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A PRE-SUBADDITIVE FUZZY MEASURE MODEL AND ITS THEORETICAL INTERPRETATION

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Abstract. In this paper a particular set function which depends on densities of singletons with interdependence coefficients and which provides redundancy among singletons is considered. The Möbius representation of this function is obtained. Then a necessary and sufficient condition is presented to attain a fuzzy measure from this set function.

Keywords: Fuzzy measure, multicriteria desicion making, pre-subadditive set function, Möbius representation.

AMS Subject Classification: 28E10, 28A25, 90B50

1. Introduction

Multicriteria decision making (MCDM) is a discipline that helps decision makers who face with conflicting alternatives to make the best decision. Since the decision makers only need to choose the alternative with the highest preference rating, decision making is extremely intuitive when decision makers consider single criterion problems whereas many problems, such as weights of criteria, preference dependence and conflicts among criteria, seem to complicate the problems and need to be overcome by more sophisticated methods as decision makers evaluate alternatives with multicriteria [26]. One of these methods is to study in fuzzy environment. MCDM problems in fuzzy environment have been studied since Bellman and Zadeh [1] proposed the concepts of decision making under fuzzy environments. In fact, making decision within a fuzzy environment to fit better many real-world applications has come into prominence.

Fuzzy measure theory is an innovate and useful tool to model the interaction of criteria in MCDM problems. There are some remarkable studies that have handled fuzzy measure theory in MCDM problems [7, 8, 13, 14, 17, 23, 25, 27, 30]. Besides, several authors have studied the identification of fuzzy measure (see e.g [9, 15, 18, 19, 21, 22, 29]). As identifying a fuzzy measure, determining the measure of each one of 2^n subsets of a finite set that has

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cardinality n is not an easy process. Grabisch [10] has proposed the concept of k-order additive fuzzy measure in order to overcome this complexity. In [15], to relieve the same complexity, a fuzzy measure which is developed based on the evaluation of interdependence coefficients between criteria has been identified. Moreover, the studies [3, 4, 6, 16, 20, 28] deal with the identification of fuzzy measure by using evolutionary or genetic algorithm.

In [15], the authors aim to identify a fuzzy measure that covers synergy among singletons. For this purpose they have mapped each subset that contains at least two points to the sum of the densities of included singletons and a nonnegative value. This nonnegative real number is obtained by choosing the maximum of the interdependence coefficients of each pair of the subset. On the other hand the structure of the set function, in fact the positivity of these coefficients, has been referring the synergy among criteria. Furthermore, the positivity again and the process of taking maximum are fortunately convenient with the monotonicity. In other words, since the densities of singletons and the interdependence coefficients are nonnegative the structure of the set function is suitable for the monotonicity and superadditivity. In contrast to this case obtaining a fuzzy measure that refers redundancy between criteria is not such an easy process.

Redundancy arises once any two criteria partially comprise each other in a MCDM problem. For instance, in an electronic equipment selection problem there may be redundancy among some criteria such as surface quality, stainlessness and water resistance. We illustrate this situation with another example at the end of Section 2. In this paper, following the idea of [15] we study a particular set function that depends on the densities of singletons and interdependence coefficients and that considers redundancy between criteria. We investigate the Möbius representation of this set function and we compare the results about Möbius representation obtained in this paper with some previous studies that consider the characterization of fuzzy measures. To get redundancy we use negative interdependence coefficients which makes the structure of the set function more complicated. The complexity of preserving monotonicity while organizing subadditivity enforces us to give conditions contrary to the superadditive case. In brief, the main goal of the present paper is to allay the difficulty of the determination of exponentially growth number of values theoretically in the case which considers redundancy between criteria.

Now let us recall some basic concepts about the fuzzy measure theory: Let X be a nonempty set and let 2^X be the class of all subsets of X. Then a set function μ over 2^X is said to be a fuzzy measure if

i) $\mu(\varnothing) = 0$ and $\mu(X) = 1$,

ii) $\mu(A) \leq \mu(B)$ whenever $A \subseteq B \subseteq X$ (monotonicity).

Recall that a set function μ is said to be

i) additive if $\mu(A \cup B) = \mu(A) + \mu(B)$,

ii) superadditive if $\mu(A \cup B) \geq \mu(A) + \mu(B)$,

iii) subadditive if $\mu(A \cup B) \leq \mu(A) + \mu(B)$,

whenever $A \cap B = \varnothing$ [8]. Moreover, if $\mu(\{x_i, x_j\}) \leq \mu(x_i) + \mu(x_j)$ for any $i, j = 1, 2, ..., n$ then we say that μ is pre-subadditive where $\mu(x) := \mu(\{x\})$ for any $x \in X$. It is clear that a subadditive set function is pre-subadditive but not conversely. Note that the superadditivity of a fuzzy measure refers to the synergy between criteria and the subadditivity of it refers to the redundancy [8]. Furthermore, observe that μ is monotone if and only if

$$\mu(A \cup \{x_j\}) \geq \mu(A) \tag{1}$$

for any $A \subset X$ and for any $j = 1, 2, ..., n$ with $x_j \notin A$.

It suffices to determine the measures of singletons to determine all combinations whenever the fuzzy measure is additive. However; if the fuzzy measure is not additive, 2^n subsets should be evaluated convenient to the definition of fuzzy measure.

In Section 2 after studying the Möbius representation, we obtain a necessary and sufficient condition for a particular set function to be a pre-subadditive fuzzy measure, so that the practical difficulty that requires the evaluation of the measures of exponentially growing numbers of subsets may be handled for problems in which there is redundancy among the singletons. Also the results are discussed and an explanatory numerical example is carried out to show the practicability of the theorem. Finally the study is concluded in Section 3.

2. Main results

In this section we deal with a set function which is pre-subadditive. Firstly, we study the Möbius representation of this function. Then we give a necessary and sufficient condition for this set function to be a fuzzy measure. Let $X = \{x_1, x_2, ..., x_n\}$ be a finite set and let $\mu : 2^X \to \mathbb{R}$ be a set function such that

$$\begin{cases} \mu(x_j) \geq 0, \text{ for all } 1 \leq j \leq n \\ \mu(G) = \sum\limits_{x_j \in G} \mu(x_j) + \min\limits_{x_i, x_j \in G, i \neq j} \lambda_{ij}, \text{ for all } G \in 2^X \text{ with } |G| \geq 2 \end{cases} \tag{2}$$

where $\{\lambda_{ij} = \lambda_{ji} : 1 \leq i, j \leq n, i \neq j\} \in [-1, 0]^{\binom{n}{2}}$. Considering each λ_{ij} as an interdependence coefficient, such a function can be used in MCDM problems in which there is redundancy between criteria. Moreover, note that if $|X| \leq 3$ such a measure is subbaditive.

The positivity of μ over singletons does not guarantee positivity over 2^X as well as $\mu(X)$ does not have to equal to 1 and such a set function does not need to be monotone. Therefore, we need some further results. For instance, some characterizations can be considered to achieve this problem (see e.g., [2, 11, 12]). One of these characterizations can be given with the help of the Möbius representation.

Let X be a finite set and let $f : 2^X \to \mathbb{R}$ be a set function. The Möbius representation m of f is a function $m : 2^X \to \mathbb{R}$ defined by

$$m(G) := \sum_{K \subset G} (-1)^{|G \setminus K|} f(K). \tag{3}$$

Now let us recall the corresponding characterization of Chateauneuf and Jaffray [2].

Theorem 2.1. *Let X be a finite set and let $\mu : 2^X \to \mathbb{R}$ be a set function. Then μ is a fuzzy measure if and only if $m(\varnothing) = 0$, $\sum\limits_{G \in 2^X} m(G) = 1$ and for any $G \in 2^X$ and for any $x \in G$*

$$\sum_{x \in K \subset G} m(K) \geq 0$$

where m is the Möbius representation of μ.

Now we give a proposition that gives the Möbius representation of a set function defined with (2).

Proposition 2.1. *Let $X = \{x_1, x_2, ..., x_n\}$ be a finite set, let μ be a set function on X defined with (2) with . Then we have*

$$m(\{x_i\}) = \mu(\{x_i\})$$

for any $i = 1, 2, ..., n$ and for any $K = \{x_{i_k}\}_{k=1}^{m} \subset X$ with $m \geq 2$ we have

$$m(K)=\sum_{j=2}^{m}\left((-1)^{m-j}\sum_{\substack{A\subset\tau(K)\\|A|=\binom{j}{2}}}\min A\right)$$

where $\tau(K)$ is the set of all interdependence coefficients of the elements of K. In particular, if $K=\{x_{i_k}\}_{k=1}^{m}$ is a subset of X such that

$$\lambda_{i_1,i_2}\leq\lambda_{i_1,i_3}\leq...\lambda_{i_1,i_m}\leq\lambda_{i_2,i_3}\leq\lambda_{i_2,i_4}\leq...\leq\lambda_{i_2,i_m}\leq...\leq\lambda_{i_{m-1},i_m}\tag{4}$$

then we get

$$m(K)=(-1)^{m}\sum_{k=1}^{m-1}\lambda_{i_k,i_m}.$$

Proof. It is obvious from the definition that $m(\{x_i\})=\mu(\{x_i\})$ holds for any for any $i=1,2,...,n$. Applying the formula (3) we get

$$\begin{aligned}
m(K)=&(-1)^{m-1}\sum_{k=1}^{m}\mu(x_{i_k})\\
&+(-1)^{m-2}\left(\binom{m-1}{1}\sum_{k=1}^{m}\mu(x_{i_k})+\sum_{\substack{A\subset\tau(K)\\|A|=\binom{2}{2}}}\min A\right)\\
&+(-1)^{m-3}\left(\binom{m-1}{2}\sum_{k=1}^{m}\mu(x_{i_k})+\sum_{\substack{A\subset\tau(K)\\|A|=\binom{3}{2}}}\min A\right)\\
&+...\\
&+(-1)^{m-m}\left(\binom{m-1}{m-1}\sum_{k=1}^{m}\mu(x_{i_k})+\sum_{\substack{A\subset\tau(K)\\|A|=\binom{m}{2}}}\min A\right)\\
&=\sum_{r=1}^{m}\left((-1)^{m-r}\binom{m-1}{r-1}\sum_{k=1}^{m}\mu(x_{i_k})\right)+\sum_{j=2}^{m}\left((-1)^{m-j}\sum_{\substack{A\subset\tau(K)\\|A|=\binom{j}{2}}}\min A\right).
\end{aligned}$$

It is easy to see that the first sum of the right hand side of the last equality is equal to zero. Hence the proof is completed. Moreover, let K be a subset such that (4) holds. Then it is easy to check that

$$\sum_{\substack{A\subset\tau(K)\\|A|=\binom{j}{2}}}\min A=\sum_{r=2}^{m+2-j}\sum_{k=r}^{m+2-j}\binom{m-k}{j-2}\lambda_{i_{r-1},i_k}.$$

Thus we have

$$
\begin{aligned}
m(K) &= \sum_{j=2}^{m}\left((-1)^{m-j}\sum_{\substack{A\subset\tau(K)\\ |A|=\binom{j}{2}}}\min A\right)\\
&= \sum_{j=2}^{m}\left((-1)^{m-j}\sum_{r=2}^{m+2-j}\sum_{k=r}^{m+2-j}\binom{m-k}{j-2}\lambda_{i_{r-1},i_k}\right)\\
&= (-1)^m\sum_{k=1}^{m-1}\lambda_{i_k,i_m}.
\end{aligned}
$$

□

Using Proposition 2.1 in Theorem 2.1 one can check whether a set function given with formula (2) is a fuzzy measure or not.

Now we give a new necessary and sufficient condition for functions defined with (2) to be monotone.

Theorem 2.2. *Let $X = \{x_1, x_2, ..., x_n\}$ be a finite set. The function $\mu : 2^X \to \mathbb{R}$ defined with (2) is monotone if and only if*

$$\mu(x_k) + \lambda_{ik} \geq 0 \tag{5}$$

for any $k = 1, 2, ..., n$ and for any $i \neq k$.

Proof. Assume that μ is monotone. Then we have for any $k = 1, 2, ..., n$ and for any $i \neq k$ that

$$\mu(\{x_k, x_j\}) \geq \mu(x_k). \tag{6}$$

On the other hand from the definition of μ we know that $\mu(\{x_k, x_j\}) = \mu(x_k)+\mu(x_k)+\lambda_{ik}$. Thus from (6) we have $\mu(x_k) + \lambda_{ik} \geq 0$.

For the converse of the statement assume that μ is not monotone. Then if we consider (1) we see that there exist $A \subset X$ and $1 \leq j_0 \leq n$ such that $x_{j_0} \notin A$ and

$$\mu\left(A \cup \{x_{j_0}\}\right) < \mu(A).$$

Therefore one can have

$$\mu(x_{j_0}) + \min_{x_i,x_j\in A\cup\left\{x_{j_0}\right\}} \lambda_{ij} - \min_{x_i,x_j\in A} \lambda_{ij} < 0. \tag{7}$$

As μ is positive now we can write from (7) that

$$\min_{x_i,x_j\in A\cup\left\{x_{j_0}\right\}} \lambda_{ij} < \min_{x_i,x_j\in A} \lambda_{ij}$$

which implies there exists $x_{i_0} \in A$ such that

$$\lambda_{i_0 j_0} = \min_{x_i,x_j\in A\cup\left\{x_{j_0}\right\}} \lambda_{ij}.$$

If we consider this fact in (7) we have

$$\mu(x_{j_0}) + \lambda_{i_0 j_0} < \min_{x_i,x_j\in A} \lambda_{ij} \leq 0.$$

Hence, (5) fails. □

The following is the promised characterization which is an immediate consequence of Theorem 2.2. Note that this condition does not depend on Möbius representation of the set function.

Corollary 2.1. *Let* $X = \{x_1, x_2, ..., x_n\}$ *be a finite set and let* $\mu : 2^X \to \mathbb{R}$ *be a set function defined as (2) such that* $\mu(\varnothing) = 0$ *and*

$$\sum_{k=1}^{n} \mu(x_k) = 1 - \min_{1 \le i,j \le n} \lambda_{ij}. \tag{8}$$

Then μ *is a fuzzy measure on* X *if and only if (5) holds.*

Proof. From (2) and (8) we see that $\mu(X) = 1$. Finally, Theorem 2.2 and the structure of μ guarantee that it is monotone and nonnegative. □

Remark 2.1. *After determining the interdependence coefficients the densities of singletons can be calculated such that*

$$\sum_{k=1}^{n} \mu(x_k) = 1 - \min_{1 \le i,j \le n} \lambda_{ij} \text{ and } \mu(x_k) \ge 0 \text{ for any } k. \tag{9}$$

In this paper we do not concentrate on determining the densities. In [15]*; as the nature of monotonicity is coherent with the proposed set function, any solution of the system (3) of* [15] *can be considered to identify a fuzzy measure that is superadditive for singletons whereas due to the structure of the set function given in the form (2) a solution of (9) may be considered to identify a fuzzy measure after checking the condition (5). To illustrate this fact the following example can be given. Note that there is redundancy among the criteria used in the example.*

Example 2.1. *Consider the following table of criteria which can be used for evaluating degree of the effect of the climate change of some selected regions that have similar climate. Each criterion is regarded for a fixed and favorable period of time.*

Table 1. Criteria

x_1	*Number of the drought years*
x_2	*Number of the years in which the rainfall is abnormal*
x_3	*Number of the natural disasters caused by weather*
x_4	*Number of the years in which average annual temperature is higher than normal*

As there is redundancy among criteria we can use a set function defined with (2). To construct the function we have two steps: The first step is determining the interdependence coefficients. Further information for this process can be found in [15]*. Now assume that the interdependence coefficients are determined as follows:*

Table 2. Interdependence coefficients

$\lambda_{1,2} = -0.1$	$\lambda_{1,3} = -0.03$	$\lambda_{1,4} = -0.07$
$\lambda_{2,3} = -0.08$	$\lambda_{2,4} = -0.1$	$\lambda_{3,4} = -0.08.$

Next step is determining the densities of singletons. For this purpose system (9) should be solved for the interdependence coefficients given above. As we mentioned before we omit *this step (see, e.g.* [15]*). It is obvious that the set*

$$\{g_1 = 0.32, g_2 = 0.4, g_3 = 0.23, g_4 = 0.15\}$$

is a solution of

$$\sum_{k=1}^{4} g_k = 1 - \min_{1\leq i,j\leq 4} \lambda_{ij}.$$

Now let define a set function $\mu : 2^X \to \mathbb{R}$ *by*

$$\mu(\varnothing) = 0,$$

$$\mu(x_j) = g_j,\ \ (j = 1, 2, 3, 4)$$

and

$$\mu(G) = \sum_{x_j\in G} \mu(x_j) + \min_{\substack{x_i,x_j\in G\\ i\neq j}} \lambda_{ij}.$$

It is obvious that (5) holds. Therefore μ *is a fuzzy measure over* X*. In fact, the following table can be checked:*

Table 3. Measures of subsets of X

$\mu(\varnothing) = 0$	$\mu(x_4) = 0.15$	$\mu(x_2, x_3) = 0.55$	$\mu(x_1, x_2, x_4) = 0.77$
$\mu(x_1) = 0.32$	$\mu(x_1, x_2) = 0.62$	$\mu(x_2, x_4) = 0.45$	$\mu(x_1, x_3, x_4) = 0.62$
$\mu(x_2) = 0.4$	$\mu(x_1, x_3) = 0.52$	$\mu(x_3, x_4) = 0.3$	$\mu(x_2, x_3, x_4) = 0.68$
$\mu(x_3) = 0.23$	$\mu(x_1, x_4) = 0.4$	$\mu(x_1, x_2, x_3) = 0.85$	$\mu(X) = 1.$

3. Conclusion

It is well known that constructing a fuzzy measure is a difficult process out of the exponential number of subsets. In this paper we encounter a more difficult process in which subadditivity is involved. We consider a particular set function which fundamentally depends on singletons and interdependence coefficients. First of all we calculate the Möbius representation of this set function. This result and Proposition 2 of [2] may be used in a MCDM problem which contains a set function that is given with (2). Moreover, we aim to give a new condition which does not depend on Möbius representation. The condition is easily testable and contributes to model the importance of criteria in MCDM problems or the importance of coalition of agents in games. We also give a numerical example to indicate the applicability of the corresponding theorem. In the example, we show that a set function satisfies the condition of Theorem 2.2 and therefore it is a fuzzy measure that is pre-subadditive. Consequently, all these arguments stress that with the help of the condition (5) one can check whether a particular set function is a fuzzy measure or not. Simultaneously, should such a measure is used as a tool in fuzzy integrals [5, 7, 8, 13, 24], this paper is a loadstar for decision makers in corresponding area.

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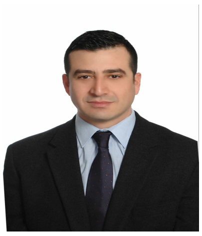

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