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A NEW SUBCLASS OF BI-UNIVALENT FUNCTIONS DEFINED BY q-DERIVATIVE

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Abstract. In this investigation we introduce, by making use of $q-$derivative operator, a new subclass which are an extension of some well-known subclasses of bi-univalent functions. Also, we give the upper bounds for the coefficients $|a_2|$ and $|a_3|$ for the functions belonging to this new subclass and its subclasses.

Keywords: $q-$derivative, bi-univalent function, coefficient inequality, subordination.

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1. Introduction and Prerequisites

Denote by $\mathcal{A}$ the class of all analytic functions f in the unit disc $\mathbb{D} = \{z : z \in \mathbb{C} \text{ and } |z| < 1\}$, with the series expansion

$$f(z) = z + \sum_{n\geq 2} a_n z^n \tag{1}$$

and normalized by $f(0) = f'(0) - 1 = 0$. Further, let $\mathcal{S}$ be the class of all functions in $\mathcal{A}$ which are univalent in $\mathbb{D}$. Because of the Koebe one-quarter theorem [6] it is well known that every function $f \in \mathcal{S}$ has an inverse f^{-1}, defined by

$$f^{-1}(f(z)) = z \quad (z \in \mathbb{D})$$

and

$$f(f^{-1}(w)) = w, \quad (|w| < r_0(f); \quad r_0(f) \geq 1/4)$$

where

$$f^{-1}(w) = w - a_2 w^2 + (2a_2^2 - a_3)w^3 - (5a_2^3 - 5a_2 a_3 + a_4)w^4 + \dots. \tag{2}$$

A function $f \in \mathcal{A}$ is said to be bi-univalent in $\mathbb{D}$ if both $f(z)$ and $f^{-1}(z)$ are univalent in $\mathbb{D}$. We denote by Σ the class of all functions $f(z)$ which are bi-univalent functions in $\mathbb{D}$. We say that f is starlike function in $\mathbb{D}$, denoted by $\mathscr{S}^\star$, if the function f is univalent in $\mathbb{D}$ and $f(\mathbb{D})$ is a starlike domain with respect to origin. Also we say that f is convex

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function in $\mathbb{D}$, denoted by $\mathscr{C}$, if f is univalent in $\mathbb{D}$ and $f(\mathbb{D})$ is a convex domain. Analytical characterizations of starlikeness and convexity are, respectively, equivalent to the conditions $\Re(zf'(z)/f(z)) > 0$ and $1 + \Re(zf''(z)/f'(z)) > 0$.

A function $f \in \mathcal{A}$ is said to be subordinate to a function $g \in \mathcal{A}$, written $f(z) \prec g(z)$, provided there exists a function ω analaytic defined on $\mathbb{D}$, with $\omega(0) = 0$ and $|\omega(z)| < 1$, and such that $f(z) = g(\omega(z))$. In view of subordination, the above mentioned conditions are, respectively, equivalent to $(zf'(z)/f(z) \prec (1+z)/(1-z)$ and $1 + zf''(z)/f'(z) \prec (1+z)/(1-z)$. It is well known that Ma and Minda [15] gave a unified presentation of various subclasses of starlike and convex functions by replacing the subordinate function $(1+z)/(1-z)$ by a more general analytic function ψ with positive real part and normalized by the conditions $\psi(0) = 1$, $\psi'(0) > 0$ and ψ maps $\mathbb{D}$ onto a region starlike with respect to 1 and symmetric with respect to the real axis. They presented and investigated the following general classes that contains several well-known classes under some special cases:

$$\mathscr{S}^\star(\psi) = \left\{ f \in \mathcal{A} \left| \frac{zf'(z)}{f(z)} \prec \psi(z) \right. \right\} \text{ and } \mathscr{C}(\psi) = \left\{ f \in \mathcal{A} \left| 1 + \frac{zf''(z)}{f'(z)} \prec \psi(z) \right. \right\}.$$

It is worth mentioning that the functions which are in these classes are said to be Ma-Minda starlike and Ma-Minda convex, respectively. Also we say that a function $f \in \mathcal{A}$ is a Ma-Minda starlike and Ma-Minda convex order γ $(\gamma \in \mathbb{C} - \{0\})$:

$$\mathscr{S}^\star(\gamma, \psi) = \left\{ f \in \mathcal{A} \left| 1 + \frac{1}{\gamma}\left(\frac{zf'(z)}{f(z)} - 1\right) \prec \psi(z) \right. \right\}$$

and

$$\mathscr{C}(\gamma, \psi) = \left\{ f \in \mathcal{A} \left| 1 + \frac{1}{\gamma}\left(\frac{zf''(z)}{f'(z)}\right) \prec \psi(z) \right. \right\},$$

respectively.

It is well known that the class Σ of bi-univalent functions was defined and studied by Lewin [14]. Since then, various subclasses the bi-univalent function class Σ were defined and *non-sharp* estimates on the coefficients $|a_2|$ and $|a_3|$ for the functions belonging to these subclasses were obtained in several recent investigations (see [2], [4], [5], [10], [16], [17], [23]). A function f is *bi-starlike* and *bi-convex* of complex order γ $(\gamma \in \mathbb{C} - \{0\})$ of Ma-Minda type if both f and f^{-1} are Ma-Minda starlike and Ma-Minda convex of complex order γ. These classes are represented respectively by $\mathscr{S}^\star_\Sigma(\gamma, \psi)$ and $\mathscr{C}_\Sigma(\gamma, \psi)$.

Recently, $q-$derivative has played a crucial role in the theory of univalent functions especially in estimating the sharp inequalities bound for various subclasses of univalent functions (see [1], [3], [8], [9], [19]). In [12, 13] for $0 < q < 1$, *the q-difference operator* denoted as $D_q f$ is defined by the equation

$$(D_q f)(z) = \frac{f(z) - f(qz)}{(1-q)z}, \quad z \neq 0, \quad (D_q f)(0) = f'(0). \tag{3}$$

It is obvious that, when $q \to 1^-$, the difference operator $D_q f$ converges to the ordinary differential operator $Df = df/dz = f'$. Further, It is clear that if $f(z)$ is of the form 1, a simple computation yields

$$D_q f(z) = 1 + \sum_{n \geq 2} \frac{1-q^n}{1-q} a_n z^{n-1}, \quad (z \in \mathbb{D}). \tag{4}$$

Previously, Ismail *et al.* [11] defined and investigated some important properties of functions f belonging to the class $\mathcal{PS}^\star{}_q$. Recently, Sahoo and Sharma [21] (see also [18]) introduced and studied the class $\mathcal{PK}_q$ of q-close-to-convex functions. In 1989, Srivastava [22] proposed the study of the class, $\mathcal{PC}_q$ of q-convex function in $\mathbb{D}$. Recently, Seoudy and Aouf [20] defined the subclasses $\mathscr{S}_q^\star(\alpha)$ and $\mathscr{C}_q(\alpha)$ of the class $\mathcal{A}$ for $0 \leq \alpha < 1$ by

$$\mathscr{S}_q^\star(\alpha) = \left\{ f \in \mathcal{A} \,\middle|\, \Re\left(\frac{zD_q(f(z))}{f(z)}\right) > \alpha, \quad z \in \mathbb{D} \right\}$$

$$\mathscr{C}_q(\alpha) = \left\{ f \in \mathcal{A} \,\middle|\, \Re\left(1 + \frac{zqD_q(D_q(f(z)))}{D_q(f(z))}\right) > \alpha, \quad z \in \mathbb{D} \right\}. \tag{5}$$

It is clear that, when $q \to 1^-$, these classes $\mathscr{S}_q^\star(\alpha)$ and $\mathscr{C}_q(\alpha)$ coincide with the classes $\mathscr{S}^\star(\alpha)$ and $\mathscr{C}(\alpha)$ of starlike and convex functions of order $\alpha \quad (0 \leq \alpha < 1)$, respectively.

Motivated by all of the above-mentioned works, we present and investigate a new subclass of bi-univalent functions by making use of $q-$derivative operator.

Definition 1.1. *Let be $0 \leq \lambda \leq 1$, $\gamma \in \mathbb{C} - \{0\}$ and $0 < q < 1$. A function $f(z)$ given by* (1) *is said to be in the class $\mathscr{H}_{\Sigma,q}(\lambda, \gamma, \psi)$ if the following subordinations are satisfied*

$$1 + \frac{1}{\gamma}\left(\frac{zD_qf(z) + \lambda z^2 qD_q\left(D_qf(z)\right)}{\lambda zD_qf(z) + (1-\lambda)f(z)} - 1\right) \prec \psi(z) \tag{6}$$

and

$$1 + \frac{1}{\gamma}\left(\frac{wD_qg(w) + \lambda w^2 qD_q\left(D_qg(w)\right)}{\lambda wD_qg(w) + (1-\lambda)g(w)} - 1\right) \prec \psi(w), \tag{7}$$

where $g(w) := f^{-1}(w)$.

A function belonging to the class $\mathscr{H}_{q,\Sigma}(\lambda, \gamma, \psi)$ is named as both $q-$bi$-\lambda-$convex function and $q-$bi$-\lambda-$starlike function of complex order γ of Ma-Minda type. This class presented in this work is inspired by the corresponding class studied in [7].

It is worth noticing that, for some values of the parameters, this class give a unified presentation of some remarkable subclasses, which the first four of these subclasses are new.

Remark 1.1. *The followings are fulfilled:*
(i) $\mathscr{H}_{q,\Sigma}(0, \gamma, \psi) \equiv \mathscr{S}^\star_{q,\Sigma}(\gamma, \psi)$.
(ii) $\mathscr{H}_{q,\Sigma}(1, \gamma, \psi) \equiv \mathscr{C}_{q,\Sigma}(\gamma, \psi)$.
(iii.) $\mathscr{H}_{q,\Sigma}(0, (1-\alpha)e^{-i\lambda}\cos\lambda, \frac{1+z}{1-z}) \equiv \mathscr{S}^\star_{q,\Sigma}[\lambda, \alpha], \quad (|\lambda| < \pi/2, 0 \leq \alpha < 1)$.
(iv.) $\mathscr{H}_{q,\Sigma}(1, (1-\alpha)e^{-i\lambda}\cos\lambda, \frac{1+z}{1-z}) \equiv \mathscr{C}_{q,\Sigma}[\lambda, \alpha], \quad (|\lambda| < \pi/2, 0 \leq \alpha < 1)$.
(v.) $\mathscr{H}_{\Sigma}(0, \gamma, \psi) \equiv \mathscr{S}^\star_{\Sigma}(\gamma, \psi)$.
For $q \to 1^-$, we arrive at the some well-known subclasses:
(vi.) $\mathscr{H}_{\Sigma}(1, \gamma, \psi) \equiv \mathscr{C}_{\Sigma}(\gamma, \psi)$.
(vii.) $\mathscr{H}_{\Sigma}(0, (1-\alpha)e^{-i\lambda}\cos\lambda, \frac{1+z}{1-z}) \equiv \mathscr{S}^\star_{\Sigma}[\lambda, \alpha], \quad (|\lambda| < \pi/2, 0 \leq \alpha < 1)$.
(viii.) $\mathscr{H}_{\Sigma}(1, (1-\alpha)e^{-i\lambda}\cos\lambda, \frac{1+z}{1-z}) \equiv \mathscr{C}_{\Sigma}[\lambda, \alpha], \quad (|\lambda| < \pi/2, 0 \leq \alpha < 1)$.

The following lemma is very useful in building our main results.

Let $\mathscr{P}$ denote the class of analytic functions p in $\mathbb{D}$ such that $p(0) = 0$ and $\Re(p(z)) > 0, \quad z \in \mathbb{D}$. It is well known that this class is usually called the Caratheodory class.

Lemma 1.1. *If the function $p \in \mathscr{P}$ is given by the following series:*

$$p(z) = 1 + p_1 z + p_2 z^2 + \ldots,$$

then the sharp estimate given as

$$|p_n| \leq 2 \quad (n = 1, 2, \ldots).$$

2. **Coefficient Estimates**

We are now in a position to establish our main result. In this section we deal with some interesting coefficient estimates for the above-mentioned class and its some subclasses. Actually, It is convenient to mention that this paper involves an extension of the results given by [7].

Theorem 2.1. *Let be $0 \leq \lambda \leq 1$, $\gamma \in \mathbb{C} - \{0\}$ and $0 < q < 1$. If a function $f(z)$ given in (1) is of the class $\mathscr{H}_{\Sigma,q}(\lambda, \gamma, \psi)$, then*

$$|a_2| \leq \frac{|\gamma|\, B_1\sqrt{2B_1}}{\sqrt{\left|\gamma\Big(q^2+1+\lambda(2q^4+3q^3-q^2+q-1)-\lambda^2 q(q^2+1)\Big)B_1^2+2\big(q(1+q\lambda)\big)^2(B_1-B_2)\right|}}$$

and

$$|a_3| \leq \frac{|\gamma|\,(\tau(q;\lambda)+\upsilon(q;\lambda))(B_1+|B_2-B_1|)}{q(1+q)(1+q(1+q)\lambda)(\tau(q;\lambda)-\upsilon(q;\lambda))}$$

where $\tau(q;\lambda) = q\big(2(1+q)(1+q(1+q)\lambda)-(1+q\lambda)^2\big)$ *and* $\upsilon(q;\lambda) = (1+q\lambda)(q(1+q)+\lambda-1)$.

Proof. Let $f \in \mathscr{H}_{\Sigma,q}(\lambda, \gamma, \psi)$. Then we have

$$1 + \frac{1}{\gamma}\left(\frac{zD_q f(z) + \lambda z^2 q D_q\left(D_q f(z)\right)}{\lambda z D_q f(z) + (1-\lambda) f(z)} - 1\right) = \psi(u(z)) \tag{8}$$

and

$$1 + \frac{1}{\gamma}\left(\frac{wD_q g(w) + \lambda w^2 q D_q\left(D_q g(w)\right)}{\lambda w D_q g(w) + (1-\lambda) g(w)} - 1\right) = \psi(v(w)), \tag{9}$$

where the function ψ is an analytic function with positive real part in the unit disc $\mathbb{D}$, with $\psi(0) = 1$ and $\psi'(0) > 0$, and $\psi(\mathbb{D})$ is symmetric with respect to the real axis. It is well known that such a function has a series expansion of the form

$$\psi(z) = 1 + B_1 z + B_2 z^2 + \ldots, \quad (B_1 > 0). \tag{10}$$

Also, $p_1, p_2 \in \mathscr{P}$ defined by

$$p_1(z) = \frac{1+u(z)}{1-u(z)} = 1 + c_1 z + c_2 z^2 + \ldots$$

and

$$p_2(w) = \frac{1+v(w)}{1-v(w)} = 1 + d_1 w + d_2 w^2 + \ldots.$$

From these equalities, we get

$$u(z) = \frac{p_1(z)-1}{p_1(z)+1} = \frac{1}{2}\left[c_1 z + \left(c_2 - \frac{c_1^2}{2}\right)z^2 + \left(c_3 - c_1 c_2 + \frac{c_1^3}{4}\right)z^3 + \ldots\right] \tag{11}$$

and

$$v(w) = \frac{p_2(w)-1}{p_2(w)+1} = \frac{1}{2}\left[c_1 w + \left(c_2 - \frac{c_1^2}{2}\right)w^2 + \left(c_3 - c_1 c_2 + \frac{c_1^3}{4}\right)w^3 + \ldots\right]. \tag{12}$$

Using (11) and (12) together with (10), It is obvious that

$$\psi(u(z)) = 1 + \frac{B_1 c_1}{2} z + \left[\frac{1}{2}\left(c_2 - \frac{c_1^2}{2}\right)B_1 + \frac{1}{4}c_1^2 B_2\right]z^2 + \ldots \tag{13}$$

and

$$\psi(v(w)) = 1 + \frac{B_1 d_1}{2} w + \left[\frac{1}{2}\left(d_2 - \frac{d_1^2}{2}\right)B_1 + \frac{1}{4}d_1^2 B_2\right]w^2 + \ldots. \tag{14}$$

Next, by considering (8), (13) and 9 and (14), after some basic calculations, we arrive at

$$\frac{1}{2}B_1c_1 = \frac{1}{\gamma}\left(q(1+q\lambda)\right)a_2, \tag{15}$$

$$\frac{1}{2}\left(c_2-\frac{c_1^2}{2}\right)B_1+\frac{1}{4}c_1^2B_2=\frac{1}{\gamma}\left\{q(q+1)(1+q(q+1)\lambda)a_3-(1+q\lambda)(q(q+1)+\lambda-1)a_2^2\right\} \tag{16}$$

and

$$\frac{1}{2}B_1d_1 = -\frac{1}{\gamma}\left(q(1+q\lambda)\right)a_2, \tag{17}$$

$$\frac{1}{2}\left(d_2-\frac{d_1^2}{2}\right)B_1+\frac{1}{4}d_1^2B_2=\frac{1}{\gamma}\left\{\Big(2q(q+1)(1+q(q+1)\lambda)-q(1+q\lambda)^2\Big)a_2^2-q(q+1)(1+q(q+1)\lambda)a_3\right\}. \tag{18}$$

From Eq. (15) and Eq. (17), we get

$$c_1=-d_1. \tag{19}$$

Also, considering (16), (17), (18) and (19)

$$a_2^2=\frac{\gamma^2B_1^3(c_2+d_2)}{2\gamma\Big(q^2+1+\lambda(2q^4+3q^3-q^2+q-1)-\lambda^2q(q^2+1)\Big)B_1^2+4\big(q(1+q\lambda)\big)^2(B_1-B_2)} \tag{20}$$

which, in light of Lemma 1.1, we obtain

$$|a_2|^2\le\frac{2\left|\gamma\right|^2B_1^3}{\left|\gamma\Big(q^2+1+\lambda(2q^4+3q^3-q^2+q-1)-\lambda^2q(q^2+1)\Big)B_1^2+2\big(q(1+q\lambda)\big)^2(B_1-B_2)\right|}.$$

Since $B_1>0$, the last inequality is the desired estimate on $|a_2|$ stated in Theorem 2.1.

Next, we are going to obtain the upper bound on $|a_3|$. From (16), (17), (18) and (19) we have

$$\begin{aligned}&\left[q(1+q)(1+q(1+q)\lambda)\Big(q\big(2(1+q)(1+q(1+q)\lambda)-(1+q\lambda)^2\big)-(1+q\lambda)\big(q(1+q)+\lambda-1\big)\Big)\right]a_3\\&=\frac{\gamma B_1}{2}\left[q(2(1+q)(1+q\lambda(1+q))-(1+q\lambda)^2)c_2+(1+q\lambda)(q(1+q)+\lambda-1)d_2\right]\\&+\frac{\gamma d_1^2}{4}\left[(1+q\lambda)\big(q(1+q)+\lambda-1\big)+q\big(2(1+q)(1+q(1+q)\lambda)-(1+q\lambda)^2\big)\right](B_2-B_1).\end{aligned}$$

In view of Lemma 1.1, and for $0<q<1, 0\le\lambda\le1$ taking into account fact that $(\tau(q;\lambda)-\upsilon(q;\lambda))>0$, we get

$$|a_3|\le\frac{|\gamma|\left((\tau(q;\lambda)+\upsilon(q,\lambda))(B_1+|B_2-B_1|\right)}{q(1+q)(1+q(1+q)\lambda)((\tau(q;\lambda)-\upsilon(q,\lambda))}$$

where $\tau(q;\lambda)=q\big(2(1+q)(1+q(1+q)\lambda)-(1+q\lambda)^2\big)$ and $\upsilon(q;\lambda)=(1+q\lambda)(q(1+q)+\lambda-1)$.

Thus, we obtain the bound on $|a_3|$ stated in Theorem 2.1. □

Now we would like to draw attention to some remarkable results obtained for some values of λ, γ and ψ in Theorem 2.1.

Corollary 2.1. *Let the function f given by* (1) *be in the class $\mathscr{S}^{\star}_{q,\Sigma}(\gamma,\psi)$. Then*

$$|a_2|\le\frac{|\gamma|\,B_1\sqrt{2B_1}}{\sqrt{\left|\gamma(q^2+1)B_1^2+2q^2(B_1-B_2)\right|}}$$

and

$$|a_3|\le\frac{|\gamma|\,(3q^2+2q-1)(B_1+|B_2-B_1|)}{q(1+q)(1+q^2)}.$$

Corollary 2.2. *Let the function f given by* (1) *be in the class $\mathscr{C}_{q,\Sigma}(\gamma,\psi)$. Then*

$$|a_2| \leq \frac{|\gamma|\, B_1\sqrt{2B_1}}{\sqrt{q\left|\gamma(2q^3+2q^2-q+1)B_1^2+2(1+q)^2(B_1-B_2)\right|}}$$

and

$$|a_3| \leq \frac{|\gamma|\left((B_1+|B_2-B_1|\right)}{q^3(1+q)}.$$

Corollary 2.3. *If the function f given by* (1) *is of the class $\mathscr{S}^{\star}_{q,\Sigma}[\lambda,\alpha]$ of $q-$bi$-\lambda-$spirallike univalent functions of order α. Then*

$$|a_2| \leq \frac{2\sqrt{(1-\alpha)\cos\lambda}}{\sqrt{1+q^2}}$$

and

$$|a_3| \leq \frac{2(3q^2+2q-1)(1-\alpha)\cos\lambda}{q(1+q)(1+q^2)}.$$

Corollary 2.4. *If the function f given by* (1) *is $q-$bi$-\lambda-$Robertson of order α, that is, $f\in\mathscr{C}_{q,\Sigma}[\lambda,\alpha]$, then*

$$|a_2| \leq \frac{2\sqrt{(1-\alpha)\cos\lambda}}{\sqrt{q(2q^3+2q^2-q+1)}}$$

and

$$|a_3| \leq \frac{2(1-\alpha)\cos\lambda}{q^3(1+q)}.$$

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