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A NUMERICAL APPROACH TO CAUDREY DODD GIBBON EQUATION VIA COLLOCATION METHOD USING QUINTIC B-SPLINE BASIS

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Abstract. In this manuscript, a numerical approach is investigated to Caudrey-Dodd-Gibbon (CDG) equation. The nonlinear CDG equation is reduced to a system of partial differential equation using $u_{xxx} = v$. The new numerical solutions are obtained with a combination of collocation method with finite element method which is one of the most important methods among all numerical approaches. In order to proceed the method, solution for each unknown is written as a linear combination of time parameters and quintic B-spline basis. Then, with the advantage of the collocation method, a system of algebraic equation systems is formulated easily. Solving the system iteratively by a method results in numerical solutions of the CDG equation. The numerical solutions together with the error norms L_2, L_∞ are tabulated. Additionally, graphical simulations of the solutions are depicted by figures.

Keywords: Finite element method, collocation, quintic B-spline basis, Caudrey-Dodd-Gibbon equation.

AMS Subject Classification: 65L60, 65N35.

1. Introduction

Calculus is one of the major branches of mathematics that deals with variables and how they change, namely as a usual description of it is "the mathematics of change". In calculus concerning to understand continuously changing quantities, differential equations are the foremost aides of mathematicians for describing the change. A process which is observed in the nature depends on several variables such as temperature, gravity, mass, string. In most of the cases, such processes require the modelling by partial differential equations. Understanding the structure of the partial differential equations and seeking their solutions are of primary importance for scholars because solutions of the equations will brighten the way of understanding the behaviour of systems or will help to predict the development of the process in nature. Therefore, many mathematicians focus their attention on solving partial differential equations. For many years, the scholars have

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offered various powerful analytical and numerical technique for solving the PDEs related with real world problems(see in [1–9]). Among these is, the finite element method, one of the quite powerful numerical approach which is revealing solutions of PDEs. Simply, the method based on subdividing a large problem into smaller, more simpler sub-problems and seek an approximate solution with polynomial interpolation on every subproblems. This procedure results in obtaining an differential equation for each subproblems. Then whole element equations are assembled into a larger system of equations for the entire problem. The collocation method on finite elements is a useful method using the idea setting the residual to zero at the selected interior collocation points of the elements. The advantage of the is the fact that; it has an easy implementation and reduces the computational cost associated with computing. For an overview of the method, consider the Caudrey Dodd Gibbon equation given as [18]

$$u_t + u_{xxxxx} + 30u\ u_{xxx} + 30u_x u_{xx} + 180u^2 u_x = 0 \tag{1}$$

with the associated initial and boundary conditions

$$u\left(\pm L, t\right) = u_x\left(\pm L, t\right) = 0, \qquad x \in \left(-L, L\right), T > 0$$

$$u\left(x, 0\right) = f\left(x\right) = \frac{k_1^2 \exp(k_1 x)}{(1+\exp(k_1 x))^2}.$$

Where $u(x,t)$ is an differentiable function and the variables x and t denotes the space and time. Subscript notations symbolize the partial derivative respect to space and time, respectively. The CDG equation is a class of fifth order Korteweg-de Vries (fKdV) equation which is the most famous equation family of mathematics which has various application in quantum mechanics and nonlinear optics.

The physical understanding of CDG equation was illustrated in [10]. The analytical solutions of the equation given in (1) have been studied in many papers using different methods (See in [11–14]). Hirota transformation of the CDGE and its bilinear forms are discussed in [15–17]. Wazwaz [18] has handled one-soliton solution by the tanh-coth method. In this study, the collocation method is examined for obtaining numerical solutions of the CDG equation. Hence, the outline of the study can be given as follow: section 1 is "introduction" and it gives an overview of the method as well CDG equation. We introduce an application of the collocation method in section 2. and section 3 gives numerical results with tables and graphical representations. At the end, in section 4 conclusions are presented.

2. Application of the collocation method to the Caudrey Dodd Gibbon equation

In this section, a collocation method is going to be applied to the Caudrey Dodd Gibbon equation with the help of quintic B-spline basis. Before we begin to obtain the numerical scheme, the above equation is split into coupled using $u_{xxx} = v$, so the Eq. (1) is replaced by

$$\begin{aligned} &u_{xxx} - v = 0 \\ &u_t + v_{xx} + 30uv + 30u_x u_{xx} + 180u^2 u_x = 0 \end{aligned} \tag{2}$$

Now, to begin the procedure, our first task for solving the boundary value problem given in Eq. (1) numerically is to discretize the solution interval. To do this, let us assume that a partition of an interval $[-L, L]$ is $P = \{x_0 = -L, x_1, x_2, ..., x_N = L\}$ that satisfies the inequalities $x_0 < x_1 < x_2 < \cdots < x_{N-1} < x_N$ with $\{x_m\}_{m=0}^{N}$ are the collocation points and $h = x_{m+1} - x_m$.

Secondly, an approximate solution is generated for the equation system. As with all classical finite element methods, approximate solutions for the each actual solutions are sought in the following forms ,

$$u=\sum_{m=-2}^{N+2}\phi_m(x)\,\delta_m(t)\,,\qquad v=\sum_{m=-2}^{N+2}\phi_m(x)\,\sigma_m(t) \tag{3}$$

where $\phi_{-2}(x),\phi_{-1}(x),\phi_0(x),...,\phi_N(x),\phi_{N+1}(x),\phi_{N+2}(x)$ are quintic B-spline basis over interval $[-L,L]$, $\delta_m(t)$ and $\sigma_m(t)$ are unknown parameters to be determined. The values of quintic B-spline basis at collocation points are given by (see in [19–21])

$$\phi_m(x)=\frac{1}{h^5}\begin{cases}(x-x_{m-3})^5, & [x_{m-3},x_{m-2}]\\ (x-x_{m-3})^5-6(x-x_{m-2})^5, & [x_{m-2},x_{m-1}]\\ (x-x_{m-3})^5-6(x-x_{m-2})^5-15(x-x_{m-1})^5, & [x_{m-1},x_m]\\ (x_{m+3}-x)^5-6(x_{m+2}-x)^5-15(x_{m+1}-x)^5, & [x_m,x_{m+1}]\\ (x_{m+3}-x)^5-6(x_{m+2}-x)^5, & [x_{m+1},x_{m+2}]\\ (x_{m+3}-x)^5 & [x_{m+2},x_{m+3}]\\ 0 & otherwise\end{cases} \tag{4}$$

Using the approximate solutions and $\phi_m(x)$ basis, approximate values at $x=x_m$ of u and v and their derivatives up to second order can be obtained in terms of time parameters $\delta_m(t)$ and $\sigma_m(t)$ as

$$\begin{aligned}
u_m &= \delta_{m-2}+26\delta_{m-1}+66\delta_m+26\delta_{m+1}+\delta_{m+2}\\
u'_m &= \tfrac{-5}{h}\left(\delta_{m-2}+10\delta_{m-1}-10\delta_{m+1}-\delta_{m+2}\right)\\
u''_m &= \tfrac{20}{h^2}\left(\delta_{m-2}+2\delta_{m-1}-6\delta_m+2\delta_{m+1}+\delta_{m+2}\right)\\
v_m &= \sigma_{m-2}+26\sigma_{m-1}+66\sigma_m+26\sigma_{m+1}+\sigma_{m+2}\\
v'_m &= \tfrac{-5}{h}\left(\sigma_{m-2}+10\sigma_{m-1}-10\sigma_{m+1}-\sigma_{m+2}\right)\\
v''_m &= \tfrac{20}{h^2}\left(\sigma_{m-2}+2\sigma_{m-1}-6\sigma_m+2\sigma_{m+1}+\sigma_{m+2}\right)
\end{aligned}$$

According to the idea of collocation method, differential equation is satisfied the at collocation points along the whole domain. Before substituting the approximate solution into Eq. (2) , we apply forward difference and Crank Nicolson formula to procees in time with the numerical scheme, Thus we obtain the following equalities

$$\begin{aligned}
&(u_{xxx})^{n+1}-v^{n+1}=-(u_{xxx})^n+v^n\\
&u^{n+1}+\tfrac{\Delta t}{2}(v_{xx})^{n+1}+15\Delta t z_m v^{n+1}+15\Delta t g_m(u_{xx})^{n+1}+90\Delta t z_m^2(u_x)^{n+1}\\
&\qquad=u^n-\tfrac{\Delta t}{2}(v_{xx})^n-15\Delta t z_m v^n-15\Delta t g_m(u_{xx})^n-90\Delta t z_m^2(u_x)^n
\end{aligned} \tag{5}$$

where Δt is time step, h is space step and values $z_m=u$ and $g_m=u_x$ are used for linearization of nonlinear terms in the numerical scheme at each time step. And, now we can begin to obtain the scheme by putting approximate solutions and their derivatives into (5) at the collocation points, thus following equation system is obtained;

$$\frac{-60}{h^3}\delta_{m-2}^{n+1}+\frac{120}{h^3}\delta_{m-1}^{n+1}-\frac{120}{h^3}\delta_{m+1}^{n+1}+\frac{60}{h^3}\delta_{m+2}^{n+1}-\sigma_{m-2}^{n+1}-26\sigma_{m-1}^{n+1}-66\sigma_{m}^{n+1}-26\sigma_{m+1}^{n+1}-\sigma_{m+2}^{n+1}$$
$$=\frac{60}{h^3}\delta_{m-2}^{n}-\frac{120}{h^3}\delta_{m-1}^{n}+\frac{120}{h^3}\delta_{m+1}^{n}-\frac{60}{h^3}\delta_{m+2}^{n}+\sigma_{m-2}^{n}+26\sigma_{m-1}^{n}+66\sigma_{m}^{n}+26\sigma_{m+1}^{n}+\sigma_{m+2}^{n}$$

$$\begin{aligned}
&\left(1+\tfrac{300}{h^2}\Delta t g_m-\tfrac{450}{h}\Delta t z_m^2\right)\delta_{m-2}^{n+1}+\left(26+\tfrac{600}{h^2}\Delta t g_m-\tfrac{4500}{h}\Delta t z_m^2\right)\delta_{m-1}^{n+1}+\left(66-\tfrac{1800}{h^2}\Delta t g_m\right)\delta_{m}^{n+1}\\
&\quad+\left(26+\tfrac{600}{h^2}\Delta t g_m+\tfrac{4500}{h}\Delta t z_m^2\right)\delta_{m+1}^{n+1}+\left(1+\tfrac{300}{h^2}\Delta t g_m+\tfrac{450}{h}\Delta t z_m^2\right)\delta_{m+2}^{n+1}\\
&\quad+\left(\tfrac{10}{h^2}\Delta t+15z_m\Delta t\right)\sigma_{m-2}^{n+1}+\left(\tfrac{20}{h^2}\Delta t+390z_m\Delta t\right)\sigma_{m-1}^{n+1}+\left(\tfrac{-60}{h^2}\Delta t+990z_m\Delta t\right)\sigma_{m}^{n+1}\\
&\quad+\left(\tfrac{20}{h^2}\Delta t+390z_m\Delta t\right)\sigma_{m+1}^{n+1}+\left(\tfrac{10}{h^2}\Delta t+15z_m\Delta t\right)\sigma_{m+2}^{n+1}\\
&=\left(1-\tfrac{300}{h^2}\Delta t g_m+\tfrac{450}{h}\Delta t z_m^2\right)\delta_{m-2}^{n}+\left(26-\tfrac{600}{h^2}\Delta t g_m+\tfrac{4500}{h}\Delta t z_m^2\right)\delta_{m-1}^{n}+\left(66+\tfrac{1800}{h^2}\Delta t g_m\right)\delta_{m}^{n}\\
&\quad+\left(26-\tfrac{600}{h^2}\Delta t g_m-\tfrac{4500}{h}\Delta t z_m^2\right)\delta_{m+1}^{n}+\left(1-\tfrac{300}{h^2}\Delta t g_m-\tfrac{450}{h}\Delta t z_m^2\right)\delta_{m+2}^{n}\\
&\quad+\left(\tfrac{-10}{h^2}\Delta t-15z_m\Delta t\right)\sigma_{m-2}^{n}+\left(\tfrac{-20}{h^2}\Delta t-390z_m\Delta t\right)\sigma_{m-1}^{n}+\left(\tfrac{60}{h^2}\Delta t-990z_m\Delta t\right)\sigma_{m}^{n}\\
&\quad+\left(\tfrac{-20}{h^2}\Delta t-390z_m\Delta t\right)\sigma_{m+1}^{n}+\left(\tfrac{-10}{h^2}\Delta t-15z_m\Delta t\right)\sigma_{m+2}^{n}.
\end{aligned}$$

2.1. **Boundary Conditions.** If we take a look to the above system, the number of equations is less than the number of unknown variables i.e the system consists of $(2N+2)$ equation with $(2N+10)$ unknown time dependent parameters. Being able to find an unique solution for the parameters $\delta_m(t)$ and $\sigma_m(t)$, the simplest way is to eliminate eight unknowns $(\delta_{-2},\delta_{-1},\sigma_{-2},\sigma_{-1},\delta_{N+2},\delta_{N+1},\sigma_{N+2},\sigma_{N+1})$ from the system. This procedure is applied using left and right boundary conditions with the values of u,v and their first order derivatives u',v' by following way;

$$\begin{aligned}
u(-L,t)&=\delta_{-2}+26\delta_{-1}+66\delta_0+26\delta_1+\delta_2\\
u_x(-L,t)&=\tfrac{-5}{h}\left(\delta_{-2}+10\delta_{-1}-10\delta_1-\delta_2\right)
\end{aligned}$$

$$\begin{aligned}
u(L,t)&=\delta_{N-2}+26\delta_{N-1}+66\delta_N+66\delta_{N+1}+\delta_{N+2}\\
u_x(L,t)&=\tfrac{-5}{h}\left(\delta_{N-2}+10\delta_{N-1}-10\delta_{N+1}-\delta_{N+2}\right)
\end{aligned}$$

$$\begin{aligned}
v(-L,t)&=\sigma_{-2}+26\sigma_{-1}+66\sigma_0+26\sigma_1+\sigma_2\\
v_x(-L,t)&=\tfrac{-5}{h}\left(\sigma_{-2}+10\sigma_{-1}-10\sigma_1-\sigma_2\right)
\end{aligned}$$

$$\begin{aligned}
v(L,t)&=\sigma_{N-2}+26\sigma_{N-1}+66\sigma_N+26\sigma_{N+1}+\sigma_{N+2}\\
v_x(L,t)&=\tfrac{-5}{h}\left(\sigma_{N-2}+10\sigma_{N-1}-10\sigma_{N+1}-\sigma_{N+2}\right)
\end{aligned}$$

Thus, this elimination provides us a solvable equation system in the form $(2N+2)\times(2N+2)$. Now, to obtain unknown parameters δ^{n+1} and σ^{n+1}, we are going to use the parameters δ^n and σ^n, iteratively since they are known. That is, once we determine δ^0 and σ^0 at initial time t_0, then we obtain $\delta^{(1)}$ and $\sigma^{(1)}$ in second loop. Similarly, the new $\delta^{(1)}$ and $\sigma^{(1)}$ are used in third loop, we will obtain $\delta^{(2)}$ and $\sigma^{(3)}$ and so on. By repeated iterations, we will have an approximation solution that converges to the actual solution.

2.2. **Initial vector.** In the previous section, we have obtained an iterative scheme. By this scheme, we have started to work with to obtain initial solution vector, for this purpose, the initial condition of the problem is used with approximate solution at initial time t_0;

$$\begin{aligned}
&\delta_{m-2}^{(0)}+26\delta_{m-1}^{(0)}+66\delta_{m}^{(0)}+26\delta_{m+1}^{(0)}+\delta_{m+2}^{(0)}=u\left(x_m,0\right),\\
&\sigma_{m-2}^{(0)}+26\sigma_{m-1}^{(0)}+66\sigma_{m}^{(0)}+26\sigma_{m+1}^{(0)}+\sigma_{m+2}^{(0)}=v\left(x_m,0\right).
\end{aligned} \tag{6}$$

when we consider the above system, it can be seen that such systems have an infinite number of solutions because of number of unknown variables is greater then the number of equations, but we need a unique solution, so the same procedure stated in boundary condition can be applied to eliminate the parameters $\delta_{-2},\delta_{-1},\sigma_{-2},\sigma_{-1},\delta_{N+2},\delta_{N+1},\sigma_{N+2}$ and σ_{N+1}. After eliminating unknowns, the system can be rewritten clearly as

$$\begin{bmatrix}
54 & 60 & 6 & 0 & 0 & 0 & 0 & 0\\
25.25 & 67.5 & 26.25 & 1 & 0 & 0 & 0 & 0\\
1 & 26 & 66 & 26 & 1 & 0 & 0 & 0\\
0 & 0 & \ddots & \ddots & \ddots & \ddots & 0 & 0\\
0 & 0 & 0 & 1 & 26 & 66 & 26 & 1\\
0 & 0 & 0 & 0 & 1 & 26.25 & 67.25 & 25.25\\
0 & 0 & 0 & 0 & 0 & 6 & 60 & 54
\end{bmatrix}
\begin{bmatrix}
\delta_0^{(0)}\\ \vdots\\ \delta_N^{(0)}\\ \sigma_0^{(0)}\\ \vdots\\ \vdots\\ \sigma_N^{(0)}
\end{bmatrix}
=
\begin{bmatrix}
u\left(x_0,0\right)\\ \vdots\\ u\left(x_N,0\right)\\ v\left(x_0,0\right)\\ \vdots\\ \vdots\\ v\left(x_N,0\right)
\end{bmatrix}$$

Now, we have a equation system in form of $(2N+2)\times(2N+2)$. the system can be solved using any methods and initial solution vector is obtained. To summarize the development thus far, we have developed an iterative algorithm for the Caudrey Dodd Gibbon equation and we reached an initial vector which will be require to start iteration using initial conditions.

3. Numerical Results

In the following experiment, we are going to present numerical solution of Caudrey Dodd Gibbon equation on interval $[-L,L]$ for various values of the number of time and space partition. For testing the accuracy of the collocation method, we compared numerical solutions with actual solution found in the literature using the error norms L_2 and L_∞ given as following formulas;

$$\begin{aligned}
L_2&=\left\|u-U_N\right\|_2=\sqrt{h\sum_{j=0}^{N}\left|u_j-\left(U_N\right)_j\right|^2},\\
L_\infty&=\left\|u-U_N\right\|_\infty=\max_{0\le j\le N}\left|u_j-\left(U_N\right)_j\right|,
\end{aligned} \tag{7}$$

The 1-soliton solution of the CDG equation given in (1) is in the following form

$$u\left(x,t\right)=\frac{k_1^2\exp\left(k_1\left(x-k_1^4t\right)\right)}{\left(1+\exp\left(k_1\left(x-k_1^4t\right)\right)\right)^2}. \tag{8}$$

In order to present numerical solutions, computational interval and final time are taken as $[-50,50]$ and $T=0.04$ for $k_1=1$. First of all, the error norms L_2 and L_∞ are computed for various values of time and step size to show what is the effect of the number of collocation points on the numerical solutions. The collocation method use a space-time grid to construct the collocation points. Thus, firstly the step size and then time size are changed for evaluating the method and newly obtained results are tabulated in Table 1-3. As expected result of the method, for Caudrey Dodd Gibbon equation, the

		$h = 0.25$		$h = 0.05$	
	t	L_2	L_∞	L_2	L_∞
	0.01	1.2150061×10^{-6}	9.212882×10^{-7}	1.9374×10^{-9}	1.4761×10^{-9}
	0.02	2.4206357×10^{-6}	1.7870691×10^{-6}	3.7735×10^{-9}	2.7646×10^{-9}
	0.03	3.6400051×10^{-6}	2.7035975×10^{-6}	5.6231×10^{-9}	3.9754×10^{-9}
	0.04	4.8726632×10^{-6}	3.6540666×10^{-6}	6.39492×10^{-8}	1.007786×10^{-7}
		$h = 0.1$		$h = 0.025$	
	0.01	2.99904×10^{-8}	2.28747×10^{-8}	4.375×10^{-10}	3.317×10^{-10}
0.1	0.02	5.95350×10^{-8}	4.33687×10^{-8}	7.672×10^{-10}	5.260×10^{-10}
	0.03	8.94379×10^{-8}	6.68890×10^{-8}	1.0495×10^{-9}	6.624×10^{-10}
	0.04	1.332525×10^{-7}	9.48677×10^{-8}	7.3959×10^{-9}	1.27734×10^{-8}

TABLE 1. The error norms L_2 and L_∞ for $k_1 = 1$,$\Delta t = 0.001$ and various values of h and t

h	L_2	L_∞
0.25	4.8726632×10^{-6}	3.6540666×10^{-6}
0.1	1.332525×10^{-7}	9.48677×10^{-8}
0.05	6.39492×10^{-8}	1.007786×10^{-7}
0.025	7.3959×10^{-9}	1.27734×10^{-8}

TABLE 2. The error norms L_2 and L_∞ for $k_1 = 1$,$\Delta t = 0.001$ and various values of h

Δt	L_2	L_∞
0.04	1.8666452×10^{-6}	1.2592529×10^{-6}
0.02	5.136933×10^{-7}	3.347566×10^{-7}
0.01	1.705244×10^{-7}	1.149466×10^{-7}
0.005	1.223963×10^{-7}	8.46205×10^{-8}
0.0025	1.195848×10^{-7}	8.83651×10^{-8}
0.00125	1.196239×10^{-7}	9.01249×10^{-8}

TABLE 3. The error norms L_2 and L_∞ for $k_1 = 1$,$h = 0.1$ and various values of Δt

best convergent results are obtained for the N=40 collocation points. It can be seen from the tables that increasing of collocation points affect the numerical solutions with a quick decreasing the error norms also increasing of the number of time step is resulted with same affect, although not as much as the effect of collocation points. Also, it can be seen from the tables that actual solutions and the numerical solution are in a good agreement and the method is efficient.

In our computations for simulations, the typical values we used are $h = 0.1$ and $\Delta = 0.001$. The movement of wave is depicted at $t = 0, 0.2$ and 0.5 in the figure 1. With the help of the numerical experiment, it is noted that the wave located at $x = 0$, with amplitude 0.250000 at the initial time $t = 0$. Then, in time,at $t = 0.5$, its location is $x = 0.5$ with the amplitude 0.250000. The figure exposes that the wave's behavior as it moves slowly and without changing its shape.

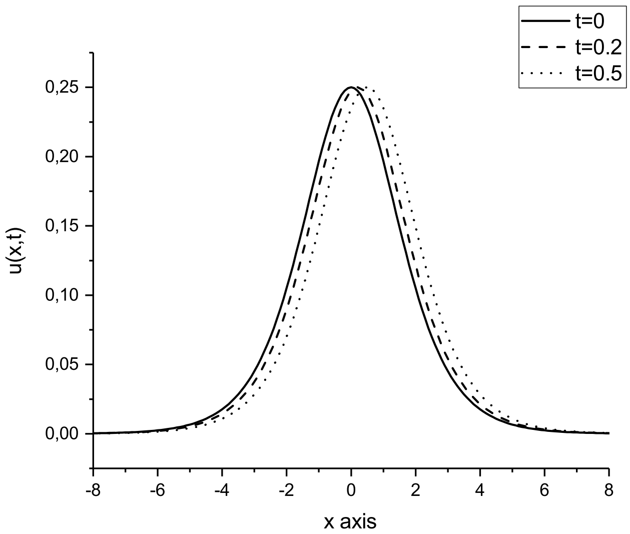

FIGURE 1. 1-solitary wave motion for Caudrey Dodd Gibbon equation

4. CONCLUSION

In this study, numerical solutions of the Caudrey Dodd Gibbon equation are investigated. For this purpose, a combination of collocation method on the finite element approach and finite difference discreziation is used to formally derive the numerical scheme. Interpolation functions are chosen as quintic B-spline basis. As numerical experiments, the motion of one soliton wave is examined. Numerical results and simulations of the solutions are presented with tables and graphics. The tables including the error norms L_2 and L_∞ coupled with the graphical representations of numerical and exact solutions reveal that the results are agreement with the exact solutions with a high accuracy. Hereby, the collocation method is powerful technique to handle several other nonlinear partial differential equations.

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