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ON (a, d)-EAT LABELING OF SUBDIVISION OF TREES

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Abstract. An (a, d)-edge antimagic total (EAT) labeling on a graph Γ with p vertices and q edges is a one-to-one function ψ from $V(\Gamma) \cup E(\Gamma)$ onto the set of integers $1, 2, ...p+q$ with the property that for each edge uv, the set $\{\psi(u) + \psi(uv) + \psi(v) : uv \in E(\Gamma)\}$ form an arithmetic progression (A. P.) starting with a and having common difference d, where $a > 0$ and $d \geq 0$ fixed integers. A (a, d)-EAT labeling is called super (a, d)-EAT labeling if the smallest numbers are labels to the vertices. In this paper, we have to show that the graph of the subdivided star and subdivided caterpillar are super (a, d)-EAT labeling.

Keywords: subdivided stars, subdivided caterpillars, super (a, d)-EAT labeling.

AMS Subject Classification: 05C78

1. Introduction and Preliminaries

In this paper all graphs are finite, simple, undirected. The graph Γ has vertex set $V(\Gamma)$ and edge $E(\Gamma)$ and we let $|V(\Gamma)| = p$ and $|E(\Gamma)| = q$. For integer $m \leq n$, we use $[m, n]$ to denote $\{m, m+1, ..., n\}$. A general reference for graph-theoretic ideas can be seen in [16]. An (a, d)-EAT labeling on a graph Γ with p vertices and q edges is a one-to-one function ψ from $V(\Gamma) \cup E(\Gamma)$ onto the set of integers $[1, p+q]$ with the property that for each edge uv, the set $\{\psi(u) + \psi(uv) + \psi(v) : uv \in E(\Gamma)\}$ form an arithmetic progression (A. P.) starting with a and having common difference d, where $a > 0$ and $d \geq 0$ fixed integers. If $d = 0$, then super (a, d)-EAT labeling is said to be super edge magic total (EMT) labeling. The notion of (EMT) labeling was introduced and studied by Kotzig and Rosa [6]. Enomoto et al. [3] introduced the name of super (EMT) labeling for (EMT) labeling with added the property that the p vertices receives the smallest labels from $[1, P]$. The definition of an (a, d)-EAT labeling was proposed by Simanjuntak et al. [13] as the natural extension of magic valuation defined by Kotzig and Rosa in [6, 7]. A super (a, d)-EATlabeling is a natural extension of the notion of super edge-magic labeling defined by Enomoto et al. Moreover, Enomoto et al. [3] proposed the following conjecture:
Conjecture 1.1. Every tree admits a super (EAT) labeling.

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In the support of this conjecture, many authors have considered a super $(a, 0)$-EAT labeling for different particular classes of trees. Lee et al. [8] verified this conjecture by a computer search for trees with at most 17 vertices. For different values of d, the results related to a super (a, d)-EAT labeling can be found for w-trees [4], stars [9], subdivided stars [5, 10, 11, 12], path-like trees [2], caterpillars [6, 7, 15] and wheels, fans and friendship graphs [14], paths and cycles [13] and complete bipartite graphs [1].

Definition 1.1. A (s, d)-edge-antimagic vertex ((s, d)-EAV) labeling of a (p, q)-graph Γ is a bijective function $\lambda : V(\Gamma) \to [1, p]$ such that the set of edge-sums of all edges in Γ, $w(uv) = \{\lambda(u) + \lambda(v) : uv \in E(\Gamma)\}$, forms an A. P. of the form $[s, s + (q - 1)d]$, where $s > 0$ and $d \geq 0$ are two fixed integers.

Definition 1.2. Let $m_i \geq 1$, $1 \leq i \leq r$, and $r \geq 3$. A subdivided star $T(m_1, m_2, ..., m_r)$ is a tree obtained by inserting $m_i - 1$ vertices to each of the ith edge of the star $K_{1,r}$. Moreover suppose that $V(\Gamma) = \{c\} \cup \{x_i^{p_i} \mid 1 \leq i \leq r \ ; \ 1 \leq p_i \leq m_i\}$ is the vertex-set and $E(\Gamma) = \{k \ x_i^1 \mid 1 \leq i \leq r\} \cup \{x_i^{p_i} x_i^{p_i+1} \mid 1 \leq i \leq r \ ; \ 1 \leq p_i \leq m_i - 1\}$ is the edge-set of the subdivided star $G \cong T(m_1, m_2, ..., m_r)$ then $p = |V(\Gamma)| = \sum\limits_{i=1}^{r} m_i + 1$ and $q = |E(\Gamma)| = \sum\limits_{i=1}^{r} m_i$.

Definition 1.3. A graph denoted by $\Gamma = G(\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_t : t, l)$, with the vertex-set is $V(\Gamma) = \{c_i : 1 \leq i \leq t\} \cup \{a_{ir}^{p_{ir}} : 1 \leq i \leq t, \ 1 \leq p_{ir} \leq s_{ir}, \ 1 \leq r \leq l\}$ and the edge-set $E(\Gamma) = \{c_i c_{i+1} : 1 \leq i \leq t - 1\} \cup \{a_{ir}^{p_{ir}} a_{ir}^{p_{ir}+1} : 1 \leq i \leq t, \ 1 \leq p_{ir} \leq s_{ir} - 1, \ 1 \leq r \leq l\} \cup \{a_{ir}^1 c_i : 1 \leq i \leq t, \ 1 \leq r \leq l\}$, where $\alpha_1 = (s_{11}, s_{12}, s_{13}, \ldots, s_{1l})$, $\alpha_2 = (s_{21}, s_{22}, s_{23}, \ldots, s_{2l}), \ldots, \alpha_t = (s_{t1}, s_{t2}, s_{t3}, \ldots, s_{tl})$ is called a subdivided caterpillar.

Ngurah et al. [10] proved that the subdivided star $T(m_1, m_2, m_3)$ is also a super $(a, 0)$-EAT graph if $m_3 = m_2 + 3$ or $m_3 = n_2 + 4$. Salman et al. [12] found a super $(a, 0)$-EAT labeling on the subdivided stars $T\underbrace{(m, m, ..., m)}_{r-times}$, where $m \in \{2, 3\}$. Moreover, Javaid et al. [5] proved the super (a, d)-EAT labeling on different subclasses of subdivided stars for $d \in \{0, 1, 2\}$. However, the investigation of the different results related to a super (a, d)-EAT labeling of the subdivided star $T(m_1, m_2, ..., m_r)$ for $m_1 \neq m_2 \neq m_3, ..., \neq m_r$ is still open.

Bača and Miller find a necessary condition far a graph to be super (a, d)-EAT, which provides an upper bound on the parameter d. Let a (p, q)-graph Γ be a super (a, d)-EAT labeling. The minimum possible edge-weight is at least $p + 4$. The maximum possible edge-weight is no more than $3p + q - 1$. Thus $a + (q - 1)d \leq 3p + q - 1$ or $d \leq \frac{2p+q-5}{q-1}$. For any subdivided star, where $p = q + 1$, it follows that $d \leq 3$.

Let us consider the following proposition which we will proven to the useful in the rest of the paper.

Proposition 2.1. [2] If a (p, q)-graph Γ has a (s, d)-EAV labeling then

(i) Γ has a super $(s + p + 1, d + 1)$-EAT labeling,
(ii) Γ has a super $(s + p + q, d - 1)$-EAT labeling. $\square$

2. Main Results

This section deals with the main results related to super (a,d)-EAT labelings on more generalized families of subdivided stars and caterpillars for all possible values of d.

Theorem 2.1. The graph $\Gamma \cong T(m, m-1, m-1, m, m_5, ..., m_r)$ admits super $(a,0)$-EAT labeling with $a = 2p + s - 1$ and super $(a_1, 2)$-EAT labeling with $a_1 = p + s + 1$ for all $m \geq 2$ and $r \geq 5$, where $p = |V(\Gamma)|$ and $s = 2m + 2 + \sum_{t=5}^{r}[2^{t-5}m]$ and $m_r = 2^{r-4}m$.

Proof. Let us denote $V(\Gamma) = \{k\} \cup \{x_i^{l_i} \mid 1 \leq i \leq r \ ; \ 1 \leq p_i \leq m_i\}$,

$E(\Gamma) = \{k\ x_i^1 \mid 1 \leq i \leq 5\} \cup \{x_i^{p_i} x_i^{p_i+1} \mid 1 \leq i \leq r \ ; \ 1 \leq p_i \leq m_i - 1\}$.

If $p = |V(\Gamma)|$ and $q = |E(\Gamma)|$ then

$$p = (4m-1) + \sum_{t=5}^{r}[2^{t-5}2m]$$

and

$$q = p - 1.$$

Now, we define the labeling $\psi : V(\Gamma) \to [1, p]$ as given below:

$$\psi(k) = 3m + \sum_{t=5}^{r}[2^{t-5}m].$$

For odd $1 \leq p_i \leq m_i$, where $i = 1, 2, 3, 4$ and $5 \leq i \leq r$, we define

$$\psi(y) = \begin{cases} \frac{p_1+1}{2}, & \text{when } \ y = x_1^{p_1}, \\ (m+1) - \frac{p_2+1}{2}, & \text{when } \ y = x_2^{p_2}, \\ (m+1) + \frac{p_3-1}{2}, & \text{when } \ y = x_3^{p_3}, \\ 2m - \frac{p_4-1}{2}, & \text{when } \ y = x_4^{p_4}. \end{cases}$$

$$\lambda(x_i^{p_i}) = 2m + \sum_{t=5}^{i}[2^{t-5}n] - \frac{p_i - 1}{2}.$$

For even $1 \leq p_i \leq m_i$, and $\alpha = 2m + \sum_{t=5}^{r}[2^{t-6}m]$

For $i = 1, 2, 3, 4$ and $5 \leq i \leq r$, we define

$$\psi(y) = \begin{cases} (\alpha+1) + \frac{p_1-2}{2}, & \text{when } \ y = x_1^{p_1}, \\ (\alpha+m-1) - \frac{p_2-2}{2}, & \text{when } \ y = x_2^{p_2}, \\ (\alpha+m+1) + \frac{l_3+2}{2}, & \text{when } \ y = x_3^{p_3}, \\ (\alpha+2m-1) + \frac{l_4-2}{2}, & \text{when } \ y = x_4^{p_4}, \end{cases}$$

and

$$\lambda(x_i^{p_i}) = (\alpha + 2m - 1) + \sum_{t=5}^{i}[2^{t-5}m] - \frac{p_i - 2}{2}.$$

By using the above scheme of labeling, we find the set of edge-sums of the form A. P. as $s = [\alpha + 2, \alpha + 1 + q]$. Thus, by Proposition 2.1, we expand ψ to a super $(a, 0)$-EAT labeling with the magic constant $a = 2p + (2m+1) + \sum\limits_{t=5}^{r}[2^{t-5}m]$. Likewise, by Proposition 2.1, ψ can be extended to a super $(a_1, 2)$-EAT labeling and we obtain the magic constant $a_1 = p + (2m+3) + \sum\limits_{t=5}^{r}[2^{t-5}m]$.

□

Theorem 2.2. For all $n \geq 2$ and $r \geq 5$, The graph $\Gamma \cong T(m, m-1, m-1, m, m_5, ..., m_r)$ admits super $(a_2, 1)$-EAT labeling with $a_2 = s + \frac{3p}{2}$ if p is even, where $p = |V(\Gamma)|$, $s = (2m+2) + \sum\limits_{t=5}^{r}[2^{t-5}m]$ and $m_r = 2^{r-5}m$.

Proof. Let us consider the vertices and edges of Γ, as defined in Theorem 2.1 Now, we define the labeling $\psi : V(\Gamma) \rightarrow [1, p]$ as in same theorem. It follows that the edge-weights of all edges of Γ form an A. P. as $s = [\alpha+2, \alpha+1+q]$ with common difference 1, where $\alpha = (5m+4) + \sum\limits_{t=6}^{r}[2^{t-6}(4m+2)+1]$. We denote it by $X = \{x_i; 1 \leq i \leq q\}$. Now for Γ we complete the edge labeling ψ for super $(a_2, 1)$-EAT labeling with values in the A. P. as $[p+1, p+q]$ with common difference 1. Let us denote it by $Y = \{y_j\ ;\ 1 \leq j \leq q\}$. Define $Z = \{x_{2i-1} + y_{e-i+1}\ ;\ 1 \leq i \leq \frac{q+1}{2}\} \cup \{x_{2j} + y_{\frac{q-1}{2}-j+1}\ ;\ 1 \leq j \leq \frac{q+1}{2} - 1\}$. It is easy to see that Z constitutes an A. P. with $d = 1$ and $a_2 = s + \frac{3(p)}{2} = 8m + \frac{1}{2} + \sum\limits_{t=5}^{r}[2^{t-3}n]$. Since all vertices receive the smallest labels, Γ is a super $(a_2, 1)$-EAT labeling. □

Theorem 2.3. For all $m \geq 2$ and and $r \geq 4$, graph $\Gamma \cong T(m, m-1, m, m_4, ..., m_r)$ admits a super $(a, 1)$-EAT labeling with $a = 2p + 2$, where $p = |V(\Gamma)|$ and $m_t = 2^{t-4}m$ for $4 \leq t \leq r$.

Proof. Let us consider $p = |V(\Gamma)|$ and $q = |E(\Gamma)|$ then $p = 3m + \sum\limits_{t=4}^{r}[2^{t-3}m]$ and $q = p-1$. Now, we define $\psi : V(\Gamma) \rightarrow [1, p]$ as following:

$$\lambda(k) = m + 1.$$

For $\quad 1 \leq p_i \leq m_i$, where $i = 1, 2, 3$ and $4 \leq i \leq r$:

$$\psi(y) = \begin{cases} (m+1) - p_1, & \text{when} \ \ y = x_1^{p_1}, \\ (m+1) + p_2, & \text{when} \ \ y = x_2^{p_2}, \\ (3m+1) - p_3, & \text{when} \ \ y = x_3^{l_3}, \end{cases}$$

and $\psi(x_i^{p_i}) = (3m+1) + \sum\limits_{t=4}^{i}[2^{t-3}m] - p_i$.

Suppose, $\alpha = 2m + \sum\limits_{t=3}^{r}[2^{t-3}m]$ and define $\psi : E(\Gamma) \rightarrow [v+1, p+q]$ as following:

For $\quad p_i = 1$, where $i = 1, 2, 3$ and, $4 \leq i \leq r$:

$$\psi(ky) = \begin{cases} 2\alpha - m, & \text{when} \ \ y = x_1^{p}, \\ 2\alpha - (m+1), & \text{when} \ \ y = x_2^{p}, \\ 2\alpha - 2m, & \text{when} \ \ y = x_3^{p}, \end{cases}$$

and $\lambda(kx_i^1) = 2\alpha - 2m - \sum_{t=4}^{i}[2^{t-3}m]$.
For $\quad 1 \leq p_i \leq m_i - 1$, $i = 1, 2, 3$ and, $4 \leq i \leq r$:

$$\psi(x_i^{p_i} x_i^{p_i+1}) = \begin{cases} 2\alpha - m + p_1, & \text{when } \ i = 1, \\ 2\alpha - (m+1) - p_2, & \text{when } \ i = 2, \\ 2\alpha - 3n + p_3, & \text{when } \ i = 3, \end{cases}$$

and $\psi(x_i^{p_i} x_i^{p_{i+1}}) = 2\alpha - 3m - \sum_{t=4}^{i}[2^{t-3}m] + l_i$.
The set of edge-weights generated by the above formulas forms a integer sequence $[(p+1)+1, (p+1)+q$ with common difference 1. Consequently, λ admits a super $(a,1)$-EAT labeling with $a = 2p + 2$. □

Theorem 2.4. For all $m \geq 2$ and $r \geq 4$, $\Gamma \cong T(m, m-1, m, m_4, ..., m_r)$ admits a super $(a,3)$-EAT labeling with $a = p + 4$, where $p = |V(\Gamma)|$ and $m_t = 2^{t-3}m$ for $4 \leq t \leq r$.

Proof. Let us consider $p = |V(\Gamma)|$, $q = |E(\Gamma)|$, α, and $\psi : V(G) \to [1, p]$ as defined in Theorem 2.3. Now, we define $\psi : E(\Gamma) \to [v+1, p+q]$ as following:
For $\quad p_i = 1$, where $i = 1, 2, 3$ and, $4 \leq i \leq r$:

$$\lambda(ky) = \begin{cases} \alpha + m, & \text{when } \ y = x_1^p, \\ \alpha + (m+1), & \text{when } \ y = x_2^p, \\ \alpha + 2m, & \text{when } \ y = x_3^1, \end{cases}$$

and $\lambda(kx_i^p) = \alpha + 2m + \sum_{t=4}^{i}[2^{t-4}m]$.
For $\quad 1 \leq p_i \leq m_i - 1$, where $i = 1, 2, 3$ and, $4 \leq i \leq r$:

$$\psi(x_i^{p_i} x_i^{p_i+1}) = \begin{cases} \alpha + m - p_1, & \text{when } \ i = 1, \\ \alpha + (m+1) + p_2, & \text{when } \ i = 2, \\ \alpha + 3m - p_3, & \text{when } \ i = 3, \end{cases}$$

and $\lambda(x_i^{1_i} x_i^{1_{i+1}}) = \alpha + 3m + \sum_{m=4}^{i}[2^{m-3}m] - l_i$, respectively.
The set of edge-weights $\{\psi(u)+\psi(uv)+\psi(v) : uv \in E(\Gamma)\}$ generated by the above formulas forms an integer sequence $[(p+1)+3(1), (p+1)+3(q)$ with difference 3. Consequently, ψ admits a super $(a,3)$-EAT labeling with $a = p + 4$. □

Theorem 2.5. For $n \geq 2$, odd $m \geq 3$, $l = 3$, $\alpha_1 = (m, m, m)$ and $\alpha_2 = \alpha_3 = ... = \alpha_n = (m, m, m)$, then the graph $G \cong \zeta(\alpha_1, \alpha_2, \alpha_3, ..., \alpha_n : n, 3)$ admits super $(a, 0)-$edge antimagic total labeling with $a = 2p + s - 1$ and super $(a, 2)-$edge antimagic total labeling with $a = p + s + 1$, where $s = \frac{1}{2}(3m-1)\lfloor\frac{n}{2}\rfloor + \frac{3}{2}(m+1)\lceil\frac{n}{2}\rceil + 2$.

Proof. If $p = |V(\Gamma)|$ and $q = |E(\Gamma)|$ then $p = n(3m+1)$ and $q = p - 1$.

Now we define the labeling $\psi : V \rightarrow [1,p]$ as following:
we shall use the expressions $\alpha = 3m+1$ *and* $\eta = s-2$ to simply the later expressions.

$$\lambda(c_i) = \begin{cases} \eta + m & when\ \ i=1, \\ \eta + \frac{\alpha}{2}(i-3) + \frac{3}{2}(3m-1) & when\ \ i \geq 3\ \ odd, \\ \frac{\alpha}{2}(i-2) + 2(m+1) & when\ \ i\ \ even. \end{cases}$$

When $i=1$ and $1 \leq r \leq 3$:

for $p_{1r} = 1,3,5,...,m_{1r}$;

$$\psi(y) = \begin{cases} \frac{p_{11}+1}{2} & \text{when} \quad y = a_{11}^{p_{11}}, \\ m+2-\frac{p_{12}+1}{2} & \text{when} \quad y = a_{12}^{p_{12}}, \\ m+1+\frac{p_{13}+1}{2} & \text{when} \quad y = a_{13}^{p_{13}}, \end{cases}$$

and for $p_{1r} = 2,4,6,...,m_{1r}-1$;

$$\psi(y) = \begin{cases} \eta + \frac{p_{11}}{2} & \text{when} \quad y = a_{11}^{p_{11}}, \\ \eta + m - \frac{p_{12}}{2} & \text{when} \quad y = a_{12}^{p_{12}}, \\ \eta + m + \frac{p_{13}}{2} & \text{when} \quad y = a_{13}^{p_{13}}. \end{cases}$$

When i is even and $1 \leq r \leq 3$:

for $p_{ir} = 1,3,5,...,m_{ir}$;

$$\psi(y) = \begin{cases} \eta + \alpha(\frac{i-2}{2}) + 2m + 1 + \frac{p_{i1}+1}{2} & \text{when} \quad y = a_{11}^{p_{i1}}, \\ \eta + \alpha(\frac{i-2}{2}) + 2(m+1) + \frac{p_{i2}+1}{2} & \text{when} \quad y = a_{12}^{p_{i2}}, \\ \eta + \alpha(\frac{i-2}{2}) + 3m + 2 + \frac{p_{i3}+1}{2} & \text{when} \quad y = a_{13}^{p_{i3}}, \end{cases}$$

and for $p_{ir} = 2,4,6,...,m_{ir}-1$;

$$\psi(y) = \begin{cases} \alpha(\frac{i-2}{2}) + 2(m+1) + \frac{p_{i1}}{2} & \text{when} \quad y = a_{11}^{p_{i1}}, \\ \alpha(\frac{i-2}{2}) + 2(m+2) + \frac{p_{i2}}{2} & \text{when} \quad y = a_{12}^{p_{i2}}, \\ \alpha(\frac{i-2}{2}) + 3m + 2 + \frac{p_{i3}}{2} & \text{when} \quad y = a_{13}^{p_{i3}}. \end{cases}$$

When $i \geq 3$ *odd* and $1 \leq r \leq 3$:

for $p_{ir} = 1,3,5,...,m_{ir}$;

$$\psi(y)=\begin{cases}\alpha(\frac{i-3}{2})+3m+2+\frac{p_{i1}+1}{2} & \text{when } y=a_{11}^{p_{i1}},\\ \alpha(\frac{i-3}{2})+(4m+3)-\frac{p_{i2}+1}{2} & \text{when } y=a_{12}^{p_{i2}},\\ \alpha(\frac{i-3}{2})+(4m+2)+\frac{p_{i3}+1}{2} & \text{when } y=a_{13}^{p_{i3}},\end{cases}$$

and for $p_{ir}=2,4,6,...,m_{ir}-1$;

$$\psi(y)=\begin{cases}\eta+\alpha(\frac{i-3}{2})+(3m+1)+\frac{p_{i1}}{2} & \text{when } y=a_{11}^{p_{i1}},\\ \eta+\alpha(\frac{i-3}{2})+4m-\frac{p_{i2}}{2} & \text{when } y=a_{12}^{p_{i2}},\\ \eta+\alpha(\frac{i-3}{2})+4m+\frac{p_{i3}}{2} & \text{when } y=a_{13}^{p_{i3}}.\end{cases}$$

The set of all edge-sums $\{\lambda(u)+\lambda(uv)+\lambda(v) : uv \in E(\Gamma)\}$ generated by the above labeling scheme forms a arithmetic sequence $s=(\eta+1)+1,(\eta+1)+2,\cdots,(\eta+1)+q$. Therefore, by proposition 2.1, ψ can be extended to a super (a,0)-EAT labeling and obtain the magic constant $a=2p+s-1=\eta+2n(3m+1)+1$. Similarly, ψ can be extended to a super (a,2)-EAT labeling and we obtain the magic constant $a=p+1+s=\eta+n(3m+1)+3$. □

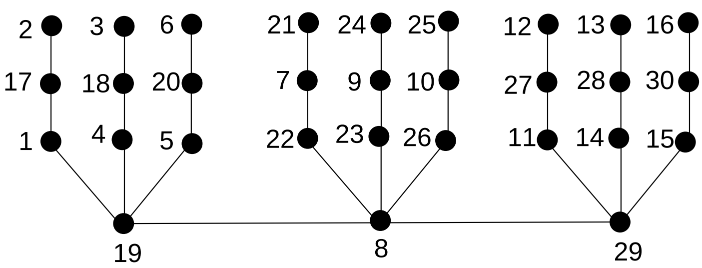

FIGURE 1. Subdivided caterpillar with $m=3$, $n=3$

Theorem 2.6. For $n\geq 2$, odd $m\geq 3$, $l=3$, $\alpha_1=(m,m,m)$ and $\alpha_2=\alpha_3=...=\alpha_n=(m,m,m)$, $G\cong\zeta(\alpha_1,\alpha_2,\alpha_3,...,\alpha_n : n,3)$ admits super $(a,1)-$edge antimagic total labeling with $a=s+\frac{3}{2}p$ if m is odd and p is even, where $s=\frac{1}{2}(3m-1)\lfloor\frac{n}{2}\rfloor+\frac{3}{2}(m+1)\lceil\frac{n}{2}\rceil+2$. and $p=|V(\Gamma)|$.

Proof. If $p=|V(\Gamma)|$ and $q=|E(\Gamma)|$ then $p=n(3m+1)$ and $q=p-1$. we use the labeling $\psi: V(\Gamma)\cup E(\Gamma)\to[1,p+q]$ as defined in Theorem 2.5. As in previous theorem the edge-weights of all edges of Γ constitute an arithmetic sequence $s=(\eta+1)+1,(\eta+1)+2,\cdots,(\eta+1)+q$, with common difference 1. We denote it by $X=\{x_i; 1\leq i\leq q\}$. Now for Γ we complete the edge labeling ψ for super $(a,1)$-EAT labeling with values in the arithmetic sequence $p+1$, $p+2$, $\cdots$,$p+q$ with common difference 1. Let us denote it by $Y=\{y_j\ ;\ 1\leq j\leq q\}$. Define $Z=\{x_{2i-1}+y_{q-i+1}\ ;\ 1\leq i\leq\frac{q+1}{2}\}\cup\{x_{2j}+y_{\frac{q-1}{2}-j+1}\ ;\ 1\leq j\leq\frac{q+1}{2}-1\}$. It is easy to see that C constitute an arithmetic sequence with $d=1$ and $a=s+\frac{3}{2}p=\eta+\frac{3}{2}n(3m+1)+2$.
Since all vertices receive the smallest labels so ψ is a super $(a,1)$-EAT labeling. □

3. Conclusion

In this paper, we have proved the super edge antimagic total labeling of subdivided stars and subdivided caterpillar for possible values of the parameter d. However, the problem of the magicness is still open for different values of magic constant (minimum edge-weight a).

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