whole operation can be mathematically expressed as follows in **Formula 1.1**:

Formula 1.1 (*deriving the frequency ratios of Arel-Ezgi-Uzdilek*)

$$\boxed{\aleph = [1 < \mathfrak{R}^n \cdot (1/2)^m < 2]} \tag{1.1}$$

$$\text{where} \quad n = \begin{cases} \{0,1,2,3,...11\} & \text{if } \mathfrak{R} = 3/2 \\ \{1,2,3,...12\} & \text{if } \mathfrak{R} = 4/3 \end{cases} \qquad \text{provided that,} \qquad m = \begin{cases} 1-6 & \text{if } \aleph > 2 \\ 0 & \text{if } \aleph < 2 \end{cases}$$

The outcome of 24 distinct pitches (not including the octave, and ordinarily sounded at *Bolahenk* with *perde yegah* or D=220 Hz) is thusly so-called *Pythagorean* due to the prime factorization of the numerators and denominators in these ratios producing only 2's and 3's – which have been held as core mystical & celestial numbers by the adherents of said ancient school of Pythagoras. (Accompanying endnotes)

[iii] Characterizable by a spectrum of superparticular *Just Intonation* ratios that proceed as 11/10, 12/11, 13/12, and 14/13 within a given whole-tone range, which are altogether absent in AEU at indispensible locations. (Accompanying endnotes)

[iv] *i.e.*, any of the countless finely calculated circulating Well-Temperaments or Modified Meantone Temperaments found in the vast literature on the historical tuning of common-practice European music, by which a chain of selectively sized perfect fifths wrap around to a full circle at the 12th step – resulting in the return to the tone of origin as well as the ability to transpose unhindered, while maximizing aurally favorable central tonalities and yielding various key-colors. (Accompanying endnotes)

[v] Small metallic levers that are altered by the performer on-the-fly to modify the vibrating length of string courses. Every *mandal* is affixed to the *Kanuns* in the construction phase, and the player does not have the option to change their default positions.

So too is the case analogous with dividing the octave into 24 equidistant parts when playing on a generalized Arabic *Qanun* or a Persian/Azeri *Tar* according to **Formula 1.2**:

Formula 1.2 (*relative frequency "ℵ" and cent value "¢" of the quarter-tone interval*)

$$\aleph = 2^{(1/24)} \tag{1.2a}$$

$$= \sqrt[24]{2} \tag{1.2b}$$

$$= exp\left(\frac{1}{24} \cdot ln\ 2\right) \tag{1.2c}$$

$$\boxed{\begin{array}{l} \text{yielding} \quad 1200 \cdot \dfrac{log_{10}\ \aleph}{log_{10}\ 2} = 50\ \text{¢} \\ \text{or otherwise,} \quad 1200 \cdot log_2\ \aleph = 50\ \text{¢} \\ \text{or still,} \quad 1200 \cdot \left(\frac{1}{24}\right) = 50\ \text{¢} \end{array}} \tag{1.2d}$$

where the result is the quarter-tone step of 50 cents, twenty-four of which synthetically added together total 1200 cents [vi] [27,40], hence, the octave.

Thus, neither the Turkish 24-tone Pythagorean, nor the Arabic/Persian quarter-tonal templates (where twelve base pitches are either commatically or quarter-tonally etched from the remaining twelve) can wholesomely house the intended music for *dastgaha/makamlar/ maqamat/mughamlar* [6] or suchlike modulations to particularly *Saba*, *Uşşak*, *Hüseyni*, *Hüzzam*, *Karcığar*, *Suzinak*, *Bayyati*, *Shur*, *Dashti*, *etc...*, without causing a dilettante of the genre to wince upon hearing them; since the aural margin of error can indeed be very narrow for certain critical microtones during modal progression [13].

In other words, it is impossible to perform authentic music in such modes based on the standardized 24 pitches to the octave systems of the diametrically opposed cultures of the geography, without detuning the strings, adding or shifting frets as required, or employing an *ad hoc* (*e.g.*, unmethodical) *mandal* configuration.

Furthermore, the absence of neutral or middle second accidentals is as glaring in AEU as the dearth of commatic nuances are in the notational symbology of Arabic and Persian music theory. A natural consequence of all this has been that, several alternative tuning models have been proposed in the past decades – particularly in Turkey, with a conscious aim to remedy the aforementioned shortcomings of the 24 pitches to the octave methodologies, including some by the first author himself [59-63].

[vi] A unit of intervallic measure in the logarithmic scale, first proposed by Alexander J. Ellis in 1885 in his revised translation of Helmholtz's *Die Lehre von den Tonempfindungen*, for determining the relative distance between two distinct pitches. Cent is defined as the 1200th root of 2, or $2^{(1/1200)}$, yielding the ratio 1 : 1.000577789506555. It follows that there are 1200 cents to an octave (~ $1.000578^{1200} = 2$). Cents are represented by the "¢" sign. The equation for calculating the cent value of a given frequency ratio is $\{1200 \cdot log_2\ \mathfrak{R} = \text{¢}\}$, or $\{(1200\ /\ \log_{10}\ 2) \cdot log_{10}\ \mathfrak{R} = \text{¢}\}$. The reverse operation is carried out by the formula $\{2^{(\text{¢}\ /\ 1200)}\}$. A hundred cents makes an "equal tempered semitone" (one degree of 12-tone Equal Temperament), hence the origin of the term. (Accompanying endnotes)

In order to overcome aforesaid intonation problems induced by mainstream Turkish and Arabic-Persian tone-systems, which fail to comprise the minimal amount of crucial intervals to satisfactorily and wholesomely represent Makam music across popular instrumental transpositions, the authors shall present herewith a novel 36-tone hybrid solution.

2. Quest For The Ideal Medium-Resolution Tuning

There is a general tendency in Turkey and the Levant, to divide the octave practically into 53 logarithmically equal parts (*i.e.*, 1200/53 = **22.64151 ¢**) via the mathematical operation shown in **Formula 2.1** below:

Formula 2.1 (*53 pitches apart by relative frequency values of the "Holderian comma"*)

$$f \cdot 2^{\,(\mathrm{n}\{1,2,3,\ldots\ 53\}\ /53)} \text{ or} \tag{2.1a}$$

$$f \cdot (\sqrt[53]{2})^{\,\mathrm{n}\{1,2,3,\ldots\ 53\}} \text{ or else,} \tag{2.1b}$$

$$f \cdot exp\ [\ (n\{1,2,3,...53\}/53) \cdot (ln\ 2)\ ], \tag{2.1c}$$

which is a voluminous resolution that embodies AEU with maximum 1 cent absolute error at any degree [58]. This "Holderian comma system" helps musicians educated according to AEU theory to conceptualize and communicate the positioning of at least two non-systematized middle seconds (1200/53·6 = **135.85 ¢** & 1200/53·7 = **158.49 ¢**), by counting comma step deviations from certain pivotal notes in a melody [46].

It goes without saying, that such microtones are not ordinarily expressed in Turkish Art music notation [vii]. Hence, musicians say, for example, that a certain pitch (*perde*) is to be sounded one or two or three commas higher or lower than written [46].

It is also significant to emphasize at this point, that *makam*-oriented Turkish music computer programs [viii] also utilize the Holderian comma resolution for true-to-the-original digital playback of traditional music scores [64].

However, there happens to be a fundamental setback with 53-tone Equal Temperament: There is not a Turkish instrument which is known to implement it faithfully or wholly. No *Kanun*, *Tanbur*, *Cümbüş*, or *Bağlama* to date is prepared to embrace Holderian commas in exactness or thoroughly.

Instead, most *Tanburs* utilize an arbitrary array of about 34 frets (*destans*) from *perde yegah* (RE) to *neva* (Re), any of which can be moved around by performers on demand. On the other hand, *Kanun*-makers haphazardly affix the half-tone *mandal* at the equal semitone (100 cents) by referring to electronic tuners, and visually partition the space between this *mandal* and the nut into 6 equal parts – arriving, to all intents and purposes, at 72-tone Equal Temperament [ix] [59,60].

[vii] However, Folk music scores do utilize comma numbers above ordinary sharps and flats to indicate the desired degree of 53-tone Equal Temperament.

[viii] "*Notist*" by Uğur Keçecioğlu, "*Nota*" by Ömer Tulgan, and "*Mus2Okur - Turkish Music Multimedia Encyclopedia*" by M. Kemal Karaosmanoğlu & Data-Soft team of developers.

[ix] Actually, only a bulky subset of 72-tone Equal Tempermant (tET) can be found on quotidian Turkish *Kanuns*, since not all degrees of 72-tET are available due to a general lack of need. Because of this setup, certain transpositions are not possible; *viz.*, the *Kanun* can only accompany an ensemble adjusted to one

Although 53-tET and 72-tET are very agreeable replacements for extended *Just Intonation*, either of which maintains approximated neutral or middle second savors sought after by executants, they are unwieldy temperaments – an observation compounded by the fact that only *ad hoc* subsets of these are applied to Turkish music instruments.

One can therefore ask, whether +50 tones per octave is really necessary as the definitive groundwork of a music theory, or when performing on an instrument such as the *Kanun* in an ensemble... In other words, is it fair to expect the *Tanbur* or *Kanun* to precisely imitate by discrete static quanta the continuously intoned voice of Singers, *Ud*, *Kemençe*, or *Ney* whose pitches are *de facto* not strictly bound to any particular theoretical grid?

We may ask this question all the more, since the first author had implemented a *79-tone tuning* on a unique *Kanun* in order to deliver a conclusive answer to the quest for the least voluminous fixed-pitch resolution required to faithfully express Makam music in every detail over all degrees of transposition [60].

Therefore, not only 53-tET and 72-tET do not anyway possess enough detail to fully represent the free-pitch capabilities of versatile Middle Eastern instruments, the authors further believe that, there should be room for subtle inflexions in a fixed-pitch tuning strategy at any case – insofar as the stabilized set of *perdeler* can serve to represent a given *makam* without altogether sounding out of place.

Accordingly, not only should the sought-after master tuning support bare instances of steady-state intonation without doing injustice to the *makam*, but the chosen framework ought not be cumbersome for the non-challenging notation and execution of temporally standstill (but even so melodically functional) microtones.

41 tones to the octave thence appears to be the upper limit for a medium-resolution fixed-pitch tuning strategy for Turkish Art music – since it is the lowest possible equal division to feature a cycle of almost pure fifths (1200/41 · 24 = **702.44 ¢**) [x], while embodying at least one minor second (1200/41 · 6 = **175.61 ¢**) and one neutral second interval (1200/41 · 5 = **146.3415 ¢**) critical to the essence of the Makam music genre [59].

However, this 41-tone resolution is still arguably a highly crowded selection. On the other hand, the lower limit for a medium-sized template can be practically determined at 24-tones to the octave. Whereas subtle nuances must inevitably be sacrificed due to the lessening of pitches, such is, unavoidably, the price to pay for a simple theoretical cast with a gentle learning-curve.

With this in mind, the first author had proposed an alternative 24-tone irregular tuning to AEU named *Yarman-24* [xi] [41], which embraces characteristic neutral seconds at crucial locations for *Saba*, *Uşşak*, *Hüseyni*, *Hüzzam*, *Karcığar*, *etc...*, while still relying on the same palette of accidentals as AEU [63]. Given enough room for pitch inflexions (1 Holderian comma berth per pitch for instance), it suffices to reasonably explain all *makams* over

of the more mainstream concert pitches (*Ahenk* or *Akort*). Sometimes, the equal semitones are observed to be asymmetrically divided into 7 parts in the lower registers owing to available space (which yields 84-tET), and into 5 parts in the upper registers because of limited space (which yields 60-tET), at the expense of pure octave complements for intra-semitonal microtones.

[x] Just 0.484 ¢ greater than the actual pure fifth (3:2) equal to $\{(1200 / log_{10} 2) \cdot log_{10} 1.5\} = 701.955$ ¢

[xi] As currently listed in the SCALA Program's (accompanying endnote) Scale Archive authored by Manuel op de Coul (YA24 notation in SCALA). It is not foreign, under the discipline of constructing tunings & temperaments, to have scales named after their creator, given that there are thousands of them to reckon in the literature, and that this procedure facilitates their cataloguing.

at least a single chosen *Ahenk* (or *Akort*). When pitted against pitch measurements from masters of Turkish Art music in our previous article [13], *Yarman-24* scored almost as high as *Mus2Okur* spearheaded by the second author, which employs the voluminous Holderian comma resolution.

Several other variants were advanced after *Yarman-24a* (christened "*b*", "*c*", "*d*"), all of which can likewise be notated using exactly the same arsenal of accidentals as AEU. In particular, *Yarman-24c* has been applied by the first author to the neck of his *bowed Tanbur* [xii], and was furthermore implemented on the fretboard of a guitar belonging to Tolgahan Çoğulu, as well as on *TouchKeys* "Capacitive Multi-Touch Sensing on a Physical Keyboard" technology by Andrew McPherson [18,39,65]. Especially, the *bowed Tanbur* and the *TouchKeys* keyboard can let a musician become quite liberal with pitch inflexions using the *Yarman-24* layout.

Nevertheless, enforcing the restricted usage of only AEU accidentals leads to an irregular mapping of notes, which results in the sanctioning of notational inconsistencies for available transpositions (*viz.*, a given number of steps do not always correspond to the same type of interval). Besides, not being able to transpose the body of *makamlar* over to at least the main *Ahenks* without a frequency shift of the whole keyboard, or altering the tuning of strings, can become a performance hindrance for certain settings.

On those accounts, a much less voluminous 36-tone alternative compared to 72-tET, 53-tET, and 41-tET shall be presented herein shortly, which features a mathematically rigorous, yet straightforward, fixed-pitch tuning strategy to the problem of adequately sounding and notating of essential Turkish *makam* genera throughout mainstream transpositions.

3. A 36-tone Replacement in place of Arel-Ezgi-Uzdilek

The so-referred *Yarman-36 makam tone-system* proposed in this paper comprises 36 tones locatable just by ear, via counting exact 0, 1, and 2 (and optionally 3) beats per second when listening to given octave, fifth and third intervals as outlined in **Fig. 2**, starting from an algebraically attained reference frequency for A at 438.41 Hertz, very near the international standard A=440 Hz.

Said tuning cast is based on a twelve-by-twelve triplex structure of exclusively tailored Modified Meantone Baroque Temperaments (each completing a fifths circle at the 12th step), with aurally pleasant shades of key-colors supporting polyphony endeavours in line with Western common-practice harmony and chordal modulation, while also accounting for hitherto omitted pitches in *Uşşak*, *Saba*, *Hüzzam*, *etc...* – in contradistinction to the praxis-mismatched *Arel-Ezgi-Uzdilek* (AEU) music theory in force – at popular transpositions that correspond to habitual *Ahenkler* (*i.e.*, 12 or more possible concert pitches, with *Bolahenk* at Re=440 Hz as the accepted default) as shown in **Table 1** and **Table 2**.

The reason for choosing 438.41 Hertz as the reference frequency for note A is to assure that the fifths cycle in Layer I is completed using only fifths with beat rates of 0, 1, and 2 per second throughout. While not at all a prerequisite of the *Yarman-36a* cast, said reference frequency can be calculated by **Formula 3.1** presented further below.

[xii] Ordinarily, open strings of *Tanbur* correspond to *Bolahenk Akort*, with *perde yegah* (melody-making open string) at A (Re=220 Hz in Turkish parlance) according to international pitch; but the instrument in question has been successfully tuned a perfect fourth sharper to *Mansur Akort* with *perde yegah* at D.

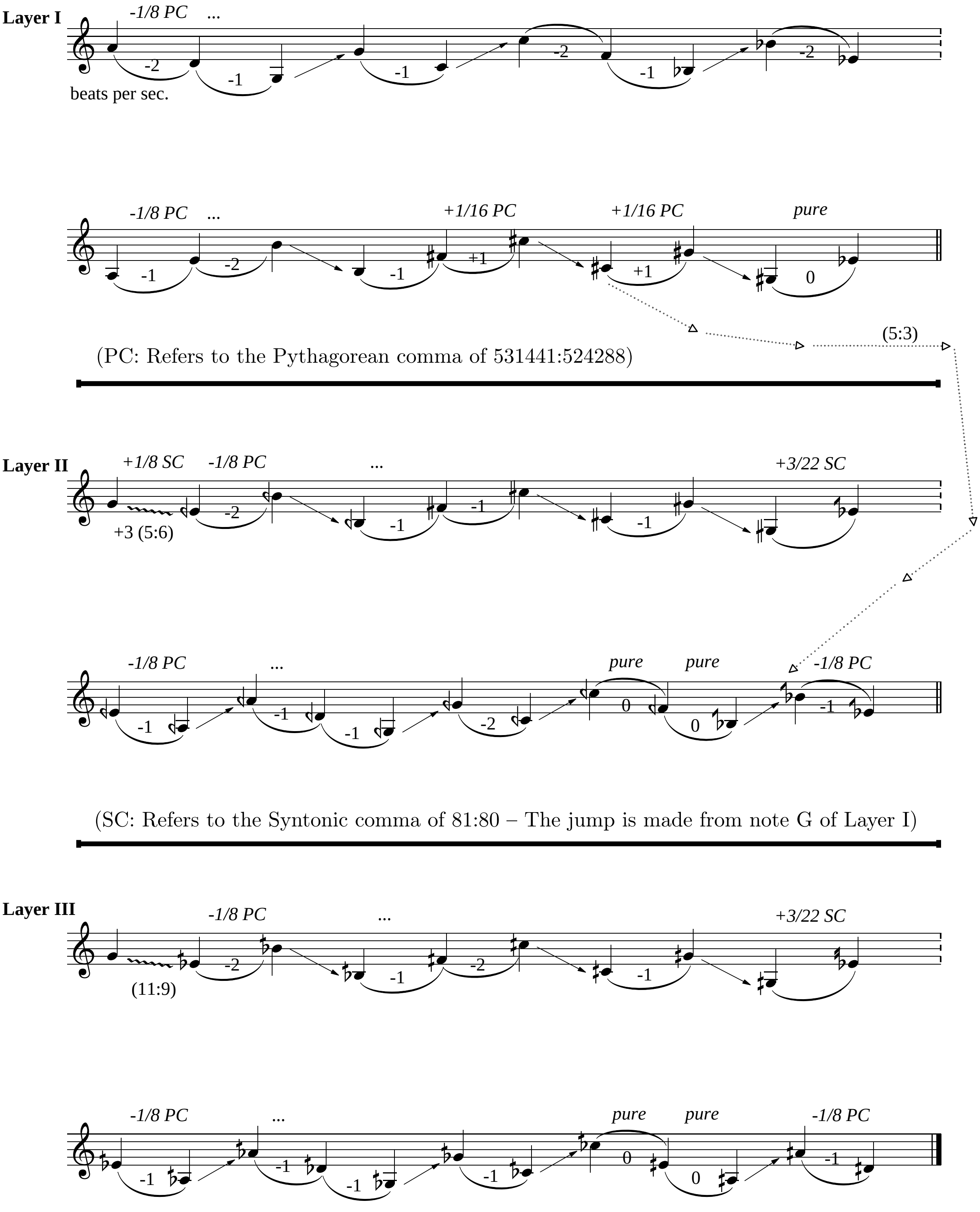
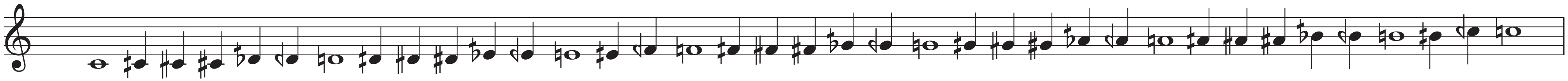


Figure 2: Tuning recipe for *Yarman-36a tone-system* via 0, 1, 2 (and optionally 3) beat counts per second from octave, fifth and third intervals; followed by 1/8 comma *Temperament Ordinaire* approximation guidelines.

Yarman-36 makam tone-system



deg.	cent	int.	SÜPÜRDE	BOLAHENK	DAVUD	MANSUR	KIZ
0	0	0	*Rast*				
1	49	49	dik rast				
2	80	31	nim zengule				
3	98	18	nerm zengule				
4	153	56	zengule				
5	182	29	dik zengule				
6	199	16	*Dügah*	*Rast*			
7	251	52	dik dügah	dik rast			
8	282	31	kürdi	nim zengule			
9	304	22	dik kürdi	nerm zengule			
10	352	49	nerm segah	zengule			
11	382	29	segah	dik zengule			
12	396	14	*Buselik*	*Dügah*	*Rast*		
13	453	57	dik buselik	dik dügah	dik rast		
14	484	31	nerm çargah	kürdi	nim zengule		
15	501	17	*Çargah*	dik kürdi	nerm zengule		
16	550	49	dik çargah	nerm segah	zengule		
17	580	29	nim hicaz	segah	dik zengule		
18	594	14	nerm hicaz	*Buselik*	*Dügah*		
19	654	60	hicaz	dik buselik	dik dügah		
20	683	29	dik hicaz	nerm çargah	kürdi		
21	700	16	*Neva*	*Çargah*	dik kürdi	*Rast*	
22	749	49	dik neva	dik çargah	nerm segah	dik rast	
23	780	31	nim hisar	nim hicaz	segah	nim zengule	
24	802	22	nerm hisar	nerm hicaz	*Buselik*	nerm zengule	
25	853	51	hisar	hicaz	dik buselik	zengule	
26	882	29	dik hisar	dik hicaz	nerm çargah	dik zengule	
27	897	14	*Hüseyni*	*Neva*	*Çargah*	*Dügah*	*Rast*
28	951	54	dik hüseyni	dik neva	dik çargah	dik dügah	dik rast
29	982	31	acem	nim hisar	nim hicaz	kürdi	nim zengule
30	1002	20	dik acem	nerm hisar	nerm hicaz	dik kürdi	nerm zengule
31	1051	49	nerm eviç	hisar	hicaz	nerm segah	zengule
32	1080	29	eviç	dik hisar	dik hicaz	segah	dik zengule
33	1095	14	*Mahur*	*Hüseyni*	*Neva*	*Buselik*	*Dügah*
34	1155	60	dik mahur	dik hüseyni	dik neva	dik buselik	dik dügah
35	1186	31	nerm gerdaniye	acem	nim hisar	nerm çargah	kürdi
36	1200	14	*Gerdaniye*	dik acem	nerm hisar	*Çargah*	dik kürdi

TABLE 1: Table of transpositions in main *Ahenks* via *Yarman-36a* tuning with corresponding microtonal notation.

Deg. & Note	Yarman-36a cent values	Frequencies	1/8 comma Temperament approximation	Difference in cents	SÜPÜRDE perde names of 1st octave	SÜPÜRDE perde names of 2nd octave	SÜPÜRDE perde names of 3rd octave
0: C	0	261.1692	0	0	KABA (PES) RAST	RAST	GERDANİYE
1: C≠	48.963	268.661	44.475	4.487	kaba dik rast	dik rast	dik gerdaniye
2: C𝄲	80.006	273.522	77.76	2.246	kaba nim zengule (şuri)	nim zengule (şuri)	nim şehnaz
3: C#/D♭	97.641	276.3223	101.955	-4.314	kaba nerm zengule	nerm zengule	nerm şehnaz
4: D♭̸	153.152	285.326	149.363	3.789	kaba zengule	zengule	şehnaz
5: D𝄳	182.378	290.1836	182.648	-0.27	kaba dik zengule	dik zengule	dik şehnaz
6: D	198.747	292.9404	198.045	0.702	KABA DÜGAH	DÜGAH	MUHAYYER
7: D≠	250.591	301.8455	248.386	2.205	kaba dik dügah	dik dügah	dik muhayyer
8: D𝄲	281.923	307.3581	281.671	0.252	kaba kürdi	kürdi	sünbüle
9: D#/E♭	303.638	311.2376	302.933	0.705	kaba dik kürdi (nihavend)	dik kürdi (nihavend)	dik sünbüle
10: E♭̸	352.336	320.1167	347.408	4.928	kaba nerm segah (uşşak)	nerm segah (uşşak)	tiz nerm segah (uşşak)
11: E𝄳	381.641	325.5816	380.693	0.948	kaba segah	segah	tiz segah
12: E	396.078	328.308	396.09	-0.012	KABA BUSELİK	BUSELİK	TİZ BUSELİK
13: E≠	452.588	339.2012	449.363	3.225	kaba dik buselik	dik buselik	tiz dik buselik
14: F𝄳	483.954	345.4028	482.648	1.306	kaba nerm çargah	nerm çargah	tiz nerm çargah
15: F	501.356	348.8922	500.978	0.378	KABA ÇARGAH	ÇARGAH	TİZ ÇARGAH
16: F≠	550.227	358.8813	545.453	4.774	kaba dik çargah	dik çargah	tiz dik çargah
17: F𝄲	579.633	365.0293	578.738	0.895	kaba nim hicaz	nim hicaz	tiz nim hicaz
18: F#/G♭	594.119	368.0965	598.534	-4.415	kaba nerm hicaz (uzzal)	nerm hicaz (uzzal)	tiz nerm hicaz (uzzal)
19: G♭̸	654.228	381.1013	650.341	3.887	kaba hicaz (saba)	hicaz (saba)	tiz hicaz (saba)
20: G𝄳	683.403	387.5782	683.626	-0.222	kaba dik hicaz	dik hicaz	tiz dik hicaz
21: G	699.744	391.2538	699.023	0.721	YEGAH	NEVA	TİZ NEVA
22: G≠	748.768	402.4915	743.498	5.27	dik yegah	dik neva	tiz dik neva
23: G𝄲	779.85	409.783	776.783	3.067	kaba nim hisar	nim hisar	tiz nim hisar
24: G#/A♭	801.683	414.9835	803.91	-2.227	kaba nerm hisar (beyati)	nerm hisar (beyati)	tiz nerm hisar (beyati)
25: A♭̸	853.083	427.489	848.386	4.697	kaba hisar (hüzzam)	hisar (hüzzam)	tiz hisar (hüzzam)
26: A𝄳	882.343	434.7755	881.671	0.672	kaba dik hisar	dik hisar	tiz dik hisar
27: A	896.757	438.4105	897.067	-0.311	HÜSEYNİAŞİRAN	HÜSEYNİ	TİZ HÜSEYNİ
28: A≠	950.633	452.2682	947.408	3.225	dik hüseyniaşiran	dik hüseyni	tiz dik hüzeyni
29: A𝄲	981.999	460.5371	980.693	1.306	acemaşiran	acem	tiz acem
30: A#/B♭	1001.88	465.8562	1001.955	-0.075	dik acemaşiran	dik acem	tiz dik acem
31: B♭̸	1050.682	479.1751	1046.43	4.251	nerm ırak	nerm eviç (nevruz)	tiz nerm eviç (nevruz)
32: B𝄳	1080.048	487.3724	1079.715	0.332	ırak	eviç	tiz eviç
33: B	1094.514	491.4619	1095.112	-0.598	GEVAŞT (REHAVİ)	MAHUR	TİZ MAHUR
34: B≠	1154.543	508.8018	1151.318	3.225	dik gevaşt	dik mahur	tiz dik mahur
35: c𝄳	1185.909	518.1043	1184.603	1.306	nerm rast	nerm gerdaniye	tiz nerm gerdaniye
36: c	1200	522.3384	1200	0	RAST	GERDANİYE	TİZ GERDANİYE
			Highest absolute : Average absolute : Root mean square : Total absolute :	5.2703 2.0902 2.7276 75.2472	*Bolahenk* = +6 deg. *Davud* = +12 deg. *Mansur* = -15 deg. *Kız* = -9 deg.	*Bolahenk* = +6 deg. *Davud* = +12 deg. *Mansur* = -15 deg. *Kız* = -9 deg.	*Bolahenk* = +6 deg. *Davud* = +12 deg. *Mansur* = -15 deg. *Kız* = -9 deg.

TABLE 2: Table of pitch data for *Yarman-36a* tuning.

Formula 3.1 (*calculation of the specific reference frequency "**f**" for note LA via the elimination of the fifth beat rate between G#–Eb of Layer I in **Fig. 2***)

$$\boxed{2\cdot\{(\boldsymbol{f}\cdot\alpha\cdot\beta\cdot\gamma\cdot\delta\cdot\varepsilon\cdot\zeta)\cdot 8\} - 3\cdot\{(\boldsymbol{f}\cdot a\cdot b\cdot c\cdot d\cdot e)/16\} = 0} \tag{3.1a}$$

Sol♯

$$\succ \quad e = \frac{3\cdot(\boldsymbol{f}\cdot(a\cdot b\cdot c\cdot d)) + 8}{2\cdot(\boldsymbol{f}\cdot(a\cdot b\cdot c\cdot d))} \tag{3.1b}$$

Do♯

$$\succ \quad d = \frac{3\cdot(\boldsymbol{f}\cdot(a\cdot b\cdot c)) + 4}{2\cdot(\boldsymbol{f}\cdot(a\cdot b\cdot c))} \tag{3.1c}$$

Fa♯

$$\succ \quad c = \frac{3\cdot(\boldsymbol{f}\cdot(a\cdot b)) - 4}{2\cdot(\boldsymbol{f}\cdot(a\cdot b))} \tag{3.1d}$$

Si

$$\succ \quad b = \frac{3\cdot(\boldsymbol{f}\cdot a) - 4}{2\cdot(\boldsymbol{f}\cdot a)} \tag{3.1e}$$

Mi

$$\succ \quad a = \frac{(3\cdot\boldsymbol{f}) - 2}{(2\cdot\boldsymbol{f})} \tag{3.1f}$$

$\boxed{\textbf{LA}\ (\boldsymbol{f})}$

$$\succ \quad \alpha = \frac{(2\cdot\boldsymbol{f}) + 2}{(3\cdot\boldsymbol{f})} \tag{3.1g}$$

Re

$$\succ \quad \beta = \frac{2\cdot(\boldsymbol{f}\cdot\alpha) + 1}{3\cdot(\boldsymbol{f}\cdot\alpha)} \tag{3.1h}$$

Sol

$$\succ \quad \gamma = \frac{2\cdot(\boldsymbol{f}\cdot(\alpha\cdot\beta)) + 0.5}{3\cdot(\boldsymbol{f}\cdot(\alpha\cdot\beta))} \tag{3.1i}$$

Do

$$\succ \quad \delta = \frac{2\cdot(\boldsymbol{f}\cdot(\alpha\cdot\beta\cdot\gamma)) + 0.5}{3\cdot(\boldsymbol{f}\cdot(\alpha\cdot\beta\cdot\gamma))} \tag{3.1j}$$

Fa

$$\succ \quad \varepsilon = \frac{2\cdot(\boldsymbol{f}\cdot(\alpha\cdot\beta\cdot\gamma\cdot\delta)) + 0.25}{3\cdot(\boldsymbol{f}\cdot(\alpha\cdot\beta\cdot\gamma\cdot\delta))} \tag{3.1k}$$

Si♭

$$\succ \quad \zeta = \frac{2\cdot(\boldsymbol{f}\cdot(\alpha\cdot\beta\cdot\gamma\cdot\delta\cdot\varepsilon)) + 0.25}{3\cdot(\boldsymbol{f}\cdot(\alpha\cdot\beta\cdot\delta\cdot\gamma\cdot\varepsilon))} \tag{3.1l}$$

Mi♭

Formula 3.1 – continued – (*calculation of the specific reference frequency "**f**" for note LA via the elimination of the fifth beat rate between G#–Eb of Layer I in **Fig. 2***)

$$\frac{16}{3}\left(\frac{2}{3}\left(\frac{2}{3}\left(\frac{2}{3}\left(\frac{2}{3}\left(\frac{2}{3}\left(2f+2\right)+1\right)+\frac{1}{2}\right)+\frac{1}{2}\right)+\frac{1}{4}\right)+\frac{1}{4}\right)- \\ \frac{3}{32}\left(\frac{3}{2}\left(\frac{3}{2}\left(\frac{3}{2}\left(\frac{3}{2}\left(3f-2\right)-4\right)-4\right)+4\right)+8\right)=0 \tag{3.1m}$$

where Formula 3.1a, via the expansion of all its associated terms, results in the equation shown in 3.1m, whose outcome is $\boldsymbol{f = 3135950\ /\ 7153}$, which makes **438.41046 Hz** for note *A*. This is simply to assure that the fifth between *G#* and *Eb* comes out pure at the end. One can, at any case, optionally disregard such a route by choosing the international standard *A*=440 Hz. Doing so does not conceptually affect the *Yarman-36a* tuning scheme in the least. On the other hand, a lower *A* is authentic not only for Western Classical music, but also for Ottoman-era music.

Whereas, the first author had formulated two more variants after his initial *Yarman-36a* (christened "*b*" and "*c*"), both of which are constructed as triple cascading quasi-equally tempered 12 tones apiece, only the original *Yarman-36a* will be undertaken in this paper. Regardless, any of the Yarman-36 variants can be implemented on a *Kanun*, *Tanbur*, *Cümbüş*, *Bağlama*, or mapped to a tripartite Halberstadt keyboard layout; and all of them readily feature approximations for both the comma nuances and one kind of critical neutral second peculiar to the essence of the genre [xiii] – that are comprised in whole by neither AEU nor the Arabic/Persian 24 tone cast.

To rephrase, *Yarman-36a* is a triple-layered "Baroque-style" 1/8 comma Modified Meantone cyclic tuning, capable of decently expressing *makams* over *Süpürde*, *Bolahenk*, *Davud*, *Mansur* and *Kız Ahenks*, while also making possible the elegant and authentic sonorities of European music, alongside several exotic microtonal chords of modern xenharmony. The proposed *Yarman-36a makam tone-system* illustrated in **Table 1** and **Table 2** not only furnishes crucial commatic, neutral, and sesquitone (augmented second) intervals demanded by traditionalist executants of the Middle East altogether in a single package, it further facilitates Western-oriented musicians' understanding of *makamlar* through the suitable

[xiii] Differences between the *Yarman-36 a, b, c* variants are minute – that is to say, a musician can swap one for the other with only slight (maximum 9 cents per degree) intervallic deformity. Such divergence ought not arouse significant aural discomfort since a few cents mistuning of intervals is observed to be indiscernable in traditional ensembles or orchestras composed of complex timbres. Besides, the "*b*" and "*c*" variants are solely the product of mathematical perfectionism as one searches for intervallic regularity.

Nevertheless, to summarize: *Yarman-36a* features pitch relations yielded by selective 0, 1, and 2 integer beats per second based on a dedicated reference frequency for A at 438.41 Hz, with 2/1 as octave; *Yarman-36b* thrice collates in identical triplex fashion equally spaced twelve pitches per layer with 441/220 (1204 cents) as the octave; and the almost entirely rational *Yarman-36c* comprises mostly pure fifths in like vein as version "*a*", with again 441/220 as octave.

Yarman-36a, subject to further elaboration hereunder, is the easiest to implement acoustically and without electronic aid. *Yarman-36b* is the closest tuning to 12-tone Equal Temperament with only 4 cents absolute difference at any degree, while possibly being the hardest to tune by ear – making it perhaps an ideal regular Temperament model when discussing theory on paper. *Yarman-36c* flaunts proportionally beating chords that ought to please the listener due to an abundance of rational pitches, rendering it the obvious choice for digitally pedantic expositions on extended *Just Intonation*. No further mention of the "*b*" and "*c*" variants are required at this point.

employment of enharmonically equivalent (*i.e.* respellable) sharps & flats at simple key signatures.

A consistent microtonal staff notation tailored to express *Yarman-36 makam tone-system* maintains all of the accidental symbols of AEU, with the addition of merely a sharp & flat pair more for degrees 2, 8, 17, 23 and 29. This specialty makes it quite easy to convert from AEU notation to the *Yarman-36 makam tone-system*, as can be seen in **Fig. 3**. The flexibility of intervals depending on the transposition means that, the accidentals occupy regions on the whole-tone continuum, as illustrated in **Fig. 3** and **Table 3**.

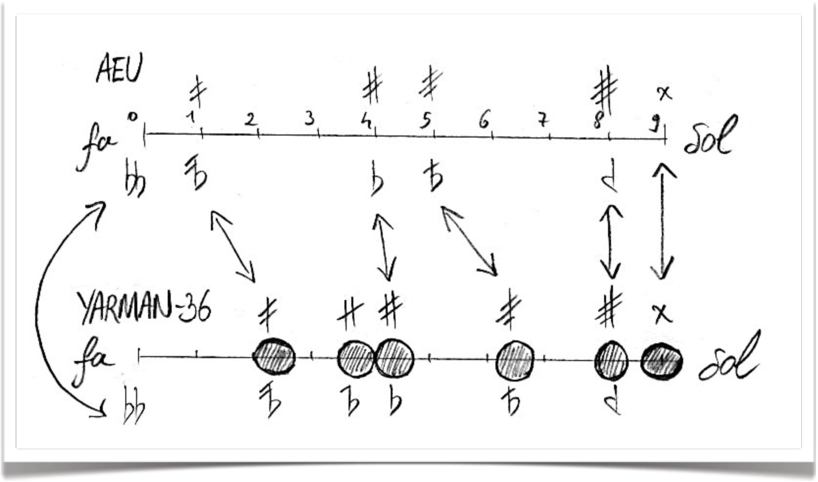


FIGURE 3. Conversion scheme from *Arel-Ezgi Uzdilek*, where accidentals of *Yarman-36a* occupy regions on the 9 Holderian commas wide whole-tone continuum, and where only one extra sharp & flat pair is needed.

TABLE 3. Extent of common microtonal accidentals from all natural notes in the *Yarman-36a* tuning.

≠	‡	#	ƀ	ɗ	x
48.9-60 ¢	78.3-91.4 ¢	92.8-105.5 ¢	-43.7 to -48.8 ¢	-14.1 to -17.4 ¢	197-203.1 ¢

An alternative palette of accidentals that are more amiable to the Persian *sori* and *koron* symbology is also possible, and perhaps more preferable for international standardization concerns. They are given in **Fig. 4**. The only change compared to Figure 3 is regarding the “lesser (≮) sharp” and the “greater (≯) sharp”.

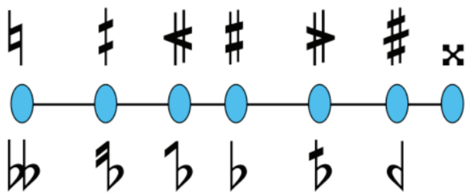


FIGURE 4. Alternative accidentals for notating Yarman-36 that are more amiable to the Persian *sori* (1/4-tone sharp) and *koron* (1/4-tone flat) symbology.

4. Analysis and Conclusions

Tetrachords and pentachords of Turkish Art music can henceforward be re-defined using the *Yarman-36a* cast. A catalogue of complete genera are attempted in **Figs. 5 & 6** throughout *Süpürde*, *Bolahenk*, *Davud*, *Mansur* and *Kız Ahenks* in the following pages. Once they have been transcribed thus, it is possible to conjoin them in the construction of characteristic *makam* scales. Due to exhaustion of space in this article, such work is postponed to a future study.

We can nevertheless engage in a comparison of select genera with their AEU counterparts in **Table 4** below, by referring each to pentachordal subsets of histogram peaks achieved from recordings by master performers [13,15]. The peaks were collated from 128 pieces in 9 makam categories and can be readily matched to 8 genera in the table. Also, since *Uşşak* and *Hüseyni* are ordinarily identified with the same intervallic structure in AEU, the average of their respective peaks are taken.

Table 4. Comparison of genera in AEU and *Yarman-36a* with pitch measurements

Genus	**AEU (Hc)**	**Measr.(Hc)**	**Measr.(¢)**	**AEU (¢)**	**Diff.**	**YA36 mean (¢)**	**Diff.**
Rast tetrachord	9	9.17	207.6	**203.9**	-3.7	197.8	-9.8
	+8	+7.47	376.7	384.4	7.7	**382**	5.3
	+5	+5.26	495.8	**498**	2.2	501.1	5.3
(Rast pentachord)	(+9)	(+9.12)	(702.3	(**702**)	-0.3	(698.9)	(-3.4)
				Average:	**3.48**	Average:	5.95
Uşşak tetrachord	8	6.32	143.1	180.4	37.3	**154.1**	11
	+5	+6.16	282.6	**294.1**	11.5	303.2	20.6
	+9	+9.36	494.5	**498**	3.5	501	6.5
(Hüseyni2 pentachord)	(+9)	(+9.36)	(706.4)	(**702**)	-4.4	699.6	(-6.8)
				Average:	14.18	Average:	**11.23**
Buselik tetrachord	9	9.38	212.4	**203.9**	-8.5	200.8	-11.6
	+4	+3.52	292.1	**294.1**	2	287.6	-4.5
	+9	+9.17	499.7	498	-1.7	**501.2**	1.5
(Buselik pentachord)	(+9)	(+8.88)	(700.8)	(702)	1.2	(**699.8**)	-1
				Average:	**3.35**	Average:	4.65
Kürdi* tetrachord	4	5.26	119.1	90.2	-28.9	**103**	-16.1
**Kürdilihicazkar*	+9	+7.45	287.8	**294.1**	6.3	303.1	15.3
	+9	+9.19	495.9	**498**	2.1	501.1	5.2
(Kürdi* pentachord)	(+9)	(+9.2)	(704.2)	(**702**)	-2.2	(698.9)	-5.3
				Average:	**9.88**	Average:	10.48
Hicaz tetrachord	5	4.65	105.3	113.7	8.4	**105.3**	0
	+12	+12.16	380.6	**384.4**	3.8	384.7	4.1
	+5	+4.98	493.4	**498**	4.6	501.2	7.8
(Hümayun pentachord)	(+9)	(+9.3)	(704)	(**702**)	-2	(699.8)	-4.2
				Average:	4.7	Average:	**4.03**
Segah tetrachord	5	4.68	106	**113.7**	7.7	119.1	13.1
	+9	+9.4	318.8	**317.6**	-1.2	316.9	-1.9
	+8	+9.03	523.3	498	-25.3	**500.5**	-22.8
(Segah pentachord)	(+9)	(+7.58)	(694.9)	(702)	7.1	(**700.1**)	5.2
				Average:	**10.33**	Average:	10.75
Saba (dim.) Tetrachord	8	7.61	172.3	**180.4**	8.1	184.2	11.9
	+5	+5.18	289.6	294.1	4.5	**287.6**	-2
	+5	+5.91	423.4	**407.8**	-15.6	403.8	-19.6
(Saba1 Pentachord)	(+13)	(+12.44)	(705.1)	(**702**)	-3.1	(699.8)	-5.3
				Average:	**7.83**	Average:	9.7
	5	4.99	113	**113.7**	0.7	119.1	6.1
	+9	+9.18	320.8	**317.6**	-3.2	316.9	-3.9
	+5	+6.28	463	431.3	-31.7	**470.8**	7.8
Hüzzam Pentachord	+12	+10.52	701.2	**702**	0.8	700	-1.2
				Average:	9.1	Average:	**4.75**
				Grand Avg.:	*7.85625*	*Grand Average:*	*7.6925*

Yarman-36 makam tetrachords

FIGURE 5: Notation of *Yarman-36a* makam tetrachords in main *Ahenks* with consecutive intervals in cents

FIGURE 6: Notation of *Yarman-36a* makam pentachords in main *Ahenks* with consecutive intervals in cents

Figure 6 – continued: Notation of *Yarman-36a* makam pentachords in main *Ahenks* with consecutive intervals in cents

Figure 6 – continued: Notation of *Yarman-36a* makam pentachords in main *Ahenks* with consecutive intervals in cents

We can right away see in **Table 4** that, the cumulative errors of AEU are slightly greater than the grand average of mean values across 5 *Ahenks* in *Yarman-36a*, despite the fact that each tuning system can be improved further through better selection of pitches for certain genera. For example, *Yarman-36a* could better approximate the 2nd steps of *Uşşak*, and *Kürdi* as well as the 3rd step of *Segah* by the occasional employment of neighboring pitches, and AEU might similarly correct for *Kürdi* as well as *Segah*. Notwithstanding, such manipulations turn out to be more advantageous overall for *Yarman-36a* and are therefore avoided.

Yet, this is about all AEU can achieve with its 24 tones, whereas our tuning proposition fares much better against problematic genera such as *Uşşak* and *Hüzzam*, and also certain known instances of *Saba* not immediately discernable from pitch measurements here – which finely fit the broad diversity of tetrachord & pentachord definitions in the *Yarman-36 makam tone-system*, with still more definitions possible.

To reflect the importance of each genera for the repertory, we can calculate a weighted arithmetic mean by referring the outcomes of **Table 4** to the percentage of pieces that belong to corresponding *makamlar*. According to Timuçin Çevikoğlu [17], 45.2 % of the total 23,592 pieces in 286 *makams* are composed in 1) *Rast* making up 1344 pieces, 2) *Uşşak* & *Hüseyni* as well as *Muhayyer* & *Bayati* making up 1242 + 987 + 359 + 309 = 2897 pieces, 3) *Buselik* making up 346 pieces [xiv], 4) *Kürdilihicazkar* making up 1275 pieces [xv], 5) *Hicaz* making up 2359 pieces [xvi], 6) *Segah* making up 601 pieces [xvii], 7) *Saba* making up 431 pieces [xviii], and 8) *Hüzzam* making up 1408 pieces [xix]. This data can now be used in **Table 5** to judge the real global distance of AEU and *Yarman-36a* from measurements:

TABLE 5. Global weighted average deviations, as referred to the repertory, of AEU and *Yarman-36a* genera from pitch measurements.

Makam	*Repertory %*	*AEU Avg.*	*Weighted Avg.*	*YA-36 Avg.*	*Weighted Avg.*
RAST	5.7 %	**3.48**	0.44	5.95	0.75
UŞŞAK-HÜS.-MUH.-BEY.	12.28 %	14.18	3.85	**11.23**	3.05
BUSELİK	1.4666 %	**3.35**	0.11	4.65	0.15
KÜRDİ (K.HİCAZKAR)	5.4 %	**9.88**	1.18	10.48	1.25
HİCAZ	9.999 %	4.7	1.04	**4.03**	0.89
SEGAH	2.55 %	**10.33**	0.58	10.75	0.61
SABA	1.83 %	**7.83**	0.32	9.7	0.39
HÜZZAM	5.97 %	9.1	1.2	**4.75**	0.63
Grand Average		7.86 cents		**7.7** cents	
Sum	45.2 %		8.72 cents		**7.72** cents

[xiv] Transposition of the genus in *Nihavend makam* is ignored, owing also to the controversy regarding how *Buselik* is structurally distinct from it.

[xv] We cannot ascertain the contributing number of pieces in *makam Kürdi* from Çevikoğlu (that are anyway outside the 72% comprising the foremost 20 *makams*), and also do not include its derivatives such as suffixed *makamlar* like *Muhayyer-Kürdi* and *Acem-Kürdi*, which only feature the genus toward the finalis.

[xvi] We cannot ascertain the contributing number of pieces in kindred *Hümayun* and *Uzzal makams* from Çevikoğlu (that are anyway outside the 72% comprising the foremost 20 *makams*).

[xvii] Transposition of the genus in kindred *Irak* and *Eviç makams* are ignored.

[xviii] We do not include derivative composite modes such as *Bestenigar* and *Çargah* (that are anyway outside the 72% comprising the foremost 20 *makams*).

[xix] *Suzinak*, which is a composite of *Hüzzam* & *Rast*, is ignored because it can belong to either category.

The calculations for **Table 5** are done by multiplying the repertory percentages in column 2 by either the AEU averages in column 3, or the *Yarman-36a* means in column 5, and then diving the resultant number by the repertorial sum 45.2 % to produce the results in columns 4 and 6. These weighted averages columns are then cumulated to yield the weighted average global outcomes – which are **8.72 ¢ overall deviation for AEU** and **7.72 ¢ overall deviation for *Yarman-36a*** – which accounts for nearly half the repertory.

As can be immediately noticed, the already poor performance of AEU at representing *Uşşak-Hüseyni-Muhayyer-Beyati* and *Hüzzam* is worsened due to the abundant usage of related genera in the repertory. In other words, characteristic and frequent occurence of middle-second interval flavors lowers the score of AEU further. While not quite discernable in the case of *Saba* here, the same situation is known to be true for *Saba*'s various auditions too, where the second step may be flattened as much as a quarter-tone in descent to finalis. All of these can be readily approximated by the available and additionally possible genera in our tone-system.

In contrast to AEU, *Yarman-36a* accomodates the problematic genera fairly enough. General intonational sacrifices such as detuned fifths, fourths, and thirds are compensated thus. Subsequently, allowing for no more than a nominal 1 Holderian comma (1200/53 = 22.6 ¢) maximum pitch-bend flexibility lets *Yarman-36a tone-system* perform admirably as a novel *makam theory* candidate.

Moreover, the Modified Meantone Temperament basis of *Yarman-36a* is agreeable with the historical 9 steps to the whole-tone, 55 steps to the octave methodology of Europe, known at the time of Georg Philipp Telemann and Leopold Mozart for approximating 1/6-comma tempered fifths tuning [42] – which was remarkably employed by Antoine de Murat at the end of the 18$^{\text{th}}$ Century to explain minute alterations of pitch in Ottoman-era Makam music to Westerners [3]. The slightly mellower 438.41 Hz reference frequency for note *A* has historicity too under such a context, way before 440 Hz became the international norm by the 20$^{\text{th}}$ Century [47].

Qualitatively speaking, *Yarman-36 makam tone-system* has greater explanatory power in terms of

1) the potential to serve Western common-practice music via a 12-tone cyclic subset easily tunable by ear and flaunting vibrant key-colors;

2) the capability to house quarter-tones, next to commatic nuances, to embrace a larger geography;

3) its hybrid functionality in notating both Western and Middle Eastern musics using a consistent array of accustomed accidentals that feature enharmonically equivalent sharps and flats;

4) its success in fairly transposing Turkish *makamlar* over to five main *Ahenks*; and

5) its support for approximated *Just Intonation* polyphony, as well as provision for substantial Xenharmonic resources.

Auditory-visual examples of some genera and chords can be discovered at the first author's website [62].

In conclusion, our hybrid tone-system proposal can appeal to not only Classical/ Contemporary Western musicians and Middle Eastern performers of traditional makam instruments, but also to avant-garde composers searching for new microtonal expression venues.

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