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INTEGRAL TRANSFORMS OF THE GALUE TYPE STRUVE FUNCTION

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Abstract. This paper refers to the study of generalized Struve type function. Using generalized Galué type Struve function (GTSF), we derive various integral transforms, including Euler transform, Laplace transform, Whittakar transform, K-transform and fractional Fourier transform. The transform images are expressed in terms of the generalized Wright function. Interesting special cases of the main results are also considered.

Keywords: Galué type Struve function, Euler transform, Laplace transform, fractional Fourier transform, generalized Wright function.

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1. Introduction and preliminaries

Integral transforms have been widely used in various problems of mathematical physics and applied mathematics (for some recent works, see, e.g., [16, 15, 6, 13]). Integral transforms with such special functions as (for example) the hypergeometric functions have been played important roles in solving numerous applied problems. This information has inspired the study of several integral transforms with verity of special functions (see [5]). The present paper deals with the evaluation of the Euler transform, Laplace transform, Whittakar transform, K-transform and Fractional Fourier transform of the Galué type Struve function recently introduced by [12]. Special cases of the results are also pointed out briefly.

For the convenience of the reader, we give here the baisc definitions and related notations which is necessary for the understanding of this study.

Definition 1.1 (Generalized Galué type Struve function [12]). *The generalized form of Struve function, named as generalized Galué type Struve function (GTSF), is defined as:*

$$ {}_a w^{\lambda,\mu}_{p,b,c,\xi}(z) = \sum_{k=0}^{\infty} \frac{(-c)^k}{\Gamma(\lambda k+\mu)\,\Gamma\left(ak+\frac{p}{\xi}+\frac{b+2}{2}\right)} \left(\frac{z}{2}\right)^{2k+p+1}, \quad a \in \mathbb{N}; z,p,b,c \in \mathbb{C}. \tag{1} $$

where $\lambda > 0, \xi > 0$ and μ is an arbitrary parameter.

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For the detailed definition of the Struve function and its more generalization, the interested reader may refer to the research papers Bhowmick [3],[4], Kanth [8], Singh [17], [18] and Singh [19].

Particularly, if we set $\lambda = a = 1, \mu = 3/2$ and $\xi = 1$ in the equation (1), it reduces to generalization of the Struve function which is defined by Orhan and Yagmur [14] as follows:

$$H_{p,b,c}(z) = \sum_{k=0}^{\infty} \frac{(-c)^k}{\Gamma\left(k+\frac{3}{2}\right)\Gamma\left(k+p+\frac{b+2}{2}\right)} \left(\frac{z}{2}\right)^{2k+p+1}, \quad z, p, b, c \in \mathbb{C}. \tag{2}$$

Details related to the function $H_{p,b,c}(z)$ and its particular cases can be seen in Baricz [1], [2], Mondal and Swaminathan [11] and Nisar *et al.* [13].

Definition 1.2 (Euler Transform [20]). *The Euler transform of a function $f(z)$ is defined as*

$$B\{f(z); a, b\} = \int_0^1 z^{a-1}(1-z)^{b-1} f(z)\,dz \qquad a, b \in \mathbb{C}, \Re(a) > 0, \Re(b) > 0. \tag{3}$$

Definition 1.3 (Laplace Transform [20]). *The Laplace transform of a function $f(z)$, denoted by $F(s)$, is defined by the equation*

$$F(s) = (Lf)(s) = L\{f(z); s\} = \int_0^{\infty} e^{-sz} f(z)\,dz \;(\Re(s) > 0), \tag{4}$$

provided the integral (4) *is convergent and that the function $f(z)$, is continous for $z > 0$ and of exponential order as $z \to \infty$. The operator* (4) *may be symbolically written as*

$$F(s) = L\{f(z); s\} \quad or \quad f(z) = L^{-1}\{F(s); z\}. \tag{5}$$

Definition 1.4 (Whittakar Transform [22]).

$$\int_0^{\infty} e^{-\frac{z}{2}} z^{\zeta-1} W_{\tau,\omega}(z)\,dz = \frac{\left(\frac{1}{2}+w+\zeta\right)\Gamma\left(\frac{1}{2}-w+\zeta\right)}{\Gamma(1-\tau+\zeta)}, \tag{6}$$

where $\Re(w \pm \zeta) > -1/2$ and $W_{\tau,\omega}(z)$ is the Whittakar confluent hypergeometric function

$$W_{\tau,\omega}(z) = \frac{\Gamma(-2\omega)}{\Gamma\left(\frac{1}{2}-\tau-\omega\right)} M_{\tau,\omega}(z) + \frac{\Gamma(2\omega)}{\Gamma\left(\frac{1}{2}+\tau+\omega\right)} M_{\tau,-\omega}(z), \tag{7}$$

where $M_{\tau,\omega}(z)$ is defined by

$$M_{\tau,\omega}(z) = z^{1/2+\omega} e^{-1/2z} {}_1F_1\left(\frac{1}{2}+\omega-\tau; 2\omega+1; z\right). \tag{8}$$

Definition 1.5 (K-Transform [7]). *K-Transform is defined by the following integral equation*

$$\Re_{\upsilon}[f(z); p] = g[p; \upsilon] = \int_0^{\infty} (pz)^{1/2} K_{\upsilon}(pz) f(z)\,dz, \tag{9}$$

where $\Re(p) > 0; K_{\upsilon}(z)$ is the Bessel function of the second kind defined by [7]

$$K_{\upsilon}(z) = \left(\frac{\pi}{2z}\right)^{1/2} W_{0,\upsilon}(2z),$$

where $W_{0,\upsilon}(.)$ is the Whittakar function defined in equation (7).

The following result given in [10], will be used in evaluating the main results:

$$\int_0^{\infty} t^{\rho-1} K_{\upsilon}(ax)\,dx = 2^{\rho-2} a^{-\rho}\, \Gamma\left(\frac{\rho \pm \upsilon}{2}\right); \Re(a) > 0; \Re(\rho \pm \upsilon) > 0. \tag{10}$$

Definition 1.6 (Fractional Fourier Transform [9]). *The fractional Fourier transform of order* $\alpha, 0 < \alpha \leq 1$ *is defined by*

$$\widehat{u}_{\alpha}(\omega) = \Im_{\alpha}[u](\omega) = \int_R e^{i\omega^{(1/\alpha)}t} u(t)\, dt. \tag{11}$$

When $\alpha = 1$, equation (11) reduces to the conventional Fourier transforms and for $\omega > 0$, it reduces to the Fractional Fourier transform defined by Luchko *et al.* [9].

Our main results are expressed in terms of the generalized Wright hypergeometric function ${}_p\psi_q(z)$ [23] (see, for detail, Srivastava and Karson [21]), for $z, a_i, b_j \in \mathbb{C}$ and $\alpha_i, \beta_j \in \mathbb{R}$, where $(\alpha_i, \beta_j \neq 0;\ i = 1, 2, \dots, p;\ j = 1, 2, \dots, q)$, is defined as below:

$${}_p\psi_q(z) = {}_p\psi_q\left[\begin{array}{c} (a_i, \alpha_i)_{1,p} \\ (b_j, \beta_j)_{1,q} \end{array} \Big| z\right] = \sum_{k=0}^{\infty} \frac{\prod_{i=1}^{p} \Gamma(a_i + \alpha_i k) z^k}{\prod_{j=1}^{q} \Gamma(b_j + \beta_j k)\, k!}, \tag{12}$$

under the condition:

$$\sum_{j=1}^{q} \beta_j - \sum_{i=1}^{p} \alpha_i > -1.$$

2. Integral Transforms of ${}_a w_{p,b,c,\xi}^{\lambda,\mu}(z)$

In this section, we evaluate the following Euler transform, Laplace transform, Whittakar transform and K-transform of Generalized Galué type Struve function.

Theorem 2.1 (Euler Transform). *For* $a \in \mathbb{N}$, $p, b, c, r, s \in \mathbb{C}$, *we have*

$$B\left\{{}_a w_{p,b,c,\xi}^{\lambda,\mu}\left(x^{1/2}z\right); r, s\right\} = \left(\frac{\sqrt{x}}{2}\right)^{p+1} \Gamma(s)$$

$$\times {}_2\psi_3\left[\begin{array}{c} (p+r+1, 2), (1,1); \\ (\mu, \lambda), \left(\frac{p}{\xi} + \frac{b}{2} + 1, a\right), (p+r+s+1, 2); \end{array} \Bigg| \frac{-cx}{4}\right], \tag{13}$$

where $\Re(r) > 0, \Re(s) > 0, \lambda > 0, \xi > 0$ *and* μ *is an arbitrary parameter.*

Proof. Using (1) and (3), we get

$$B\left\{{}_a w_{p,b,c,\xi}^{\lambda,\mu}\left(x^{1/2}z\right); r, s\right\} = \int_0^1 z^{r-1}(1-z)^{s-1}\, {}_a w_{p,b,c,\xi}^{\lambda,\mu}\left(x^{1/2}z\right) dz$$

$$= \sum_{k=0}^{\infty} \frac{(-c)^k}{\Gamma(\lambda k + \mu)\Gamma\left(ak + \frac{p}{\xi} + \frac{b+2}{2}\right)} \left(\frac{x^{1/2}}{2}\right)^{2k+p+1} \int_0^1 z^{2k+p+r+1-1}(1-z)^{s-1} dz,$$

$$= \sum_{k=0}^{\infty} \frac{(-c)^k}{\Gamma(\lambda k + \mu)\Gamma\left(ak + \frac{p}{\xi} + \frac{b+2}{2}\right)} \left(\frac{x^{1/2}}{2}\right)^{2k+p+1} B(p+r+1+2k, s),$$

$$= \left(\frac{x^{1/2}}{2}\right)^{p+1} \sum_{k=0}^{\infty} \frac{1}{\Gamma(\lambda k + \mu)\Gamma\left(\frac{p}{\xi} + \frac{b+2}{2} + ak\right)} \frac{\Gamma(p+r+1+2k)\Gamma(s)}{\Gamma(p+r+s+1+2k)} \left(\frac{(-cx)^{1/2}}{2}\right)^k.$$

In accordance with the definition of (12), we obtain the result (13). This completes the proof of the theorem. □

Corollary 2.1. *For $\lambda = a = 1$, $\mu = 3/2$ and $\xi = 1$, equation (13) reduces in the following form*

$$B\left\{H_{p,b,c}\left(x^{1/2}z\right);r,s\right\} = \left(\frac{x^{1/2}}{2}\right)^{p+1}\Gamma(s)$$
$$\times {}_2\psi_3\left[\begin{array}{c}(p+r+1,2),(1,1);\\ \left(p+\frac{b}{2}+1,1\right),(p+r+s+1,2),(3/2,1);\end{array}\middle|\frac{-cx}{4}\right]. \quad (14)$$

Theorem 2.2 (Laplace Transform)**.** *For $a \in \mathbb{N}$, $p,b,c \in \mathbb{C}$, the following formula holds:*

$$L\left\{{}_aw^{\lambda,\mu}_{p,b,c,\xi}\left(x^{1/2}z\right);s\right\} = \left(\frac{\sqrt{x}}{2}\right)^{p+1}s^{-(p+2)}{}_2\psi_3\left[\begin{array}{c}(p+2,2),(1,1);\\ (\mu,\lambda),\left(\frac{p}{\xi}+\frac{b}{2}+1,a\right);\end{array}\middle|\frac{-cx}{4s^k}\right], \quad (15)$$

where $\lambda > 0, \xi > 0$ and μ is an arbitrary parameter.

Proof. Using (1) and (4), it gives

$$L\left\{{}_aw^{\lambda,\mu}_{p,b,c,\xi}\left(x^{1/2}z\right);s\right\} = \int_0^\infty e^{-sz}{}_aw^{\lambda,\mu}_{p,b,c,\xi}\left(x^{1/2}z\right)dz,$$

$$= \sum_{k=0}^{\infty}\frac{(-c)^k}{\Gamma(\lambda k+\mu)\Gamma\left(ak+\frac{p}{\xi}+\frac{b+2}{2}\right)}\left(\frac{x^{1/2}}{2}\right)^{2k+p+1}\int_0^\infty z^{2k+p+1+1-1}e^{zs}dz,$$

$$= \left(\frac{x^{1/2}}{2}\right)^{p+1}\sum_{k=0}^{\infty}\frac{\Gamma(p+2+2k)}{\Gamma(\lambda k+\mu)\Gamma\left(ak+\frac{p}{\xi}+\frac{b+2}{2}\right)}\left(\frac{-cx^{1/2}}{2}\right)^{2k}L\left\{\frac{z^{p+2+2k-1}}{\Gamma(p+2+2k)};s\right\},$$

which in view of definition (12), yield to the result (15). □

Again, for $\lambda = a = 1, \mu = 3/2$ and $\xi = 1$, result (15) reduces in the following form:

Corollary 2.2. *The Laplace trsnform of the generalized Struve function, defined by (2), is given by*

$$L\left\{H_{p,b,c}\left(x^{1/2}z\right);s\right\} = \left(\frac{\sqrt{x}}{2}\right)^{p+1}s^{-(p+2)}{}_2\psi_3\left[\begin{array}{c}(p+2,2),(1,1);\\ \left(p+\frac{b}{2}+1,1\right),(3/2,1);\end{array}\middle|\frac{-cx}{4s^k}\right]. \quad (16)$$

Theorem 2.3 (Whittakar Transform)**.** *If $a \in \mathbb{N}$, $p,b,c \in \mathbb{C}$ and $\Re(\zeta) > 0$, $\Re(w \pm \zeta) > -1/2$, then*

$$\int_0^\infty t^{\zeta-1}e^{-\frac{t}{2}}W_{\tau,\omega}(t)\,{}_aw^{\lambda,\mu}_{p,b,c,\xi}\left(x^{1/2}t\right)dt = \left(\frac{\sqrt{x}}{2}\right)^{p+1}$$
$$\times {}_3\psi_3\left[\begin{array}{c}(w+\zeta+p+3/2,2),(-w+\zeta+p+3/2,2),(1,1);\\ (\mu,\lambda),\left(\frac{p}{\xi}+\frac{b}{2}+1,a\right),(-\tau+\zeta+p+2,2);\end{array}\middle|\frac{-cx}{4}\right], \quad (17)$$

provided $\Re(e) > |\Re(\omega)| - 1/2, \Re(p) > 0, \lambda > 0, \xi > 0$ and μ is an arbitrary parameter.

Proof. On using (1), the left hand side of (17) (say L) can be written as

$$L = \int_0^\infty t^{\zeta-1}e^{-\frac{t}{2}}W_{\tau,\omega}(t)\sum_{k=0}^{\infty}\frac{(-c)^k}{\Gamma(\lambda k+\mu)\,\Gamma\left(ak+\frac{p}{\xi}+\frac{b+2}{2}\right)}\left(\frac{x^{1/2}t}{2}\right)^{2k+p+1}dt,$$

$$= \sum_{k=0}^{\infty}\frac{(-c)^k}{\Gamma(\lambda k+\mu)\,\Gamma\left(ak+\frac{p}{\xi}+\frac{b+2}{2}\right)}\left(\frac{x^{1/2}}{2}\right)^{2k+p+1}\int_0^\infty e^{-\frac{t}{2}}t^{\zeta+p+2k+1-1}W_{\tau,\omega}(t)\,dt.$$

Now, on using (6), we obtain

$$L=\sum_{k=0}^{\infty}\frac{(-c)^k}{\Gamma(\lambda k+\mu)\,\Gamma\left(ak+\frac{p}{\xi}+\frac{b+2}{2}\right)}$$

$$\times\left(\frac{x^{1/2}}{2}\right)^{2k+p+1}\frac{\left(\frac{1}{2}+w+\zeta+p+2k+1\right)\Gamma\left(\frac{1}{2}-w+\zeta+p+2k+1\right)}{\Gamma(1-\tau+\zeta+p+2k+1)},$$

$$=\left(\frac{x^{1/2}}{2}\right)^{p+1}\sum_{k=0}^{\infty}\frac{\left(\frac{1}{2}+w+\zeta+p+2k+1\right)\Gamma\left(\frac{1}{2}-w+\zeta+p+2k+1\right)}{\Gamma(\lambda k+\mu)\ \ \Gamma(1-\tau+\zeta+p+2k+1)\,\Gamma\left(ak+\frac{p}{\xi}+\frac{b+2}{2}\right)}\left(\frac{(-cx)^{1/2}}{2}\right)^{2k},$$

which in accordance with the definition (12), yield the desired result (17). □

Corollary 2.3. *If we set,* $\lambda=a=1,\mu=3/2$ *and* $\xi=1$, *then formula* (17) *reduces in the following form*

$$\int_0^{\infty}t^{\zeta-1}\,e^{-\frac{t}{2}}\,W_{\tau,\omega}(t)\;H_{p,b,c}\left(x^{1/2}t\right)\,dt=\left(\frac{\sqrt{x}}{2}\right)^{p+1}$$

$$\times{}_3\psi_3\left[\begin{matrix}(w+\zeta+p+3/2,2),(-w+\zeta+p+3/2,2),(1,1)\,;\\ \left(\frac{3}{2},1\right),\left(p+\frac{b}{2}+1,1\right),(-\tau+\zeta+p+2,2)\,;\end{matrix}\;\middle|\;\frac{-cx}{4}\right].\tag{18}$$

Theorem 2.4 (K-Transform)**.** *Let us assume* $a\in\mathbb{N};p,b,c,\rho\in\mathbb{C}$, *then the following result holds true:*

$$\int_0^{\infty}t^{\rho-1}K_{\upsilon}(\omega t)\,{}_aw_{p,b,c,\xi}^{\lambda,\mu}\left(x^{1/2}t\right)dt=2^{\rho+p-1}\omega^{1-\rho-p}\left(\frac{\sqrt{x}}{2}\right)^{p+1}$$

$$\times{}_3\psi_2\left[\begin{matrix}\left(\frac{\rho+p+\upsilon+1}{2},1\right),\left(\frac{\rho+p-\upsilon+1}{2},1\right),(1,1)\,;\\ (\mu,\lambda),\left(\frac{p}{\xi}+\frac{b}{2}+1,a\right);\end{matrix}\;\middle|\;\frac{-cx}{\omega^2}\right].\tag{19}$$

where $\Re(\omega)>0,\Re(\rho\pm\upsilon)>0.$ $\lambda>0,\xi>0$ *and* μ *is an arbitrary parameter.*

Proof. Using definition (1), the left hand side of (19) (say L) can be expressed as

$$L=\int_0^{\infty}t^{\rho-1}K_{\upsilon}(\omega t)\sum_{k=0}^{\infty}\frac{(-c)^k}{\Gamma(\lambda k+\mu)\,\Gamma\left(ak+\frac{p}{\xi}+\frac{b+2}{2}\right)}\left(\frac{x^{1/2}t}{2}\right)^{2k+p+1}dt,$$

$$=\sum_{k=0}^{\infty}\frac{(-c)^k}{\Gamma(\lambda k+\mu)\,\Gamma\left(ak+\frac{p}{\xi}+\frac{b+2}{2}\right)}\left(\frac{x^{1/2}}{2}\right)^{2k+p+1}\int_0^{\infty}t^{\rho+p+2k+1-1}K_{\upsilon}(\omega t)\,dt,$$

$$=\sum_{k=0}^{\infty}\frac{(-c)^k}{\Gamma(\lambda k+\mu)\,\Gamma\left(ak+\frac{p}{\xi}+\frac{b+2}{2}\right)}\left(\frac{x^{1/2}}{2}\right)^{2k+p+1}$$

$$\times2^{\rho+p+2k-1}\omega^{1-\rho-p-2k}\Gamma\left(\frac{(\rho+p+2k+1)\pm\upsilon}{2}\right),$$

$$=2^{\rho+p-1}\omega^{1-\rho-p}\left(\frac{\sqrt{x}}{2}\right)^{p+1}\sum_{k=0}^{\infty}\frac{\Gamma\left(\frac{\rho+p+\upsilon+1}{2}+k\right)\Gamma\left(\frac{\rho+p-\upsilon+1}{2}+k\right)}{\Gamma(\lambda k+\mu)\,\Gamma\left(ak+\frac{p}{\xi}+\frac{b+2}{2}\right)}\left(\frac{(-cx)^{1/2}}{\omega}\right)^{2k}.$$

Hence, on using (12), we obtain the required result (19). □

Further, if we take $\lambda = a = 1, \mu = 3/2$ and $\xi = 1$, equation (19) reduces in the following result:

Corollary 2.4. *The following integral holds true:*

$$\int_0^\infty t^{\rho-1} K_\upsilon(\omega t)\, H_{p,b,c}\left(x^{1/2}\, t\right) dt = 2^{\rho+p-1} \omega^{1-\rho-p} \left(\frac{\sqrt{x}}{2}\right)^{p+1}$$

$$\times {}_3\psi_2 \left[\begin{array}{l} \left(\frac{\rho+p+\upsilon+1}{2}, 1\right), \left(\frac{\rho+p-\upsilon+1}{2}, 1\right), (1,1)\,; \\ \left(\frac{3}{2}, 1\right), \left(p + \frac{b}{2} + 1, 1\right); \end{array} \left| \frac{-cx}{\omega^2} \right. \right]. \quad (20)$$

3. Fractional Fourier Transforms (FFT) of ${}_a w^{\lambda,\mu}_{p,b,c,\xi}(z)$

Now, we present the fractional Fourier transform of the generalized Struve type function as follows:

Theorem 3.1. *Suppose $a \in \mathbb{N}$, $p, b, c \in \mathbb{C}$, then*

$$\Im_\zeta \left[{}_a w^{\lambda,\mu}_{p,b,c,\xi}\left(x^{1/2}\, t\right) \right] = \left(\frac{\sqrt{x}}{2}\right)^{p+1} \sum_{k=0}^\infty \frac{1}{(i)^{2k+p+2}\, \omega^{(2k+p+2)/\zeta}\, (-1)^{2k+p+1}}$$

$$\times \frac{\Gamma(2k+p+2)}{\Gamma(\lambda k + \mu)\, \Gamma\left(ak + \frac{p}{\xi} + \frac{b+2}{2}\right)} \left(\frac{(-cx)^{1/2}}{2}\right)^{2k}, \quad (21)$$

provided $\zeta > 0, \lambda > 0, \xi > 0$ and μ is an arbitrary parameter.

Proof. Using (1) and (11), it gives

$$\Im_\zeta \left[{}_a w^{\lambda,\mu}_{p,b,c,\xi}\left(x^{1/2} t\right) \right] (\omega)$$

$$= \int_R e^{i\,\omega^{(1/\zeta)} t} \sum_{k=0}^\infty \frac{(-c)^k}{\Gamma(\lambda k + \mu)\, \Gamma\left(ak + \frac{p}{\xi} + \frac{b+2}{2}\right)} \left(\frac{x^{1/2} t}{2}\right)^{2k+p+1} dt,$$

$$= \sum_{k=0}^\infty \frac{(-c)^k}{\Gamma(\lambda k + \mu)\, \Gamma\left(ak + \frac{p}{\xi} + \frac{b+2}{2}\right)} \left(\frac{x^{1/2}}{2}\right)^{2k+p+1} \int_R e^{i\,\omega^{(1/\zeta)} t} t^{2k+p+1} dt,$$

If we set $i\omega^{(1/\zeta)} t = \eta$, then

$$= \sum_{k=0}^\infty \frac{(-c)^k}{\Gamma(\lambda k + \mu)\, \Gamma\left(ak + \frac{p}{\xi} + \frac{b+2}{2}\right)} \left(\frac{x^{1/2}}{2}\right)^{2k+p+1} \int_{-\infty}^0 e^{-\eta} \left(\frac{-\eta}{i\,\omega^{1/\zeta}}\right)^{2k+p+1} \left(\frac{-d\eta}{i\,\omega^{1/\zeta}}\right),$$

$$= \sum_{k=0}^\infty \frac{(-c)^k}{\Gamma(\lambda k + \mu)\, \Gamma\left(ak + \frac{p}{\xi} + \frac{b+2}{2}\right)} \left(\frac{x^{1/2}}{2}\right)^{2k+p+1} \frac{1}{(i)^{2k+p+2}\, \omega^{(2k+p+2)/\zeta}\, (-1)^{2k+p+1}}$$

$$\times \int_0^\infty e^{-\eta} \eta^{2k+p+1} d\eta,$$

$$= \sum_{k=0}^\infty \frac{(-c)^k}{\Gamma(\lambda k + \mu)\, \Gamma\left(ak + \frac{p}{\xi} + \frac{b+2}{2}\right)} \left(\frac{x^{1/2}}{2}\right)^{2k+p+1} \frac{\Gamma(2k+p+2)}{(i)^{2k+p+2}\, \omega^{(2k+p+2)/\zeta}\, (-1)^{2k+p+1}},$$

$$= \left(\frac{\sqrt{x}}{2}\right)^{p+1} \sum_{k=0}^{\infty} \frac{1}{(i)^{2k+p+2}\,\omega^{\left(2k+p+2\right)/\zeta}\,(-1)^{2k+p+1}}$$

$$\times \frac{\Gamma\left(2k+p+2\right)}{\Gamma(\lambda k+\mu)\,\Gamma\left(ak+\frac{p}{\xi}+\frac{b+2}{2}\right)} \left(\frac{(-cx)^{1/2}}{2}\right)^{2k}.$$

This completes the proof of the theorem. □

Finally, if we put $\lambda = a = 1, \mu = 3/2$ and $\xi = 1$, then result (21) reduces to the following corollary:

Corollary 3.1. *Fractional Fourier transform of the generalized Struve type function is given by*

$$\Im_\zeta\left[H_{p,\,b,\,c}\left(x^{1/2}\,t\right)\right] = \left(\frac{\sqrt{x}}{2}\right)^{p+1}$$

$$\sum_{k=0}^{\infty} \frac{1}{(i)^{2k+p+2}\,\omega^{\left(2k+p+2\right)/\zeta}\,(-1)^{2k+p+1}} \frac{\Gamma\left(2k+p+2\right)}{\Gamma\left(k+\frac{3}{2}\right)\,\Gamma\left(k+P+\frac{b+2}{2}\right)} \left(\frac{(-cx)^{1/2}}{2}\right)^{2k}. \quad (22)$$

Lastly, we conclude this paper by remarking that, the integral transform formulas deduced in this paper, for generalized Galué type Struve function (GTSF), are significant and can lead to yield numerous transforms for variety of Struve functions. The transforms established here are general in nature and are likely to find useful in applied problem of sciences, engineering and technology.

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