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LINEAR OPTIMIZATION METHOD ON SINGLE VALUED NEUTROSOPHIC SET AND ITS SENSITIVITY ANALYSIS

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Abstract. Recently, decision making problems has prompted extensive awareness, especially multi-attribute decision-making problem in single valued neutrosophic sets. Given the inherent characteristics of this case, a multi-attribute decision-making problem with a single valued neutrosophic sets(SVN-sets) is explored with both weights and attribute ratings expressed by single valued neutrosophic information. Firstly, some basic concepts concerning SVN-sets are reviewed for the subsequent analysis. Secondly, a linear optimization method of SVN-sets are developed to describe the sensitivity analysis of attribute weights which give changing intervals of attribute weights in which the ranking order of the alternatives is required to remain unchanging. Finally, we presented an illustrative example to show its applicability and effectiveness.

Keywords: Single valued neutrosophic set, linear optimization, sensitivity analysis, multi-attribute decision making.

AMS Subject Classification: 03B52, 03E72

1. Introduction

Since the increasing lack of knowledge or data about multi attribute decision-making (MADM) problems, decicion makers are more and more overwhelmed to make a sound decision. To model the uncertain information some set theory developed such fuzzy set theory [39], intuitionistic fuzzy set theory [1] and neutrosophic sets [31] introduced. The neutrosophic set theory which is characterized by a truth-membership degree, indeterminacy-membership degree and falsity-membership degree to describe the uncertainty and fuzziness more objectively than fuzzy set theory [39] and intuitionistic fuzzy set. Up to now, researches on neutrosophic set theory roughly fall into two groups: theory and application. A lot of work on the neutrosophic set theory has been done such as; on the theory [11, 12, 16, 17, 19, 21, 34] and on application [4, 5, 9, 18, 33, 20]. Also, Nguyen et al. [27] presented an application based on biomedical diagnoses, Liu et al. [25] developed some aggregation operators including score and accuracy functions and Peng et al. [30] proposed outranking approach for single-valued neutrosophic sets.

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Single valued neutrosophic numbers and their application to multi-criteria decision making problems proposed in [13, 37]. Based on the single valued neutrosophic numbers, various applications have been proposed for fusing neutrosophic number information such triangular neutrosophic numbers [2, 6, 23], trapezoidal neutrosophic numbers [3, 5, 10, 22] and interval trapezoidal neutrosophic numbers [8, 15]. During the last five years, the researchers are paying more attention to this neutrosophic numbers and have effectively applied it to the different situations in applications; on critical path problem[24], on Maclaurin symmetric mean operators[26, 35], on power aggregation operators[36], on VIKOR method [29], on intuitionistic fuzzy multi objective LPP into LCP [28] and so on.

Li[14] gave sensitivity analysis and a linear weighted averaging method based on intuitionistic fuzzy sets. Neutrosophic sets sensitivity analysis based on linear weighted averaging method is yet to appear in the literature. Therefore, in this study we developed a method and sensitivity analysis by expanding linear weighted averaging method and sensitivity analysis of intuitionistic fuzzy sets [14] to neutrosophic sets.

2. Preliminaries

We start by introducing the concepts that are connected with the present paper.

Definition 2.1. [39] *Let $X = \{x_1, x_2, ..., x_n\}$ be a fixed set. A fuzzy set K in X is an object having the form*

$$K = \{\mu_X(x)/x : x \in X\}$$

which is characterized by a function: membership function $\mu_X : X \to [0,1]$ with the condition for all $x \in X$.

Definition 2.2. [1] *Let $X = \{x_1, x_2, ..., x_n\}$ be a fixed set. A intuitionistic fuzzy set L in X is an object having the form*

$$L = \{\langle x, \mu_L(x), \gamma_L(x)\rangle : \ x \in X\}$$

which are characterized by two functions: membership function $\mu_L : X \to [0,1]$ and non-membership function $\gamma_L : X \to [0,1]$, with the condition $0 \le \mu_L(x) + \gamma_L(x) \le 1$, for all $x \in X$.

Definition 2.3. [34] *Let $X = \{x_1, x_2, ..., x_n\}$ be a fixed set. A single valued neutrosophic set(SVN-set) A in X is an object having the form*

$$A = \{\langle x, (T_A(x), I_A(x), F_A(x))\rangle : x \in X\}.$$

which are characterized by three functions: truth-membership function $T_A : X \to [0,1]$, indeterminacy-membership function $I_A : X \to [0,1]$ and falsity-membership function $F_A : X \to [0,1]$, with the condition $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$, for all $x \in X$.

Peng et al. [30] and Ye [38] gave the operations of SVN-sets as: Assume that A and B be two SVN-sets. Then

(1)
$$A + B = \{\langle x, (T_A(x) + T_B(x) - T_A(x)T_B(x), I_A(x)I_B(x), F_A(x)F_B(x))\rangle : x \in X\}$$

(2)
$$A.B = \{\langle x, (T_A(x)T_B(x), I_A(x)+I_B(x)-I_A(x)I_B(x), F_A(x)+F_B(x)-F_A(x)F_B(x))\rangle : x \in X\}$$

(3)
$$\xi A = \{\langle x, (1-(1-T_A(x))^{\xi}, I_A(x)^{\xi}, F_A(x)^{\xi})\rangle : x \in X\}$$

(4)
$$A^{\xi} = \langle \{x, (T_A(x)^{\xi}, 1-(1-I_A(x))^{\xi}, 1-(1-F_A(x))^{\xi})\rangle : x \in X\}$$
where $\xi \in R$.

For convenience, [30] used the notation $\langle T, I, F\rangle$ instead of $\langle x, (T_A(x), I_A(x), F_A(x))\rangle$ for a single valued neutrosophic element of $x \in X$.

Definition 2.4. [30] *Let $X = \{x_1, x_2, ..., x_n\}$ be a set of alternatives, $U = \{o_1, o_2, ..., o_m\}$ be the set of attributes. The ratings (or evaluations) of alternatives $x_j \in X (j = 1, 2, ..., n)$ on attributes $o_i \in U$ are expressed with SVN-number $A_{ij} = \langle T_{ij}, I_{ij}, F_{ij}\rangle$. Then*

$$[A_{ij}]_{m\times n} = \begin{array}{c} \\ o_1 \\ o_2 \\ \vdots \\ o_m \end{array} \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & \cdots & x_n \end{array} \\ \begin{pmatrix} \langle T_{11}, I_{11}, F_{11}\rangle & \langle T_{12}, I_{12}, F_{12}\rangle & \cdots & \langle T_{1n}, I_{1n}, F_{1n}\rangle \\ \langle T_{21}, I_{21}, F_{21}\rangle & \langle T_{22}, I_{22}, F_{22}\rangle & \cdots & \langle T_{2n}, I_{2n}, F_{2n}\rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle T_{m1}, I_{m1}, F_{m1}\rangle & \langle T_{m2}, I_{m2}, F_{m2}\rangle & \cdots & \langle T_{mn}, I_{mn}, F_{mn}\rangle \end{pmatrix} \end{array}$$

is called a decision making matrix.

By using [32]*, If we get weighted vector of attribute set U as*
$$\omega = (\omega_1, \omega_2, ..., \omega_m) = (\langle \alpha_1, \beta_1, \gamma_1\rangle, \langle \alpha_2, \beta_2, \gamma_2\rangle, ..., \langle \alpha_m, \beta_m, \gamma_m\rangle)$$
then, we present weighted decision making matrix $[\bar{A}_{ij}]_{m\times n} = \omega[A_{ij}]_{m\times n}$ as;

$$[\bar{A}_{ij}]_{m\times n} = \begin{array}{c} \\ o_1 \\ o_2 \\ \vdots \\ o_m \end{array} \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & \cdots & x_n \end{array} \\ \begin{pmatrix} \langle \bar{T}_{11}, \bar{I}_{11}, \bar{F}_{11}\rangle & \langle \bar{T}_{12}, \bar{I}_{12}, \bar{F}_{12}\rangle & \cdots & \langle T_{1n}, \bar{I}_{1n}, \bar{F}_{1n}\rangle \\ \langle \bar{T}_{21}, \bar{I}_{21}, \bar{F}_{21}\rangle & \langle \bar{T}_{22}, \bar{I}_{22}, \bar{F}_{22}\rangle & \cdots & \langle T_{2n}, \bar{I}_{2n}, \bar{F}_{2n}\rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle \bar{T}_{m1}, \bar{I}_{m1}, \bar{F}_{m1}\rangle & \langle \tilde{T}_{m2}, \bar{I}_{m2}, \bar{F}_{m2}\rangle & \cdots & \langle T_{mn}, \bar{I}_{mn}, \bar{F}_{mn}\rangle \end{pmatrix} \end{array}$$

where

$$\langle \bar{T}_{ij}, \bar{I}_{ij}, \bar{F}_{ij}\rangle = \omega_i A_{ij} = \langle \alpha_i, \beta_i, \gamma_i\rangle\langle T_{ij}, I_{ij}, F_{ij}\rangle = \langle \alpha_i T_{ij}, \beta_i + I_{ij} - \beta_i I_{ij}, \gamma_i + F_{ij} - \gamma_i F_{ij}\rangle$$

Based on arithmetic average operator of Ye [38] *we defined comprehensive evaluation of each alternative $x_j \in X(j = 1, 2, ..., n)$, denoted V_j, is given by;*

$$V_j = \sum_{i=1}^{m} \langle \bar{T}_{ij}, \bar{I}_{ij}, \bar{F}_{ij}\rangle = \langle T_j, I_j, F_j\rangle$$

Then Liu et al. [25] *proposed score and accuracy function to compare two alternatives as;*

(1) *score function of V_j (j=1,2,...,n), denoted $s(V_j)$, defined as;*
$$s(V_j) = 2 + T_j - F_j - I_j$$
(2) *accuracy function of V_j (j=1,2,...,n), denoted $a(V_j)$, defined as;*
$$a(V_j) = T_j - F_j$$
and then for $s, t \in \{1, 2, ..., n\}$,

(a) *If $s(V_s) < s(V_t)$, then V_s is smaller than V_t, denoted by $V_s < V_t$*

(b) *If $s(V_s) = s(V_t)$;*

(i) *If $a(V_s) < a(V_t)$, then V_t is smaller than V_s, denoted by $V_t < V_s$*

(ii) *If* $s(V_t) = s(V_s)$*, then* V_t *and* V_s *are the same, denoted by* $V_t = V_s$

3. Sensitivity analysis

In this section, we present a method is called sensitivity analysis by inspiration from Li [14].

Definition 3.1. ***(Sensitivity analysis)*** *Let* $[A_{ij}]_{m\times n}$ *be a decision making matrix,* $\omega = (\omega_1, \omega_2, ..., \omega_m) = (\langle\alpha_1, \beta_1, \gamma_1\rangle, \langle\alpha_2, \beta_2, \gamma_2\rangle, ..., \langle\alpha_m, \beta_m, \gamma_m\rangle)$ *be a weighted vector and* $\omega' = (\langle\alpha_1, \beta_1, \gamma_1\rangle, \langle\alpha_2, \beta_2, \gamma_2\rangle, ..., \langle\alpha_k+\Delta\alpha_k, \beta_k+\Delta\beta_k, \gamma_k+\Delta\gamma_k\rangle, ..., \langle\alpha_m, \beta_m, \gamma_m\rangle)^T$ *be a changed weighted vector where* $\Delta\alpha_k$*,* $\Delta\beta_k$ *and* $\Delta\gamma_k$ *are increments of* α_k*,* β_k *and* γ_k*, respectively. Then, comprehensive evaluation* V_j *of the alternative* x_j *is given as:*

$$\begin{aligned} V_j &= \textstyle\sum_{i=1,i\neq k}^{m} \omega_i A_{ij} + \omega_k A_{kj} \\ &= \langle x_j, y_j, z_j\rangle + \langle \alpha_k T_{kj}, \beta_k + I_{kj} - \beta_k I_{kj}, \gamma_k + F_{kj} - \gamma_k F_{kj}\rangle \\ &= \langle x_j + \alpha_k T_{kj} - x_j\alpha_k T_{kj}, y_j(\beta_k + I_{kj} - \beta_k I_{kj}), z_j(\gamma_k + F_{kj} - \gamma_k F_{kj})\rangle \end{aligned}$$

where

$$\langle x_j, y_j, z_j\rangle = \sum_{i=1,i\neq k}^{m} \omega_i A_{ij}$$

and

$$\omega_k A_{kj} = \langle\alpha_k, \beta_k, \gamma_k\rangle\langle T_{kj}, I_{kj}, F_{kj}\rangle = \langle\alpha_k T_{kj}, \beta_k + I_{kj} - \beta_k I_{kj}, \gamma_k + F_{kj} - \gamma_k F_{kj}\rangle$$

Therefore, we have:

$$\begin{aligned} T_j =&\ x_j + \alpha_k T_{kj} - x_j\alpha_k T_{kj}, \\ I_j =&\ y_j(\beta_k + I_{kj} - \beta_k I_{kj}) \\ &\text{and} \\ F_j =&\ z_j(\gamma_k + F_{kj} - \gamma_k F_{kj}). \end{aligned}$$

Likewise, the changed comprehensive evaluation V_j' of the alternative x_j with the weight change of the attribute o_k can be calculated as follows;

$$\begin{aligned} V_j' =&\ \langle x_j, y_j, z_j\rangle + \langle(\alpha_k + \Delta\alpha_k)T_{kj}, \beta_k + \Delta\beta_k + \\ &I_{kj} - (\beta_k + \Delta\beta_k)I_{kj}, \gamma_k + \Delta\gamma_k + F_{kj} - (\gamma_k + \Delta\gamma_k)F_{kj}\rangle \\ =&\ \langle x_j + \alpha_k T_{kj} + \Delta\alpha_k T_{kj} - x_j\alpha_k T_{kj} - x_j\Delta\alpha_k T_{kj}, y_j(\beta_k + I_{kj} - \beta_k I_{kj}) + \\ &y_j(\Delta\beta_k - \Delta\beta_k I_{kj}), z_j(\gamma_k + F_{kj} - \gamma_k F_{kj}) + z_j(\Delta\gamma_k - \Delta\gamma_k F_{kj})\rangle \\ =&\ \langle T_j + \Delta\alpha_k T_{kj}(1 - x_j), I_j + \Delta\beta_k y_j(1 - I_{kj}), F_j + \Delta\gamma_k z_j(1 - F_{kj})\rangle \end{aligned}$$

where

$$\begin{aligned} \omega_k A_{kj} =&\ \langle\alpha_k + \Delta\alpha_k, \beta_k + \Delta\beta_k, \gamma_k + \Delta\gamma_k\rangle\langle T_{kj}, I_{kj}, F_{kj}\rangle \\ =&\ \langle(\alpha_k + \Delta\alpha_k)T_{kj}, \beta_k + \Delta\beta_k + I_{kj} - (\beta_k + \Delta\beta_k)I_{kj}, \\ &\gamma_k + \Delta\gamma_k + F_{kj} - (\gamma_k + \Delta\gamma_k)F_{kj}\rangle \end{aligned}$$

Similarly, the changed comprehensive evaluations V_s' and V_t' of the alternatives x_s and x_t with the weight change of the attribute o_k is given as:

$$\begin{aligned} V_s' =&\ \langle x_s, y_s, z_s\rangle + \langle(\alpha_k + \Delta\alpha_k)T_{ks}, \beta_k + \Delta\beta_k + I_{ks} - (\beta_k + \Delta\beta_k)I_{ks}, \\ &\gamma_k + \Delta\gamma_k + F_{ks} - (\gamma_k + \Delta\gamma_k)F_{ks}\rangle \\ =&\ \langle T_s + \Delta\alpha_k T_{ks}(1 - x_s), I_s + \Delta\beta_k y_s(1 - I_{ks}), F_s + \Delta\gamma_k z_s(1 - F_{ks})\rangle \end{aligned}$$

and

$$\begin{aligned} V_t' =&\ \langle x_t, y_t, z_t\rangle + \langle(\alpha_k + \Delta\alpha_k)T_{kt}, \beta_k + \Delta\beta_k + I_{kt} - (\beta_k + \Delta\beta_k)I_{kt}, \\ &\gamma_k + \Delta\gamma_k + F_{kt} - (\gamma_k + \Delta\gamma_k)F_{kt}\rangle \\ =&\ \langle T_t + \Delta\alpha_k T_{kt}(1 - x_t), I_t + \Delta\beta_k y_t(1 - I_{kt}), F_t + \Delta\gamma_k z_t(1 - F_{kt})\rangle \end{aligned}$$

respectively, where

$$\begin{array}{rl} T_s = & x_s + \alpha_k T_{ks} - x_s \alpha_k T_{ks}, \\ I_s = & y_s(\beta_k + I_{ks} - \beta_k I_{ks}) \\ F_s = & z_s(\gamma_k + F_{ks} - \gamma_k F_{ks}), \\ T_t = & x_t + \alpha_k T_{kt} - x_t \alpha_k T_{kt}, \\ I_t = & y_t(\beta_k + I_{kt} - \beta_k I_{kt}) \\ and & \\ F_t = & z_t(\gamma_k + F_{kt} - \gamma_k F_{kt}). \end{array}$$

Then, we can calculate the scores of $V_j^{'}$, $V_s^{'}$, and $V_t^{'}$ as follows:

$$\begin{array}{ll} s(V_j^{'}) & = 2 + T_j - I_j - F_j + \Delta\alpha_k T_{kj}(1 - x_j) - \Delta\beta_k y_j(1 - I_{kj}) - \Delta\gamma_k z_j(1 - F_{kj}) \\ s(V_s^{'}) & = 2 + T_s - I_s - F_s + \Delta\alpha_k T_{ks}(1 - x_s) - \Delta\beta_k y_s(1 - I_{ks}) - \Delta\gamma_k z_s(1 - F_{ks}) \\ s(V_t^{'}) & = 2 + T_t - I_t - F_t + \Delta\alpha_k T_{kt}(1 - x_t) - \Delta\beta_k y_t(1 - I_{kt}) - \Delta\gamma_k z_t(1 - F_{kt}) \end{array}$$

Also, we can obtain the accuracies of $V_j^{'}$, $V_s^{'}$, and $V_t^{'}$ as follows:

$$\begin{array}{ll} a(V_j^{'}) & = T_j - F_j + \Delta\alpha_k T_{kj}(1 - x_j) - \Delta\gamma_k z_j(1 - F_{kj}) \\ a(V_s^{'}) & = T_s - F_s + \Delta\alpha_k T_{ks}(1 - x_s) - \Delta\gamma_k z_s(1 - F_{ks}) \\ a(V_t^{'}) & = T_t - F_t + \Delta\alpha_k T_{kt}(1 - x_t) - \Delta\gamma_k z_t(1 - F_{kt}) \end{array}$$

Soppose that the ranking the alternatives x_j, x_s and x_t is $x_j > x_s > x_t$. When the weight ω_t of the attribute o_k is changed to $\omega_t^{'}$, if the ranking order of the alternatives x_j, x_s and x_t are required to remain unchanging, then $V_j^{'}$, $V_s^{'}$ and $V_t^{'}$ should satisfy either

(1) $s(V_j^{'}) > s(V_s^{'})$ and $s(V_s^{'}) > s(V_t^{'})$
or
(2) $s(V_j^{'}) = s(V_s^{'})$, $s(V_s^{'}) = s(V_t^{'})$, $a(V_j^{'}) > a(V_s^{'})$,and $a(V_s^{'}) > a(V_t^{'})$.

Therefore, we have following inequalities;

(1)

$$\begin{array}{l} s(V_j^{'}) > s(V_s^{'}) \\ s(V_s^{'}) > s(V_t^{'}) \\ 0 \le \alpha_k + \Delta\alpha_k + \beta_k + \Delta\beta_k + \gamma_k + \Delta\gamma_k \le 3, \\ 0 \le \alpha_k + \Delta\alpha_k \le 1 \\ 0 \le \beta_k + \Delta\beta_k \le 1 \\ 0 \le \gamma_k + \Delta\gamma_k \le 1 \end{array}$$

(2)

$$\begin{array}{l} s(V_j^{'}) = s(V_s^{'}) \\ s(V_s^{'}) = s(V_t^{'}) \\ a(V_j^{'}) > a(V_s^{'}) \\ a(V_s^{'}) > a(V_t^{'}) \\ 0 \le \alpha_k + \Delta\alpha_k + \beta_k + \Delta\beta_k + \gamma_k + \Delta\gamma_k \le 3, \\ 0 \le \alpha_k + \Delta\alpha_k \le 1 \\ 0 \le \beta_k + \Delta\beta_k \le 1 \\ 0 \le \gamma_k + \Delta\gamma_k \le 1 \end{array}$$

and

Solving either 1. or 2., we can obtain the changing ranges $\Delta\alpha_k$, $\Delta\beta_k$ and $\Delta\gamma_k$ of the weight ω_k of the attribute o_k. Namely, if the weight ω_k takes any value between $\langle\alpha_k, \beta_k, \gamma_k\rangle$; and $\langle\alpha_k+\Delta\alpha_k, \beta_k+\Delta\beta_k, \gamma_k+\Delta\gamma_k\rangle$, then, the ranking order of the alternatives still remains unchanging.

4. A Linear Optimization Method based on sensitivity analysis of SVN-sets

In this section, we give a method, which is called linear weighted averaging method, for sensitivity analysis of SVN-weights of the attributes;

Let $X = \{x_1, x_2, ..., x_n\}$ be a set of alternatives, $O = \{o_1, o_2, ..., o_m\}$ be the set of attributes.

Algorithm:

***Step 1*:** Input decision making matrix $[A_{ij}]_{m\times n}$;

***Step 2*:** Determine the weighted vector for attributes $\omega = (\langle \alpha_i, \beta_i, \gamma_i\rangle)_{mx1}$

***Step 3*:** Find the weighted decision making matrix $[\bar{A}_{ij}]_{m\times n}$;

***Step 4*:** Calculate $V_j = \sum_{i=1}^{m} \langle \bar{T}_{ij}, \bar{I}_{ij}, \bar{F}_{ij}\rangle = \langle T_j, I_j, F_j\rangle$ of the alternatives $x_j \in X(j = 1, 2, ..., n)$;

***Step 5*:** Rank the alternatives by using score and accuracy functions based $V_j(j = 1, 2, ..., n)$ according to Definition 2.4;

***Step 6*:** Compute the sensitivity analysis of weights of the attributes in the linear optimization method based Definition 3.1;

5. Application

Asssume that $X = \{x_1, x_2, x_3\}$ be a set of alternatives and $O = \{o_1, o_2, o_3\}$ be the set of attributes. Then, a decision maker wants to select the best alternative considering three attribute. Therefore decision progress is given as;

***Step 1*:** Decision making matrix $[A_{ij}]_{3\times 3}$ entered as;

$$[A_{ij}]_{3\times 3} = \begin{pmatrix} \langle 0.7, 0.1, 0.8\rangle & \langle 0.7, 0.6, 0.8\rangle & \langle 0.1, 0.4, 0.7\rangle \\ \langle 0.5, 0.2, 0.8\rangle & \langle 0.4, 0.2, 0.3\rangle & \langle 0.2, 0, 1, 0.9\rangle \\ \langle 0.1, 0.1, 0.6\rangle & \langle 0.8, 0.5, 0.4\rangle & \langle 0, 6, 0.3, 0.7\rangle \end{pmatrix}$$

***Step 2*:** The weighted vector is determined as

$$\omega = (\langle 0.2, 0.9, 0.8\rangle, \langle 0.8, 0.4, 0.9\rangle, \langle 0.7, 0.6, 0.3\rangle)$$

***Step 3*:** Weighted decision making matrix $[\bar{A}_{ij}]_{3\times 3}$ found as;

$$[\bar{A}_{ij}]_{3\times 3} = \begin{pmatrix} \langle 0.14, 0.91, 0.96\rangle & \langle 0.14, 0.96, 0.96\rangle & \langle 0.02, 0.94, 0.94\rangle \\ \langle 0.40, 0.52, 0.98\rangle & \langle 0.32, 0.52, 0.93, \rangle & \langle 0.16, 0, 46, 0.99\rangle \\ \langle 0.07, 0.64, 0.72\rangle & \langle 0.56, 0.80, 0.58\rangle & \langle 0, 42, 0.72, 0.79\rangle \end{pmatrix}$$

***Step 4*:** The V_j of the alternatives $x_j \in X(j = 1, 2, ..., n)$ calculated as;

$$\begin{aligned} V_1 &= \langle 1-(1-0.14)(1-0.40)(1-0.07), 0.91\times 0.52\times 0.64, 0.96\times 0.98\times 0.72\rangle \\ &= \langle 0.52012, 0.30285, 0.67738\rangle \end{aligned}$$

$$\begin{aligned} V_2 &= \langle 1-(1-0.14)(1-0.32)(1-0.56), 0.96\times 0.52\times 0.80, 0.96\times 0.93\times 0.58\rangle \\ &= \langle 0.74269, 0.39936, 0.51782\rangle \end{aligned}$$

and

$$\begin{aligned} V_3 &= \langle 1-(1-0.02)(1-0.16)(1-0.42), 0.94\times 0.46\times 0.72, 0.94\times 0.99\times 0.79\rangle \\ &= \langle 0.52254, 0.31133, 0.73517\rangle \end{aligned}$$

respectively.

***Step 5*:** The scores of $V_j (j = 1, 2, 3)$ are calculated as;

$$s(V_1) = 1.53990$$
$$s(V_2) = 1.82550$$
$$and$$
$$s(V_3) = 1.47604$$

respectively. Then we have get $x_2 > x_1 > x_3$.

***Step 6*:** We computed the sensitivity analysis of weight ω_2 of the attribute o_2 in the linear optimization method based Definition 3.1 as;

(Similarly, the analysis can be make for ω_1 and ω_3)

Firstly, we assume that only weight $\omega_2 = \langle \alpha_2, \beta_2, \gamma_2 \rangle$ of the attribute o_2 is changed to the weight $\bar{\omega}_2 = \langle \alpha_2 + \Delta\alpha_2, \beta_2 + \Delta\beta_2, \gamma_2 + \Delta\gamma_2 \rangle$ and the weights of other attributes $o_i (i = 1, 3)$ remain the same as the original weights ω_1 and ω_3. Then, we have the systems of inequalities as follows:

$$\begin{array}{l} s(V_2') > s(V_1') \\ s(V_1') > s(V_3') \\ 0 \le \alpha_2 + \Delta\alpha_2 + \beta_2 + \Delta\beta_2 + \gamma_2 + \Delta\gamma_2 \le 3, \\ 0 \le 0.8 + \Delta\alpha_2 \le 1 \\ 0 \le 0.4 + \Delta\beta_2 \le 1 \\ 0 \le 0.9 + \Delta\gamma_2 \le 1 \end{array}$$

where

$$\begin{aligned} s(V_1') &= 2 + T_1 - I_1 - F_1 + \Delta\alpha_2 T_{21}(1 - x_1) - \Delta\beta_2 y_1 (1 - I_{21}) - \Delta\gamma_2 z_1 (1 - F_{21}) \\ &= 1.35176 + 0.31992\Delta\alpha_2 - 0.27955\Delta\beta_2 - 0.01382\Delta\gamma_2 \\ s(V_2') &= 2 + T_2 - I_2 - F_2 + \Delta\alpha_2 T_{22}(1 - x_2) - \Delta\beta_2 y_2 (1 - I_{22}) - \Delta\gamma_2 z_2 (1 - F_{22}) \\ &= 1.38793 + 0.12109\Delta\alpha_2 - 0.36864\Delta\beta_2 - 0.03898\Delta\gamma_2 \\ s(V_3') &= 2 + T_3 - I_3 - F_3 + \Delta\alpha_2 T_{23}(1 - x_3) - \Delta\beta_2 y_3 (1 - I_{23}) - \Delta\gamma_2 z_3 (1 - F_{23}) \\ &= 1.30498 + 0.09094\Delta\alpha_2 - 0.36547\Delta\beta_2 - 0.00743\Delta\gamma_2 \end{aligned}$$

$$\begin{aligned} T_1 &= 0.45614 \\ I_1 &= 0.41467 \\ F_1 &= 0.68982 \\ T_2 &= 0.71847 \\ I_2 &= 0.46694 \\ F_2 &= 0.86360 \\ T_3 &= 0.50436 \\ I_3 &= 0.45752 \\ F_3 &= 0.74186 \end{aligned}$$

which can be simplified into the system of inequalities as follows:

$$\begin{array}{c} 0.03628 - 0.19883\Delta\alpha_2 - 0.08909\Delta\beta_2 - 0.02515\Delta\gamma_2 > 0 \\ 0,04667 + 0,22898\Delta\alpha_2 + 0,08592\Delta\beta_2 - 0,00640\Delta\gamma_2 > 0 \\ -0.8 \le \Delta\alpha_2 \le 0.2 \\ -0.4 \le \Delta\beta_2 \le 0.6 \\ -0.9 \le \Delta\gamma_2 \le 0.1 \end{array}$$

Some solutions of the system is given by Fig. 1.

Likewise,we assume that only weight $\omega_1 = \langle \alpha_1, \beta_1, \gamma_1 \rangle$ of the attribute o_1 is changed to the weight $\bar{\omega}_1 = \langle \alpha_1 + \Delta\alpha_1, \beta_1 + \Delta\beta_1, \gamma_1 + \Delta\gamma_1 \rangle$ and the weights of other attributes

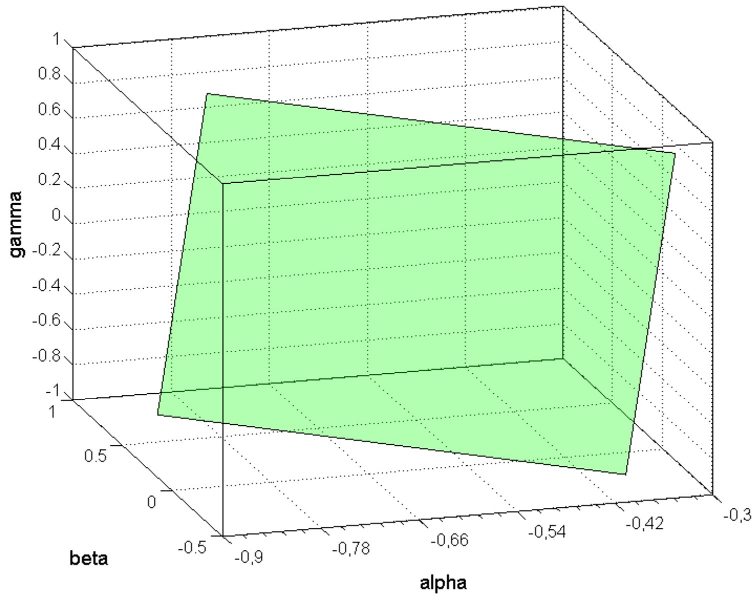

FIGURE 1. Some solutions of the system

$o_i(i = 2, 3)$ remain the same as the original weights or that only weight $\omega_3 = \langle \alpha_3, \beta_3, \gamma_3 \rangle$ of the attribute o_3 is changed to the weight $\bar{\omega}_3 = \langle \alpha_3 + \Delta\alpha_3, \beta_3 + \Delta\beta_3, \gamma_3 + \Delta\gamma_3 \rangle$ and the weights of other attributes $o_i(i = 1, 2)$ remain the same as the original weights, then the solutions can easily be made in a similar way for o_1 and o_3.

6. Conclusion

In this study, we proposed a a linear optimization method of SVN-sets to describe the sensitivity analysis of attribute weights. Also we an application which show its applicability and effectiveness. Finally, we applied our proposed method to a multi-attribute decision-making problem to demonstrate its feasibility and stability in solution. Since our paper still has some limitations, in future studies we will study on different methods by combining other objective methods for determining criteria weights in netrosophic sets.

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