TWMS J. App. and Eng. Math. V.10, N.4, 2020, pp. 827-845

GENERALIZED BIPOLAR NEUTROSOPHIC HYPERGRAPHS

A. HASSAN[1], M. A. MALIK[2], §

Abstract. The generalization of the concept of single valued neutrosophic hypergraph (SVNHG) and bipolar single valued neutrosophic hypergraph (BSVNHG) to generalized SVNHG and BSVNHG by considering SVN-Vertices and BSVN-Vertices instead of crisp vertices set and interrelations between SVN-Vertices and BSVN-Vertices with family of SVN-Edges and BSVN-Edges are introduced here. A few properties and operations of such hypergraphs are established here.
Keywords: Generalized BSVNHG, generalized strong BSVNHG, generalized BSVN sub hypergraph, spanning generalized BSVN sub hypergraph.
AMS Subject Classification: 99A00

1. Introduction

Neutrosopic sets were introduced by Smarandache [10] which are the generalization of fuzzy sets and intuitionistic fuzzy sets. Some studies in neutrosophic graphs introduced by Nasir in [8]. Further Yang, Guo, She and Liao in [11] studied on single valued neutrosophic relations. The bipolar single valued neutrosophic graphs were introduced by Broumi, Talea, Bakali and Smarandache [1]. Recently in [2] proposed some algorithms dealt with shortest path problem in a network (graph) where edge weights are characterized by a neutrosophic numbers including single valued neutrosophic numbers, bipolar neutrosophic numbers and interval valued neutrosophic numbers.

In graph edges are pairs of nodes, hyperedges are arbitrary sets of nodes, and can therefore contain an arbitrary number of nodes. However, it is often desirable to study hypergraphs where all hyperedges have the same cardinality. Hyperedges are absurdly general, likewise the notion of data. To make this useful, one needs to constrain the form hyper edges take. There are many research papers on fuzzy hypergraph in [3, 7] based on vertex set as a crisp set. In fact, in the definition of fuzzy graph, both the concepts of vertices and edges are fuzzy and there is an interrelation between the fuzzy vertices and fuzzy edges. The generalized strong intuitionistic fuzzy hypergraphs were discussed by Samanta and Mohinta [9].

[1] Department of Mathematics, University of Karachi, University road, Karachi, Pakistan.
e-mail: alihassan.iiui.math@gmail.com; ORCID: https://orcid.org/0000-0003-1648-337X.
[2] Department of Mathematics, University of Punjab, Quaid-e-Azam Campus, Lahore-54590, Pakistan.
e-mail: malikpu@yahoo.com; ORCID: https://orcid.org/0000-0003-4434-7893.
§ Manuscript received: March 6, 2017; accepted: September 20, 2017.

In this paper, we generalize the concept of SVNHG and BSVNHG by considering SVN-Vertex and BSVN-Vertex instead of crisp vertex set and interrelation between SVN-Vertices and BSVN-Vertices with family of SVN-Edges and BSVN-Edges. The GSVNHG, GBSVNHG, generalized strong SVNHG, generalized strong BSVNHG and a few operations on them are defined here. Also some of their properties are studied.

2. Preliminaries

Definition 2.1. [10] *Let X be a crisp set, the single valued neutrosophic set (SVNS) Z is characterized by three membership functions $T_Z(x), I_Z(x)$ and $F_Z(x)$, which are truth, indeterminacy and falsity membership functions, i.e $\forall x \in X,\ T_Z(x), I_Z(x), F_Z(x) \in [0,1]$. The support of Z is denoted and defined by $Supp(Z) = \{x : x \in X, T_Z(x) > 0, I_Z(x) > 0, F_Z(x) > 0\}$.*

Definition 2.2. [1] *Let X be a crisp set, the bipolar single valued neutrosophic set (BSVNS) Z is characterized by membership functions $T_Z^+(x), I_Z^+(x), F_Z^+(x), T_Z^-(x), I_Z^-(x)$, and $F_Z^-(x)$. That is $\forall x \in X,\ T_Z^+(x), I_Z^+(x), F_Z^+(x) \in [0,1]$ and $T_Z^-(x), I_Z^-(x), F_Z^-(x) \in [-1,0]$. The support of Z, which is denoted by $Supp(Z)$, is defined by $Supp(Z) = \{x : T_Z^+(x) > 0, I_Z^+(x) > 0, F_Z^+(x) > 0, T_Z^-(x) < 0, I_Z^-(x) < 0, F_Z^-(x) < 0\}$.*

Definition 2.3. [6] *A bipolar single valued neutrosophic graph (BSVNG) is a pair $G = (Y, Z)$ of G^*, where Y is BSVNS on V and Z is BSVNS on E such that*

$$T_Z^+(\beta\gamma) \leq \min(T_Y^+(\beta), T_Y^+(\gamma)),\ I_Z^+(\beta\gamma) \geq \max(I_Y^+(\beta), I_Y^+(\gamma)),$$
$$I_Z^-(\beta\gamma) \leq \min(I_Y^-(\beta), I_Y^-(\gamma)),\ F_Z^-(\beta\gamma) \leq \min(F_Y^-(\beta), F_Y^-(\gamma)),$$
$$F_Z^+(\beta\gamma) \geq \max(F_Y^+(\beta), F_Y^+(\gamma)),\ T_Z^-(\beta\gamma) \geq \max(T_Y^-(\beta), T_Y^-(\gamma)),$$

where

$$0 \leq T_Z^+(\beta\gamma) + I_Z^+(\beta\gamma) + F_Z^+(\beta\gamma) \leq 3$$
$$-3 \leq T_Z^-(\beta\gamma) + I_Z^-(\beta\gamma) + F_Z^-(\beta\gamma) \leq 0$$

$\forall\ \beta, \gamma \in V$. In this case D is bipolar single valued neutrosophic relation (BSVNR) on C. The BSVNG $G = (Y, Z)$ is complete (strong) BSVNG, if

$$T_Z^+(\beta\gamma) = \min(T_Y^+(\beta), T_Y^+(\gamma)),\ I_Z^+(\beta\gamma) = \max(I_Y^+(\beta), I_Y^+(\gamma)),$$
$$I_Z^-(\beta\gamma) = \min(I_Y^-(\beta), I_Y^-(\gamma)),\ F_Z^-(\beta\gamma) = \min(F_Y^-(\beta), F_Y^-(\gamma)),$$
$$F_Z^+(\beta\gamma) = \max(F_Y^+(\beta), F_Y^+(\gamma)),\ T_Z^-(\beta\gamma) = \max(T_Y^-(\beta), T_Y^-(\gamma)),$$

$\forall\ \beta, \gamma \in V (\forall\ \beta\gamma \in E)$. The order of G, which is denoted by $O(G)$, is defined by

$$O(G) = (O_T^+(G), O_I^+(G), O_F^+(G), O_T^-(G), O_I^-(G), O_F^-(G))$$

where,

$$O_T^+(G) = \sum_{\alpha \in V} T_A^+(\alpha),\ O_I^+(G) = \sum_{\alpha \in V} I_A^+(\alpha),\ O_F^+(G) = \sum_{\alpha \in V} F_A^+(\alpha),$$
$$O_T^-(G) = \sum_{\alpha \in V} T_A^-(\alpha),\ O_I^-(G) = \sum_{\alpha \in V} I_A^-(\alpha),\ O_F^-(G) = \sum_{\alpha \in V} F_A^-(\alpha).$$

The size of G, which is denoted by $S(G)$, is defined by

$$S(G) = (S_T^+(G), S_I^+(G), S_F^+(G), S_T^-(G), S_I^-(G), S_F^-(G))$$

where

$$S_T^+(G) = \sum_{\beta\gamma \in E} T_B^+(\beta\gamma),\ S_T^-(G) = \sum_{\beta\gamma \in E} T_B^-(\beta\gamma),$$

$$S_I^+(G) = \sum_{\beta\gamma \in E} I_B^+(\beta\gamma),\ S_I^-(G) = \sum_{\beta\gamma \in E} I_B^-(\beta\gamma),$$
$$S_F^+(G) = \sum_{\beta\gamma \in E} F_B^+(\beta\gamma),\ S_F^-(G) = \sum_{\beta\gamma \in E} F_B^-(\beta\gamma).$$

The degree of a vertex β in G, which is denoted by $d_G(\beta)$, is defined by

$$d_G(\beta) = (d_T^+(\beta), d_I^+(\beta), d_F^+(\beta), d_T^-(\beta), d_I^-(\beta), d_F^-(\beta))$$

where

$$d_T^+(\beta) = \sum_{\beta\gamma \in E} T_B^+(\beta\gamma),\ d_T^-(\beta) = \sum_{\beta\gamma \in E} T_B^-(\beta\gamma),$$
$$d_I^+(\beta) = \sum_{\beta\gamma \in E} I_B^+(\beta\gamma),\ d_I^-(\beta) = \sum_{\beta\gamma \in E} I_B^-(\beta\gamma),$$
$$d_F^+(\beta) = \sum_{\beta\gamma \in E} F_B^+(\beta\gamma),\ d_F^-(\beta) = \sum_{\beta\gamma \in E} F_B^-(\beta\gamma).$$

Definition 2.4. [6] *The bipolar single valued neutrosophic subgraph of BSVNG $G = (C, D)$ of $G^* = (V, E)$ is a BSVNG $H = (C', D')$ on a $H^* = (V', E')$, such that $C' = C$, and $D' = D$.*

Definition 2.5. [6] *Let $G_1 = (C_1, D_1)$ and $G_2 = (C_2, D_2)$ be two BSVNGs of $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$, respectively. Then the homomorphism $\chi : V_1 \to V_2$ is a mapping from V_1 into V_2 satisfying following conditions*

$$T_{C_1}^+(p) \le T_{C_2}^+(\chi(p)),\ I_{C_1}^+(p) \ge I_{C_2}^+(\chi(p)),\ F_{C_1}^+(p) \ge F_{C_2}^+(\chi(p)),$$
$$T_{C_1}^-(p) \ge T_{C_2}^-(\chi(p)),\ I_{C_1}^-(p) \le I_{C_2}^-(\chi(p)),\ F_{C_1}^-(p) \le F_{C_2}^-(\chi(p)),$$

$\forall\ p \in V_1$.

$$T_{D_1}^+(pq) \le T_{D_2}^+(\chi(p)\chi(q)),\ I_{D_1}^+(pq) \ge I_{D_2}^+(\chi(p)\chi(q)),\ F_{D_1}^+(pq), \ge F_{D_2}^+(\chi(p)\chi(q)),$$
$$T_{D_1}^-(pq) \ge T_{D_2}^-(\chi(p)\chi(q)),\ I_{D_1}^-(pq) \le I_{D_2}^-(\chi(p)\chi(q)),\ F_{D_1}^-(pq) \le F_{D_2}^-(\chi(p)\chi(q)),$$

$\forall\ pq \in E_1$. The weak isomorphism $\upsilon : V_1 \to V_2$ is a bijective homomorphism from V_1 into V_2 satisfying following conditions

$$T_{C_1}^+(p) = T_{C_2}^+(\upsilon(p)),\ I_{C_1}^+(p) = I_{C_2}^+(\upsilon(p)),\ F_{C_1}^+(p) = F_{C_2}^+(\upsilon(p)),$$
$$T_{C_1}^-(p) = T_{C_2}^-(\upsilon(p)),\ I_{C_1}^-(p) = I_{C_2}^-(\upsilon(p)),\ F_{C_1}^-(p) = F_{C_2}^-(\upsilon(p)),$$

$\forall\ p \in V_1$. The co-weak isomorphism $\kappa : V_1 \to V_2$ is a bijective homomorphism from V_1 into V_2 satisfying following conditions

$$T_{D_1}^+(pq) = T_{D_2}^+(\kappa(p)\kappa(q)),\ I_{D_1}^+(pq) = I_{D_2}^+(\kappa(p)\kappa(q)),\ F_{D_1}^+(pq), = F_{D_2}^+(\kappa(p)\kappa(q)),$$
$$T_{D_1}^-(pq) = T_{D_2}^-(\kappa(p)\kappa(q)),\ I_{D_1}^-(pq) = I_{D_2}^-(\kappa(p)\kappa(q)),\ F_{D_1}^-(pq) = F_{D_2}^-(\kappa(p)\kappa(q)),$$

$\forall\ pq \in E_1$. An isomorphism $\psi : V_1 \to V_2$ is a bijective homomorphism from V_1 into V_2 satisfying following conditions

$$T_{C_1}^+(p) = T_{C_2}^+(\psi(p)),\ I_{C_1}^+(p) = I_{C_2}^+(\psi(p)),\ F_{C_1}^+(p) = F_{C_2}^+(\psi(p)),$$
$$T_{C_1}^-(p) = T_{C_2}^-(\psi(p)),\ I_{C_1}^-(p) = I_{C_2}^-(\psi(p)),\ F_{C_1}^-(p) = F_{C_2}^-(\psi(p)),$$

$\forall\ p \in V_1$.

$$T_{D_1}^+(pq) = T_{D_2}^+(\psi(p)\psi(q)),\ I_{D_1}^+(pq) = I_{D_2}^+(\psi(p)\psi(q)),\ F_{D_1}^+(pq), = F_{D_2}^+(\psi(p)\psi(q)),$$
$$T_{D_1}^-(pq) = T_{D_2}^-(\psi(p)\psi(q)),\ I_{D_1}^-(pq) = I_{D_2}^-(\psi(p)\psi(q)),\ F_{D_1}^-(pq) = F_{D_2}^-(\psi(p)\psi(q)),$$

$\forall\ pq \in E_1$.

Remark 2.1. *One can see the following.*

(1) *The weak isomorphism between two BSVNGs preserves the orders.*
(2) *The weak isomorphism between BSVNGs is a partial order relation.*
(3) *The co-weak isomorphism between two BSVNGs preserves the sizes.*
(4) *The co-weak isomorphism between BSVNGs is a partial order relation.*
(5) *The isomorphism between two BSVNGs is an equivalence relation.*
(6) *The isomorphism between two BSVNGs preserves the orders and sizes.*
(7) *The isomorphism between two BSVNGs preserves the degrees of their vertices's.*

Definition 2.6. [7] *A hypergraph is an ordered pair* $H = (Z, \Theta)$, *where*
(**1**) $Z = \{\eta_1, \eta_2, \dots, \eta_n\}$ *be a finite set of vertices.*
(**2**) $\Theta = \{\Theta_1, \Theta_2, \dots, \Theta_m\}$ *be a family of subsets of* Z.
(**3**) $\Theta_j \neq \phi$, $\forall j = 1, 2, 3, \dots, m$ *and* $\bigcup_j \Theta_j = Z$.
A hypergraph is also called a set system or a family of sets drawn from the universal set X.

3. Generalized strong SVNHGs

Definition 3.1. *The single valued neutrosophic hypergraph (SVNHG) be a* $H = (Z, \Theta)$, *where*
(**1**) $Z = \{\eta_1, \eta_2, \dots, \eta_n\}$ *be a finite set of vertices.*
(**2**) $\Theta = \{\Theta_1, \Theta_2, \dots, \Theta_m\}$ *be a family of SVNSs of* Z.
(**3**) $\Theta_j \neq O = (0, 0, 0)$ $\forall j = 1, 2, 3, \dots, m$ *and* $\bigcup_j Supp(\Theta_j) = Z$.

Definition 3.2. *A generalized single valued neutrosophic hypergraph (GSVNHG)* $H = (Z, \Theta)$, *where*
(**1**) $Z = \{\eta_1, \eta_2, \dots, \eta_n\}$ *be a finite set of vertices.*
(**2**) $A, B, C : Z \to [0, 1]$ *be the SVNS of vertices.*
(**3**) $\Theta = \{\Theta_1, \Theta_2, \dots, \Theta_m\}$ *be set of SVNSs of* Z, *where*

$$\Theta_j = \{(\eta_i, T_{\Theta_j}(\eta_i), I_{\Theta_j}(\eta_i), F_{\Theta_j}(\eta_i)) : T_{\Theta_j}(\eta_i), I_{\Theta_j}(\eta_i), F_{\Theta_j}(\eta_i) : Z \to [0, 1]\}$$

with

$$\bigvee_{j=1}^{m} T_{\Theta_j}(\eta_i) \leq A(\eta_i), \ \bigwedge_{j=1}^{m} I_{\Theta_j}(\eta_i) \geq B(\eta_i), \ \bigwedge_{j=1}^{m} F_{\Theta_j}(\eta_i) \geq C(\eta_i)$$

$\forall i = 1, 2, 3, \dots, n$ *and* $\forall j = 1, 2, 3, \dots, m$.
(**4**) $\Theta_j \neq O = (0, 0, 0)$, $j = 1, 2, 3, \dots, m$ *and* $\bigcup_j Supp(\Theta_j) = Z$.

Remark 3.1. *The generalized single valued neutrosophic hypergraph is the generalization of generalized intuitionistic fuzzy hypergraph.*

Example 3.1. *Consider the* $H = (X, E)$, *where* $X = \{\alpha, \beta, \gamma, \delta\}$ *and* $E = \{E_1, E_2, E_3, E_4\}$. *Also* $A, B, C : X \to [0, 1]$ *defined by* $A(\alpha) = .5$, $A(\beta) = .9$, $A(\gamma) = .8$, $A(\delta) = .6$, $B(\alpha) = .0$, $B(\beta) = .1$, $B(\gamma) = .1$, $B(\delta) = .0$, $C(\alpha) = .1$, $C(\beta) = .1$, $C(\gamma) = .2$, $C(\delta) = .3$,

$$\begin{aligned}
E_1 &= \{(\alpha, .2, .3, .4), (\beta, .5, .3, .6), (\gamma, .5, .3, .2), (\delta, .0, .1, .3)\}, \\
E_2 &= \{(\alpha, .5, .0, .2), (\beta, .6, .7, .4), (\gamma, .1, .6, .9), (\delta, .2, .3, .6)\}, \\
E_3 &= \{(\alpha, .1, .3, .5), (\beta, .8, .1, .3), (\gamma, .3, .8, .9), (\delta, .5, .0, .9)\}, \\
E_4 &= \{(\alpha, .1, .6, .2), (\beta, .2, .1, .6), (\gamma, .6, .1, .3), (\delta, .3, .2, .6)\}.
\end{aligned}$$

Then by routine calculations H *is GSVNHG.*

Definition 3.3. *The GSVNHG $H = (X, E)$ is said to be generalized strong single valued neutrosophic hypergraph (GSSVNHG), if*

$$\bigvee_{j=1}^{m} T_{E_j}(x_i) = A(x_i),\ \bigwedge_{j=1}^{m} I_{E_j}(x_i) = B(x_i),\ \bigwedge_{j=1}^{m} F_{E_j}(x_i) = C(x_i)$$

$\forall i = 1, 2, 3, \ldots, n$ *and* $j = 1, 2, 3, \ldots, m$.

Example 3.2. *Consider the GSVNHG $H = (X, E)$, where $X = \{\alpha, \beta, \gamma\}$ and $E = \{E_1, E_2, E_3, E_4\}$. Also $A, B, C : X \to [0, 1]$ defined by $A(\alpha) = .5$, $A(\beta) = .6$, $A(\gamma) = .8$, $B(\alpha) = .2$, $B(\beta) = .2$, $B(\gamma) = .0$, $C(\alpha) = .3$, $C(\beta) = .2$, $C(\gamma) = .1$,*

$$\begin{aligned} E_1 &= \{(\alpha, .5, .2, .3), (\beta, .5, .2, .9), (\gamma, .3, .9, .1)\}, \\ E_2 &= \{(\alpha, .1, .6, .5), (\beta, .3, .2, .6), (\gamma, .0, .3, .2)\}, \\ E_3 &= \{(\alpha, .3, .6, .9), (\beta, .1, .3, .2), (\gamma, .1, .0, .9)\}, \\ E_4 &= \{(\alpha, .2, .3, .6), (\beta, .6, .5, .2), (\gamma, .8, .6, .4)\}. \end{aligned}$$

Then by routine calculations H is GSSVNHG.

Definition 3.4. *Let $H = (X, E)$ be a GSVNHG, where $A, B, C : X \to [0, 1]$,*

$$E = \{(T_{E_j}, I_{E_j}, F_{E_j}) : X \to [0, 1]^3 : j = 1, 2, 3, \ldots, m\}$$

and let $H' = (X, E')$, where $A', B', C' : X \to [0, 1]$,

$$E' = \{(T'_{E_j}, I'_{E_j}, F'_{E_j}) : X \to [0, 1]^3 : j = 1, 2, 3, \ldots, m\}$$

H' is said to be a generalized single valued neutrosophic sub hypergraph (GSVNSHG) of H, whenever

$$\bigvee_{j=1}^{m} T'_{E_j}(x_i) \le \bigvee_{j=1}^{m} T_{E_j}(x_i),\ \bigwedge_{j=1}^{m} I'_{E_j}(x_i) \ge \bigwedge_{j=1}^{m} I_{E_j}(x_i),\ \bigwedge_{j=1}^{m} F'_{E_j}(x_i) \ge \bigwedge_{j=1}^{m} F_{E_j}(x_i)$$

$$A'(x_i) \le A(x_i),\ B'(x_i) \ge B(x_i),\ C'(x_i) \ge C(x_i)$$

$\forall i = 1, 2, 3, \ldots, n$. *The GSVNHG $H' = (X, E')$ is said to be a spanning generalized single valued neutrosophic sub hypergraph (SGSVNSHG) of $H = (X, E)$, if*

$$A'(x_i) = A(x_i),\ B'(x_i) = B(x_i),\ C'(x_i) = C(x_i)$$

$\forall i = 1, 2, 3, \ldots, n$.

Definition 3.5. *Let $H = (X, E)$ be a GSSVNHG, where $A, B, C : X \to [0, 1]$,*

$$E = \{(T_{E_j}, I_{E_j}, F_{E_j}) : X \to [0, 1]^3 : j = 1, 2, 3, \ldots, m\}$$

and let $H' = (X, E')$, where $A', B', C' : X \to [0, 1]$, and

$$E' = \{(T'_{E_j}, I'_{E_j}, F'_{E_j}) : X \to [0, 1]^3 : j = 1, 2, 3, \ldots, m\}$$

H' is is said to be a generalized strong single valued neutrosophic sub hypergraph (GSSVN-SHG) of H, whenever

$$\bigvee_{j=1}^{m} T'_{E_j}(x_i) = \bigvee_{j=1}^{m} T_{E_j}(x_i),\ \bigwedge_{j=1}^{m} I'_{E_j}(x_i) = \bigwedge_{j=1}^{m} I_{E_j}(x_i),\ \bigwedge_{j=1}^{m} F'_{E_j}(x_i) = \bigwedge_{j=1}^{m} F_{E_j}(x_i)$$

$$A'(x_i) = A(x_i),\ B'(x_i) = B(x_i),\ C'(x_i) = C(x_i)$$

$\forall i = 1, 2, 3, \ldots, n$. *The GSVNHG* $H^{'} = (X, E^{'})$ *is said to be a spanning generalized strong single valued neutrosophic sub hypergraph (SGSSVNSHG) of* $H = (X, E)$, *if*

$$A^{'}(x_i) = A(x_i),\ B^{'}(x_i) = B(x_i),\ C^{'}(x_i) = C(x_i)$$

$\forall i = 1, 2, 3, \ldots, n$.

Example 3.3. *Consider the GSVNHGs* $G = (X, E)$, $H = (X, E^{'})$ *and* $S = (X, E^{''})$, *where* $X = \{\alpha, \beta, \gamma\}$, $E = \{E_1, E_2\}$, $E^{'} = \{E_1^{'}, E_2^{'}\}$ *and* $E^{''} = \{E_1^{''}, E_2^{''}\}$. *Also* $A, B, C : X \to [0, 1]$ *defined by* $A(\alpha) = .4$, $A(\beta) = .5$, $B(\alpha) = .2$, $B(\beta) = .2$, $C(\alpha) = .3$, $C(\beta) = .0$, $A^{'}(\alpha) = .4$, $A^{'}(\beta) = .4$, $B^{'}(\alpha) = .1$, $B^{'}(\beta) = .1$, $C^{'}(\alpha) = .3$, $C^{'}(\beta) = .0$, $A^{''}(\alpha) = .4$, $A^{''}(\beta) = .5$, $B^{''}(\alpha) = .2$, $B^{''}(\beta) = .2$, $C^{''}(\alpha) = .3$, $C^{''}(\beta) = .0$,

$$E_1 = \{(\alpha, .2, .3, .6), (\beta, .5, .6, .2)\},\ E_2 = \{(\alpha, .4, .2, .3), (\beta, .3, .2, .5)\},$$
$$E_1^{'} = \{(\alpha, .2, .3, .5), (\beta, .4, .3, .5)\},\ E_2^{'} = \{(\alpha, .3, .2, .3), (\beta, .3, .4, .3)\},$$
$$E_1^{''} = \{(\alpha, .2, .3, .5), (\beta, .5, .3, .5)\},\ E_2^{''} = \{(\alpha, .4, .2, .3), (\beta, .3, .4, .3)\}.$$

Then by routine calculations H *is GSVNSHG of* G *but* S *is SGSVNSHG of* G.

Definition 3.6. *Let* $H_1 = (X_1, E_1)$ *and* $H_2 = (X_2, E_2)$ *be two GSVNHGs, where* $X_1 = \{x_1, x_2, \ldots, x_n\}$, $X_2 = \{y_1, y_2, \ldots, y_n\}$, $A_1, B_1, C_1 : X_1 \to [0, 1]$, $A_2, B_2, C_2 : X_2 \to [0, 1]$ *and*

$$\begin{aligned} E_1 &= \{(T_{E_{11}}, I_{E_{11}}, F_{E_{11}}), (T_{E_{12}}, I_{E_{12}}, F_{E_{12}}), \ldots, (T_{E_{1k}}, I_{E_{1k}}, F_{E_{1k}})\} \\ E_2 &= \{(T_{E_{21}}, I_{E_{21}}, F_{E_{21}}), (T_{E_{22}}, I_{E_{22}}, F_{E_{22}}), \ldots, (T_{E_{2p}}, I_{E_{2p}}, F_{E_{2p}})\} \end{aligned}$$

where

$$T_{E_{1i}}, I_{E_{1i}}, F_{E_{1i}} : X_1 \to [0, 1],$$
$$T_{E_{2j}}, I_{E_{2j}}, F_{E_{2j}} : X_2 \to [0, 1],$$

$\forall i = 1, 2, 3, \ldots, k$ *and* $j = 1, 2, 3, \ldots, p$. *The union* $H_1 \cup H_2 = (X_1 \cup X_2, E_1 \cup E_2)$ *of* H_1 *and* H_2 *is defined by*

$$(A_1 \cup A_2)(x) = \begin{cases} A_1(x) & x \in X_1 - X_2 \\ A_2(x) & x \in X_2 - X_1 \\ \max(A_1(x), A_2(x)) & x \in X_1 \cap X_2 \end{cases}$$

$$(B_1 \cup B_2)(x) = \begin{cases} B_1(x) & x \in X_1 - X_2 \\ B_2(x) & x \in X_2 - X_1 \\ \min(B_1(x), B_2(x)) & x \in X_1 \cap X_2 \end{cases}$$

$$(C_1 \cup C_2)(x) = \begin{cases} C_1(x) & x \in X_1 - X_2 \\ C_2(x) & x \in X_2 - X_1 \\ \min(C_1(x), C_2(x)) & x \in X_1 \cap X_2 \end{cases}$$

$$(T_{E_{1i}} \cup T_{E_{2j}})(x) = \begin{cases} T_{E_{1i}}(x) & x \in X_1 - X_2 \\ T_{E_{2j}}(x) & x \in X_2 - X_1 \\ \max(T_{E_{1i}}(x), T_{E_{2j}}(x)) & x \in X_1 \cap X_2 \end{cases}$$

$$(I_{E_{1i}} \cup I_{E_{2j}})(x) = \begin{cases} I_{E_{1i}}(x) & x \in X_1 - X_2 \\ I_{E_{2j}}(x) & x \in X_2 - X_1 \\ \min(I_{E_{1i}}(x), I_{E_{2j}}(x)) & x \in X_1 \cap X_2 \end{cases}$$

$$(F_{E_{1i}} \cup F_{E_{2j}})(x) = \begin{cases} F_{E_{1i}}(x) & x \in X_1 - X_2 \\ F_{E_{2j}}(x) & x \in X_2 - X_1 \\ \min(F_{E_{1i}}(x), F_{E_{2j}}(x)) & x \in X_1 \cap X_2 \end{cases}$$

Remark 3.2. *If* $H_1 = (X_1, E_1)$ *and* $H_2 = (X_2, E_2)$ *be two GSVNHGs, then* $H_1 \cup H_2$ *is also GSVNHG.*

Remark 3.3. *If* $H_1 = (X_1, E_1)$ *and* $H_2 = (X_2, E_2)$ *be two GSSVNHGs, then* $H_1 \cup H_2$ *is also GSSVNHG.*

Definition 3.7. *Let* $H_1 = (X_1, E_1)$ *and* $H_2 = (X_2, E_2)$ *be two GSVNHGs, where* $X_1 = \{x_1, x_2, \ldots, x_n\}$, $X_2 = \{y_1, y_2, \ldots, y_n\}$, $A_1, B_1, C_1 : X_1 \to [0,1]$, $A_2, B_2, C_2 : X_2 \to [0,1]$,

$$E_1 = \{(T_{E_{11}}, I_{E_{11}}, F_{E_{11}}), (T_{E_{12}}, I_{E_{12}}, F_{E_{12}}), \ldots, (T_{E_{1k}}, I_{E_{1k}}, F_{E_{1k}})\},$$
$$E_2 = \{(T_{E_{21}}, I_{E_{21}}, F_{E_{21}}), (T_{E_{22}}, I_{E_{22}}, F_{E_{22}}), \ldots, (T_{E_{2p}}, I_{E_{2p}}, F_{E_{2p}})\},$$

where

$$T_{E_{1i}}, I_{E_{1i}}, F_{E_{1i}} : X_1 \to [0,1],$$
$$T_{E_{2j}}, I_{E_{2j}}, F_{E_{2j}} : X_2 \to [0,1],$$

$\forall i = 1, 2, 3, \ldots, k$ *and* $j = 1, 2, 3, \ldots, p$. *The cartesian product* $H_1 \times H_2$ *of* H_1 *and* H_2 *is defined by an ordered pair* $H_1 \times H_2 = (X_1 \times X_2, E_1 \times E_2)$, *where*

$$\begin{aligned}(A_1 \times A_2)(x,y) &= \min(A_1(x), A_2(x))\\(B_1 \times B_2)(x,y) &= \max(B_1(x), B_2(x))\\(C_1 \times C_2)(x,y) &= \max(C_1(x), C_2(x))\end{aligned}$$

$$\begin{aligned}(T_{E_{1i}} \times T_{E_{2j}})(x,y) &= \min(T_{E_{1i}}(x), T_{E_{2j}}(y))\\(I_{E_{1i}} \times I_{E_{2j}})(x,y) &= \max(I_{E_{1i}}(x), I_{E_{2j}}(y))\\(F_{E_{1i}} \times F_{E_{2j}})(x,y) &= \max(F_{E_{1i}}(x), F_{E_{2j}}(y))\end{aligned}$$

$\forall x \in X_1,\ y \in X_2,\ i = 1, 2, 3, \ldots, k$ *and* $j = 1, 2, 3, \ldots, p$.

Remark 3.4. *If both* H_1 *and* H_2 *are not GSSVNHGs, then* $H_1 \times H_2$ *may or may not be GSSVNHG.*

Example 3.4. *Consider a GSVNHGs* $H_1 = (X_1, E_1)$ *and* $H_2 = (X_2, E_2)$ *where* $X_1 = \{a, b\}$, $X_2 = \{p, q\}$, $E_1 = \{P, Q\}$ $E_2 = \{P', Q'\}$. *Also* $A_1, B_1, C_1 : X_1 \to [0,1]$ *defined by* $A_1(a) = .3$, $A_1(b) = .5$, $B_1(a) = .2$, $B_1(b) = .4$, $C_1(a) = .5$, $C_1(b) = .5$ *and* $A_2, B_2, C_2 : X_2 \to [0,1]$ *defined by* $A_2(p) = .5$, $A_2(q) = .9$, $B_2(p) = .1$, $B_2(q) = .5$, $C_2(p) = .5$, $C_2(q) = .5$,

$$P = \{(a, .1, .2, .5), (b, .5, .4, .5)\},\ Q = \{(a, .3, .4, .5), (b, .4, .6, .5)\},$$
$$P' = \{(p, .5, .3, .5), (q, .8, .5, .5)\},\ Q' = \{(p, .4, .6, .5), (q, .1, .5, .5)\}.$$

Then by routine calculations H_1 *is GSSVNHG and* H_2 *is GSVNHG. Let* $H = (X_1 \times X_2, E_1 \times E_2)$, $A = A_1 \times A_2$, $B = B_1 \times B_2$, $C = C_1 \times C_2$. *Then by routine calculations,* $A((a,p)) = .3$, $A((a,q)) = .3$, $A((b,p)) = .5$, $A((b,q)) = .5$, $B((a,p)) = .2$, $B((a,q)) = .5$, $B((b,p)) = .4$, $B((b,q)) = .5$, $C((a,p)) = .5$ $C((a,q)) = .5$, $C((b,p)) = .5$, $C((b,q)) = .5$,

$$\begin{aligned}P \times P' &= \{((a,p), .1, .3, .5), ((a,q), .1, .5, .5), ((b,p), .5, .4, .5), ((b,q), .5, .5, .5)\},\\P \times Q' &= \{((a,p), .1, .6, .5), ((a,q), .1, .5, .5), ((b,p), .4, .6, .5), ((b,q), .1, .5, .5)\},\\Q \times P' &= \{((a,p), .3, .4, .5), ((a,q), .3, .5, .5), ((b,p), .4, .6, .5), ((b,q), .4, .6, .5)\},\\Q \times Q' &= \{((a,p), .3, .6, .5), ((a,q), .1, .5, .5), ((b,p), .4, .6, .5), ((b,q), .1, .6, .5)\}.\end{aligned}$$

By calculations H *is not GSSVNHG.*

Example 3.5. *Consider the GSVNHGs $H_1 = (X_1, E_1)$ and $H_2 = (X_2, E_2)$ where $X_1 = \{a, b\}$, $X_2 = \{p, q\}$, $E_1 = \{P, Q\}$, $E_2 = \{P', Q'\}$. Also $A_1, B_1, C_1 : X_1 \to [0, 1]$ defined by $A_1(a) = .3$, $A_1(b) = .5$, $B_1(a) = .3$, $B_1(b) = .4$, $C_1(a) = .5$, $C_1(b) = .5$ and $A_2, B_2, C_2 : X_2 \to [0, 1]$ defined by $A_2(p) = .5$, $A_2(q) = .9$, $B_2(p) = .1$, $B_2(q) = .5$, $C_2(p) = .5$, $C_2(q) = .5$,*

$$P = \{(a, .1, .3, .5), (b, .5, .4, .5)\},\ Q = \{(a, .3, .4, .5), (b, .4, .6, .5)\},$$
$$P' = \{(p, .5, .3, .5), (q, .8, .5, .5)\},\ Q' = \{(p, .4, .6, .5), (q, .1, .5, .5)\}.$$

Then by routine calculations H_1 is GSSVNHG and H_2 is GSVNHG. Let $H = (X_1 \times X_2, E_1 \times E_2)$, $A = A_1 \times A_2$, $B = B_1 \times B_2$, $C = C_1 \times C_2$, then by routine calculations, $A((a, p)) = .3$, $A((a, q)) = .3$, $A((b, p)) = .5$, $A((b, q)) = .5$, $B((a, p)) = .3$, $B((a, q)) = .5$, $B((b, p)) = .4$, $B((b, q)) = .5$, $C((a, p)) = .5$, $C((a, q)) = .5$, $C((b, p)) = .5$, $C((b, q)) = .5$,

$$P \times P' = \{((a, p), .1, .3, .5), ((a, q), .1, .5, .5), ((b, p), .5, .4, .5), ((b, q), .5, .5, .5)\},$$
$$P \times Q' = \{((a, p), .1, .6, .5), ((a, q), .1, .5, .5), ((b, p), .4, .6, .5), ((b, q), .1, .5, .5)\},$$
$$Q \times P' = \{((a, p), .3, .4, .5), ((a, q), .3, .5, .5), ((b, p), .4, .6, .5), ((b, q), .4, .6, .5)\},$$
$$Q \times Q' = \{((a, p), .3, .6, .5), ((a, q), .1, .5, .5), ((b, p), .4, .6, .5), ((b, q), .1, .6, .5)\}.$$

By calculations H is GSSVNHG.

Proposition 3.1. *If both H_1 and H_2 are GSVNHGs, then $H_1 \times H_2$ is also GSVNHG.*

Proof. Let $H_1 = (X_1, E_1)$ and $H_2 = (X_2, E_2)$ be two GSVNHGs, where $X_1 = \{x_1, x_2, \ldots, x_n\}$, $X_2 = \{y_1, y_2, \ldots, y_n\}$, $A_1, B_1, C_1 : X_1 \to [0, 1]$, $A_2, B_2, C_2 : X_2 \to [0, 1]$,

$$\begin{aligned} E_1 &= \{(T_{E_{11}}, I_{E_{11}}, F_{E_{11}}), (T_{E_{12}}, I_{E_{12}}, F_{E_{12}}), \ldots, (T_{E_{1k}}, I_{E_{1k}}, F_{E_{1k}})\} \\ E_2 &= \{(T_{E_{21}}, I_{E_{21}}, F_{E_{21}}), (T_{E_{22}}, I_{E_{22}}, F_{E_{22}}), \ldots, (T_{E_{2p}}, I_{E_{2p}}, F_{E_{2p}})\} \end{aligned}$$

where

$$T_{E_{1i}}, I_{E_{1i}}, F_{E_{1i}} : X_1 \to [0, 1],$$
$$T_{E_{2j}}, I_{E_{2j}}, F_{E_{2j}} : X_2 \to [0, 1],$$

$\forall i = 1, 2, 3, \ldots, k$ and $j = 1, 2, 3, \ldots, p$. Then the cartesian product $H_1 \times H_2 = (X_1 \times X_2, E_1 \times E_2)$, where

$$E_1 \times E_2 = \{((T_{E_{11}} \times T_{E_{21}}), (I_{E_{11}} \times I_{E_{21}}), (F_{E_{11}} \times F_{E_{21}})), \ldots, ((T_{E_{11}} \times T_{E_{2p}}), (I_{E_{11}} \times I_{E_{2p}}), (F_{E_{11}} \times F_{E_{2p}})), \ldots, ((T_{E_{1k}} \times T_{E_{2p}}), (I_{E_{1k}} \times I_{E_{2p}}), (F_{E_{1k}} \times F_{E_{2p}}))\}$$

with

$$\bigvee_{r=1}^{k} T_{E_{1r}}(x_i) \leq A_1(x_i), \bigvee_{s=1}^{p} T_{E_{2s}}(y_j) \leq A_2(y_j)$$
$$\bigwedge_{r=1}^{k} I_{E_{1r}}(x_i) \geq B_1(x_i), \bigwedge_{s=1}^{p} I_{E_{2s}}(y_j) \geq B_2(y_j)$$
$$\bigwedge_{r=1}^{k} F_{E_{1r}}(x_i) \geq C_1(x_i), \bigwedge_{s=1}^{p} F_{E_{2s}}(y_j) \geq C_2(y_j)$$

$\forall i = 1, 2, 3, \dots, n$ and $j = 1, 2, 3, \dots, m$. Now consider

$$\begin{aligned}\bigvee_{s=1}^{p}\bigvee_{r=1}^{k}(T_{E_{1r}} \times T_{E_{2s}})(x_i, y_j) &= \bigvee_{s=1}^{p}\bigvee_{r=1}^{k}(T_{E_{1r}}(x_i), T_{E_{2s}}(y_j)) \\ &= (\bigvee_{r=1}^{k} T_{E_{1r}}(x_i)) \wedge (\bigvee_{s=1}^{p} T_{E_{2s}}(y_j)) \\ &\leq A_1(x_i) \wedge A_2(y_j) = (A_1 \times A_2)(x_i, y_j)\end{aligned}$$

$\forall i$ and j. Similarly others can be proved. Thus $H_1 \times H_2$ is the GSVNHG.

□

Proposition 3.2. *If both H_1 and H_2 are GSSVNHGs, then $H_1 \times H_2$ is also GSSVNHG.*

Proposition 3.3. *If $H_1 \times H_2$ is GSSVNHG, then at least H_1 or H_2 must be GSSVNHG.*

Proof. Suppose $H_1 \times H_2$ is GSSVNHG, but H_1 and H_2 are not GSSVNHGs, then by definition

$$\begin{aligned}&\bigvee_{r=1}^{k} T_{E_{1r}}(x_i) < A_1(x_i), \ \bigvee_{s=1}^{p} T_{E_{2s}}(y_j) < A_2(y_j) \\ &\bigwedge_{r=1}^{k} I_{E_{1r}}(x_i) > B_1(x_i), \ \bigwedge_{s=1}^{p} I_{E_{2s}}(y_j) > B_2(y_j) \\ &\bigwedge_{r=1}^{k} F_{E_{1r}}(x_i) > C_1(x_i), \ \bigwedge_{s=1}^{p} F_{E_{2s}}(y_j) > C_2(y_j)\end{aligned}$$

$\forall i = 1, 2, 3, \dots, n$ and $j = 1, 2, 3, \dots, m$. Therefore

$$\begin{aligned}\bigvee_{s=1}^{p}\bigvee_{r=1}^{k}(T_{E_{1r}} \times T_{E_{2s}})(x_i, y_j) &= \bigvee_{s=1}^{p}\bigvee_{r=1}^{k}(T_{E_{1r}}(x_i), T_{E_{2s}}(y_j)) \\ &= (\bigvee_{r=1}^{k} T_{E_{1r}}(x_i)) \wedge (\bigvee_{s=1}^{p} T_{E_{2s}}(y_j))) \\ &< A_1(x_i) \wedge A_2(y_j) = (A_1 \times A_2)(x_i, y_j)\end{aligned}$$

$\forall i$ and j. Similarly

$$\begin{aligned}&\bigwedge_{s=1}^{p}\bigwedge_{r=1}^{k}(I_{E_{1r}} \times I_{E_{2s}})(x_i, y_j) > (B_1 \times B_2)(x_i, y_j) \\ &\bigwedge_{s=1}^{p}\bigwedge_{r=1}^{k}(F_{E_{1r}} \times F_{E_{2s}})(x_i, y_j) > (C_1 \times C_2)(x_i, y_j)\end{aligned}$$

$\forall i$ and j. Therefore $H_1 \times H_2$ is not GSSVNHG, hence at least one of H_1 or H_2 must be GSSVNHG. □

4. Generalized strong BSVNHGs

Definition 4.1. *The bipolar single valued neutrosophic hypergraph (BSVNHG) $H = (Z, \Theta)$, where*

(1) *$Z = \{\eta_1, \eta_2, \dots, \eta_n\}$ be a finite set of vertices.*

(2) $\Theta = \{\Theta_1, \Theta_2, \ldots, \Theta_m\}$ *be a set of BSVNSs of* Z.
(3) $\Theta_j \neq O = (0,0,0,0,0,0)$ $\forall j = 1,2,3,\ldots,m$ *and* $\bigcup_j Supp(\Theta_j) = Z$.

Definition 4.2. *A generalized bipolar single valued neutrosophic hypergraph (GBSVNHG) be a* $H = (X, E)$, *where*
(1) $X = \{x_1, x_2, \ldots, x_n\}$ *be a finite set of vertices.*
(2) $A^+, B^+, C^+ : X \to [0,1]$ *and* $A^-, B^-, C^- : X \to [-1,0]$ *be the BSVNSs of vertices.*
(3) $E = \{E_1, E_2, \ldots, E_m\}$ *be the set of BSVNSs of* X, *where*
$E_j = \{(x_i, T^+_{E_j}(x_i), I^+_{E_j}(x_i), F^+_{E_j}(x_i), T^-_{E_j}(x_i), I^-_{E_j}(x_i), F^-_{E_j}(x_i)) : T^+_{E_j}(x_i), I^+_{E_j}(x_i), F^+_{E_j}(x_i) : X \to [0,1], T^-_{E_j}(x_i), I^-_{E_j}(x_i), F^-_{E_j}(x_i) : X \to [-1,0]\}$, *with*

$$\bigvee_{j=1}^{m} T^+_{E_j}(x_i) \leq A^+(x_i), \ \bigwedge_{j=1}^{m} I^+_{E_j}(x_i) \geq B^+(x_i), \ \bigwedge_{j=1}^{m} F^+_{E_j}(x_i) \geq C^+(x_i)$$

$$\bigwedge_{j=1}^{m} T^-_{E_j}(x_i) \geq A^-(x_i), \ \bigvee_{j=1}^{m} I^-_{E_j}(x_i) \leq B^-(x_i), \ \bigvee_{j=1}^{m} F^-_{E_j}(x_i) \leq C^-(x_i)$$

$\forall i = 1,2,3,\ldots,n$ *and* $\forall j = 1,2,3,\ldots,m$.
(4) $E_j \neq O = (0,0,0,0,0,0)$, $\forall j = 1,2,3,\ldots,m$ *and* $\bigcup_j Supp(E_j) = X$.

Remark 4.1. *The generalized bipolar single valued neutrosophic hypergraph is the generalization of generalized intuitionistic fuzzy hypergraphs and generalized single valued neutrosophic hyper graphs.*

Example 4.1. *Consider the* $H = (X, E)$, *where* $X = \{\alpha, \beta, \gamma\}$ *and* $E = \{E_1, E_2, E_3\}$. *The BSVN-Vertices and BSVN-Edges are defined in Tables. 1 and 2.*

	ϕ^+	φ^+	χ^+	ϕ^-	φ^-	χ^-
α	.7	.2	.2	$-.6$	$-.2$	.0
β	.6	.5	.2	$-.3$	$-.1$	$-.2$
γ	.9	.1	.2	$-.7$	$-.2$	.0

Table 1. BSVN-Vertices of GBSVNHG.

	E_1	E_2	E_3
α	$(.2, .3, .5, -.6, -.2, -.9)$	$(.3, .5, .6, -.2, -.3, -.2)$	$(.6, .2, .3, -.1, -.2, .0)$
β	$(.5, .6, .3, -.1, -.2, -.3)$	$(.5, .6, .2, -.3, -.1, -.2)$	$(.6, .8, .2, -.1, -.5, -.2)$
γ	$(.8, .2, .3, -.1, -.2, -.8)$	$(.3, .1, .8, -.1, -.2, -.3)$	$(.6, .2, .8, -.7, -.8, .0)$

Table 2. BSVN-Hyperedges of GBSVNHG.

Then by routine calculations H *is GBSVNHG.*

Definition 4.3. *The GBSVNHG* $H = (X, E)$ *is said to be generalized strong bipolar single valued neutrosophic hypergraph (GSBSVNHG), if*

$$\bigvee_{j=1}^{m} T^+_{E_j}(x_i) = A^+(x_i), \ \bigwedge_{j=1}^{m} I^+_{E_j}(x_i) = B^+(x_i), \ \bigwedge_{j=1}^{m} F^+_{E_j}(x_i) = C^+(x_i)$$

$$\bigwedge_{j=1}^{m} T^-_{E_j}(x_i) = A^-(x_i), \ \bigvee_{j=1}^{m} I^-_{E_j}(x_i) = B^-(x_i), \ \bigvee_{j=1}^{m} F^-_{E_j}(x_i) = C^-(x_i)$$

$\forall i = 1,2,3,\ldots,n$ *and* $\forall j = 1,2,3,\ldots,m$.

Example 4.2. *Consider the GBSVNHG $H = (X, E)$, where $X = \{\alpha, \beta, \gamma\}$ and $E = \{E_1, E_2, E_3, E_4\}$. The BSVN-Vertices and BSVN-Edges are defined in Tables. 3 and 4.*

	ϕ^+	φ^+	χ^+	ϕ^-	φ^-	χ^-
α	.6	.2	.2	−.6	−.2	.0
β	.6	.6	.2	−.3	−.1	−.2
γ	.8	.1	.2	−.7	−.2	.0

Table 3. BSVN-Vertices of GSBSVNHG.

	E_1	E_2	E_3
α	(.2, .3, .5, −.6, −.2, −.3)	(.3, .5, .8, −.2, −.3, −.2)	(.6, .2, .3, −.1, −.2, .0)
β	(.5, .6, .3, −.1, −.2, −.3)	(.5, .6, .2, −.3, −.1, −.2)	(.6, .8, .2, −.1, −.5, −.2)
γ	(.8, .2, .3, −.1, −.2, −.8)	(.3, .1, .8, −.1, −.2, −.3)	(.6, .2, .8, −.7, −.8, .0)

Table 4. BSVN-Hyperedges of GSBSVNHG.

Then by routine calculations H is GSBSVNHG.

Definition 4.4. *Let $H = (X, E)$ be a GBSVNHG, let $A^+, B^+, C^+ : X \to [0, 1]$, $A^-, B^-, C^- : X \to [-1, 0]$*

$$E = \{(T^+_{E_j}, I^+_{E_j}, F^+_{E_j}, T^-_{E_j}, I^-_{E_j}, F^-_{E_j}) : X \to [0, 1]^3 \times [-1, 0]^3 : j = 1, 2, 3, \dots, m\}$$

and let $H' = (X, E')$ where $A'^+, B'^+, C'^+ : X \to [0, 1]$, $A'^-, B'^-, C'^- : X \to [-1, 0]$

$$E' = \{(T'^+_{E_j}, I'^+_{E_j}, F'^+_{E_j}, T'^-_{E_j}, I'^-_{E_j}, F'^-_{E_j}) : X \to [0, 1]^3 \times [-1, 0]^3 : j = 1, 2, 3, \dots, m\}$$

H' is said to be a generalized bipolar single valued neutrosophic sub hypergraph (GBSVNSHG) of H, whenever

$$\bigvee_{j=1}^{m} T'^+_{E_j}(x_i) \leq \bigvee_{j=1}^{m} T^+_{E_j}(x_i), \ \bigwedge_{j=1}^{m} I'^+_{E_j}(x_i) \geq \bigwedge_{j=1}^{m} I^+_{E_j}(x_i), \ \bigwedge_{j=1}^{m} F'^+_{E_j}(x_i) \geq \bigwedge_{j=1}^{m} F^+_{E_j}(x_i)$$

$$\bigwedge_{j=1}^{m} T'^-_{E_j}(x_i) \geq \bigwedge_{j=1}^{m} T^-_{E_j}(x_i), \ \bigvee_{j=1}^{m} I'^-_{E_j}(x_i) \leq \bigvee_{j=1}^{m} I^-_{E_j}(x_i), \ \bigvee_{j=1}^{m} F'^-_{E_j}(x_i) \leq \bigvee_{j=1}^{m} F^-_{E_j}(x_i)$$

$$A'^+(x_i) \leq A^+(x_i), \ B'^+(x_i) \geq B^+(x_i), \ C'^+(x_i) \geq C^+(x_i)$$

$$A'^-(x_i) \geq A^-(x_i), \ B'^-(x_i) \leq B^+(x_i), \ C'^-(x_i) \leq C^-(x_i)$$

$\forall i = 1, 2, 3, \dots, n$. The GBSVNHG $H' = (X, E')$ is said to be a spanning generalized bipolar single valued neutrosophic sub hypergraph (SGBSVNSHG) of $H = (X, E)$, whenever

$$A'^+(x_i) = A^+(x_i), \ B'^+(x_i) = B^+(x_i), \ C'^+(x_i) = C^+(x_i)$$

$$A'^-(x_i) = A^-(x_i), \ B'^-(x_i) = B^-(x_i), \ C'^-(x_i) = C^-(x_i)$$

$\forall i = 1, 2, 3, \dots, n$.

Definition 4.5. *Let $H = (X, E)$ be a GBSVNHG where $A^+, B^+, C^+ : X \to [0, 1]$, $A^-, B^-, C^- : X \to [-1, 0]$,*

$$E = \{(T^+_{E_j}, I^+_{E_j}, F^+_{E_j}, T^-_{E_j}, I^-_{E_j}, F^-_{E_j}) : X \to [0, 1]^3 \times [-1, 0]^3 : j = 1, 2, 3, \dots, m\}$$

and let $H^{'} = (X, E^{'})$ *where* $A^{'+}, B^{'+}, C^{'+} : X \to [0,1]$, $A^{'-}, B^{'-}, C^{'-} : X \to [-1,0]$

$$E^{'} = \{(T^{'+}_{E_j}, I^{'+}_{E_j}, F^{'+}_{E_j}, T^{'-}_{E_j}, I^{'-}_{E_j}, F^{'-}_{E_j}) : X \to [0,1]^3 \times [-1,0]^3 : j = 1,2,3,\ldots,m\}$$

$H^{'}$ *is said to be be a generalized strong bipolar single valued neutrosophic sub hypergraph (GSBSVNSHG) of* H, *if*

$$\bigvee_{j=1}^{m} T^{'+}_{E_j}(x_i) = \bigvee_{j=1}^{m} T^{+}_{E_j}(x_i),\ \bigwedge_{j=1}^{m} I^{'+}_{E_j}(x_i) = \bigwedge_{j=1}^{m} I^{+}_{E_j}(x_i),\ \bigwedge_{j=1}^{m} F^{'+}_{E_j}(x_i) = \bigwedge_{j=1}^{m} F^{+}_{E_j}(x_i)$$

$$\bigwedge_{j=1}^{m} T^{'-}_{E_j}(x_i) = \bigwedge_{j=1}^{m} T^{-}_{E_j}(x_i),\ \bigvee_{j=1}^{m} I^{'-}_{E_j}(x_i) = \bigvee_{j=1}^{m} I^{-}_{E_j}(x_i),\ \bigvee_{j=1}^{m} F^{'-}_{E_j}(x_i) = \bigvee_{j=1}^{m} F^{-}_{E_j}(x_i)$$

$$A^{'+}(x_i) = A^{+}(x_i),\ B^{'+}(x_i) = B^{+}(x_i),\ C^{'+}(x_i) = C^{+}(x_i)$$

$$A^{'-}(x_i) = A^{-}(x_i),\ B^{'-}(x_i) = B^{-}(x_i),\ C^{'-}(x_i) = C^{-}(x_i)$$

$\forall i = 1,2,3,\ldots,n$ *and the GBSVNHG* $H^{'} = (X, E^{'})$ *is said to be a spanning generalized strong bipolar single valued neutrosophic sub hypergraph (SGSBSVNSHG) of* $H = (X, E)$ *if*

$$A^{'+}(x_i) = A^{+}(x_i),\ B^{'+}(x_i) = B^{+}(x_i),\ C^{'+}(x_i) = C^{+}(x_i)$$

$$A^{'-}(x_i) = A^{-}(x_i),\ B^{'-}(x_i) = B^{-}(x_i),\ C^{'-}(x_i) = C^{-}(x_i)$$

$\forall i = 1,2,3,\ldots,n$.

Example 4.3. *Consider the GBSVNHGs* $G = (X, E)$, $H = (X, E^{'})$ *and* $S = (X, E^{''})$ *where* $X = \{\alpha, \beta, \gamma\}$, $E = \{E_1, E_2\}$, $E^{'} = \{E^{'}_1, E^{'}_2\}$ *and* $E^{''} = \{E^{''}_1, E^{''}_2\}$. *Also* $\phi^+, \varphi^+, \chi^+ : V \to [0,1]$ *defined by* $\phi^+(\alpha) = .4$, $\phi^+(\beta) = .5$, $\varphi^+(\alpha) = .2$, $\varphi^+(\beta) = .2$, $\chi^+(\alpha) = .3$, $\chi^+(\beta) = .0$, $\phi^{'+}(\alpha) = .4$, $\phi^{'+}(\beta) = .4$, $\varphi^{'+}(\alpha) = .1$, $\varphi^{'+}(\beta) = .1$, $\chi^{'+}(\alpha) = .3$, $\chi^{'+}(\beta) = .0$, $\phi^{''+}(\alpha) = .4$, $\phi^{''+}(\beta) = .5$, $\varphi^{''+}(\alpha) = .2$, $\varphi^{''+}(\beta) = .2$, $\chi^{''+}(\alpha) = .3$, $\chi^{''+}(\beta) = .0$ *and* $\phi^-, \varphi^-, \chi^- : V \to [-1,0]$ *defined by* $\phi^-(\alpha) = -.1$, $\phi^-(\beta) = -.1$, $\varphi^-(\alpha) = -.2$, $\varphi^-(\beta) = -.2$, $\chi^-(\alpha) = -.3$, $\chi^-(\beta) = -.3$, $\phi^{'-}(\alpha) = -.1$, $\phi^{'-}(\beta) = -.1$, $\varphi^{'-}(\alpha) = -.2$, $\varphi^{'-}(\beta) = -.2$, $\chi^{'-}(\alpha) = -.3$, $\chi^{'-}(\beta) = -.3$, $\phi^{''-}(\alpha) = -.1$, $\phi^{''-}(\beta) = -.1$, $\varphi^{''-}(\alpha) = -.2$, $\varphi^{''-}(\beta) = -.2$, $\chi^{''-}(\alpha) = -.3$, $\chi^{''-}(\beta) = -.3$,

$$\begin{aligned}
E_1 &= \{(\alpha, .2, .3, .6, -.1, -.2, -.3), (\beta, .5, .6, .2, -.1, -.2, -.3)\},\\
E_2 &= \{(\alpha, .4, .2, .3, -.1, -.2, -.3), (\beta, .3, .2, .5, -.1, -.2, -.3)\},\\
E^{'}_1 &= \{(\alpha, .2, .3, .5, -.1, -.2, -.3), (\beta, .4, .3, .5, -.1, -.2, -.3)\},\\
E^{'}_2 &= \{(\alpha, .3, .2, .3, -.1, -.2, -.3), (\beta, .3, .4, .3, -.1, -.2, -.3)\},\\
E^{''}_1 &= \{(\alpha, .2, .3, .5, -.1, -.2, -.3), (\beta, .5, .3, .5, -.1, -.2, -.3)\},\\
E^{''}_2 &= \{(\alpha, .4, .2, .3, -.1, -.2, -.3), (\beta, .3, .4, .3, -.1, -.2, -.3)\}.
\end{aligned}$$

Then by routine calculations H *is* $GBSVNSHG$ *of* G *but* S *is* $SGBSVNSHG$ *of* G.

Definition 4.6. *Let* $H_1 = (Z_1, E_1)$ *and* $H_2 = (Z_2, E_2)$ *be two GBSVNHGs, where* $Z_1 = \{x_1, x_2, \ldots, x_n\}$, $Z_2 = \{y_1, y_2, \ldots, y_n\}$, $A_1^+, B_1^+, C_1^+ : Z_1 \to [0,1]$, $A_1^-, B_1^-, C_1^- : Z_1 \to [-1,0]$, $A_2^+, B_2^+, C_2^+ : Z_2 \to [0,1]$, $A_2^-, B_2^-, C_2^- : Z_2 \to [-1,0]$ *and*

$$\begin{aligned}
E_1 &= \{(T^+_{E_{11}}, I^+_{E_{11}}, F^+_{E_{11}}, T^-_{E_{11}}, I^-_{E_{11}}, F^-_{E_{11}}), \ldots, (T^+_{E_{1k}}, I^+_{E_{1k}}, F^+_{E_{1k}}, T^-_{E_{1k}}, I^-_{E_{1k}}, F^-_{E_{1k}})\}\\
E_2 &= \{(T^+_{E_{21}}, I^+_{E_{21}}, F^+_{E_{21}}, T^-_{E_{21}}, I^-_{E_{21}}, F^-_{E_{21}}), \ldots, (T^+_{E_{2p}}, I^+_{E_{2p}}, F^+_{E_{2p}}, T^-_{E_{2p}}, I^-_{E_{2p}}, F^-_{E_{2p}})\}
\end{aligned}$$

where

$$T^+_{E_{1i}}, I^+_{E_{1i}}, F^+_{E_{1i}} : Z_1 \to [0,1],\ T^-_{E_{1i}}, I^-_{E_{1i}}, F^-_{E_{1i}} : Z_1 \to [-1,0]$$
$$T^+_{E_{2j}}, I^+_{E_{2j}}, F^+_{E_{2j}} : Z_2 \to [0,1],\ T^-_{E_{2j}}, I^-_{E_{2j}}, F^-_{E_{2j}} : Z_2 \to [-1,0]$$

$\forall i = 1,2,3,\ldots,k$ and $j = 1,2,3,\ldots,p$. The union $H_1 \cup H_2 = (Z_1 \cup Z_2, E_1 \cup E_2)$ of H_1 and H_2 are defined as follows

$$(A_1^+ \cup A_2^+)(\xi) = \begin{cases} A_1^+(\xi) & \xi \in Z_1 - Z_2 \\ A_2^+(\xi) & \xi \in Z_2 - Z_1 \\ \max(A_1^+(\xi), A_2^+(\xi)) & \xi \in Z_1 \cap Z_2 \end{cases}$$

$$(B_1^+ \cup B_2^+)(\xi) = \begin{cases} B_1^+(\xi) & \xi \in Z_1 - Z_2 \\ B_2^+(\xi) & \xi \in Z_2 - Z_1 \\ \min(B_1^+(\xi), B_2^+(\xi)) & \xi \in Z_1 \cap Z_2 \end{cases}$$

$$(C_1^+ \cup C_2^+)(\xi) = \begin{cases} C_1^+(\xi) & \xi \in Z_1 - Z_2 \\ C_2^+(\xi) & \xi \in Z_2 - Z_1 \\ \min(C_1^+(\xi), C_2^+(\xi)) & \xi \in Z_1 \cap Z_2 \end{cases}$$

$$(A_1^- \cup A_2^-)(\xi) = \begin{cases} A_1^-(\xi) & \xi \in Z_1 - Z_2 \\ A_2^-(\xi) & \xi \in Z_2 - Z_1 \\ \min(A_1^-(\xi), A_2^-(\xi)) & \xi \in Z_1 \cap Z_2 \end{cases}$$

$$(B_1^- \cup B_2^-)(\xi) = \begin{cases} B_1^-(\xi) & \xi \in Z_1 - Z_2 \\ B_2^-(\xi) & \xi \in Z_2 - Z_1 \\ \max(B_1^-(\xi), B_2^-(\xi)) & \xi \in Z_1 \cap Z_2 \end{cases}$$

$$(C_1^- \cup C_2^-)(\xi) = \begin{cases} C_1^-(\xi) & \xi \in Z_1 - Z_2 \\ C_2^-(\xi) & \xi \in Z_2 - Z_1 \\ \max(C_1^-(\xi), C_2^-(\xi)) & \xi \in Z_1 \cap Z_2 \end{cases}$$

$$(T^+_{E_{1i}} \cup T^+_{E_{2j}})(\xi) = \begin{cases} T^+_{E_{1i}}(\xi) & \xi \in Z_1 - Z_2 \\ T^+_{E_{2j}}(\xi) & \xi \in Z_2 - Z_1 \\ \max(T^+_{E_{1i}}(\xi), T^+_{E_{2j}}(\xi)) & \xi \in Z_1 \cap Z_2 \end{cases}$$

$$(I^+_{E_{1i}} \cup I^+_{E_{2j}})(\xi) = \begin{cases} I^+_{E_{1i}}(\xi) & \xi \in Z_1 - Z_2 \\ I^+_{E_{2j}}(\xi) & \xi \in Z_2 - Z_1 \\ \min(I^+_{E_{1i}}(\xi), I^+_{E_{2j}}(\xi)) & \xi \in Z_1 \cap Z_2 \end{cases}$$

$$(F^+_{E_{1i}} \cup F^+_{E_{2j}})(\xi) = \begin{cases} F^+_{E_{1i}}(\xi) & \xi \in Z_1 - Z_2 \\ F^+_{E_{2j}}(\xi) & \xi \in Z_2 - Z_1 \\ \min(F^+_{E_{1i}}(\xi), F^+_{E_{2j}}(\xi)) & \xi \in Z_1 \cap Z_2 \end{cases}$$

$$(T^-_{E_{1i}} \cup T^-_{E_{2j}})(\xi) = \begin{cases} T^-_{E_{1i}}(\xi) & \xi \in Z_1 - Z_2 \\ T^-_{E_{2j}}(\xi) & \xi \in Z_2 - Z_1 \\ \min(T^-_{E_{1i}}(\xi), T^-_{E_{2j}}(\xi)) & \xi \in Z_1 \cap Z_2 \end{cases}$$

$$(I^-_{E_{1i}} \cup I^-_{E_{2j}})(\xi) = \begin{cases} I^-_{E_{1i}}(\xi) & \xi \in Z_1 - Z_2 \\ I^-_{E_{2j}}(\xi) & \xi \in Z_2 - Z_1 \\ \max(I^-_{E_{1i}}(\xi), I^-_{E_{2j}}(\xi)) & \xi \in Z_1 \cap Z_2 \end{cases}$$

$$(F^-_{E_{1i}} \cup F^-_{E_{2j}})(\xi) = \begin{cases} F^-_{E_{1i}}(\xi) & \xi \in Z_1 - Z_2 \\ F^-_{E_{2j}}(\xi) & \xi \in Z_2 - Z_1 \\ \max(F^-_{E_{1i}}(\xi), F^-_{E_{2j}}(\xi)) & \xi \in Z_1 \cap Z_2 \end{cases}$$

Remark 4.2. *If $H_1 = (Z_1, E_1)$ and $H_2 = (Z_2, E_2)$ be two GBSVNHGs, then $H_1 \cup H_2$ is also GBSVNHG.*

Remark 4.3. *If $H_1 = (Z_1, E_1)$ and $H_2 = (Z_2, E_2)$ be two GSBSVNHGs, then $H_1 \cup H_2$ is also GSBSVNHG.*

Definition 4.7. *Let $H_1 = (Z_1, E_1)$ and $H_2 = (Z_2, E_2)$ be two GBSVNHGs, where $Z_1 = \{x_1, x_2, \ldots, x_n\}$, $Z_2 = \{y_1, y_2, \ldots, y_n\}$, $A_1^+, B_1^+, C_1^+ : Z_1 \to [0,1]$, $A_1^-, B_1^-, C_1^- : Z_1 \to [-1,0]$, $A_2^+, B_2^+, C_2^+ : Z_2 \to [0,1]$, $A_2^-, B_2^-, C_2^- : Z_2 \to [-1,0]$,*

$$\begin{aligned} E_1 &= \{(T^+_{E_{11}}, I^+_{E_{11}}, F^+_{E_{11}}, T^-_{E_{11}}, I^-_{E_{11}}, F^-_{E_{11}}), \ldots, (T^+_{E_{1k}}, I^+_{E_{1k}}, F^+_{E_{1k}}, T^-_{E_{1k}}, I^-_{E_{1k}}, F^-_{E_{1k}})\}, \\ E_2 &= \{(T^+_{E_{21}}, I^+_{E_{21}}, F^+_{E_{21}}, T^-_{E_{21}}, I^-_{E_{21}}, F^-_{E_{21}}), \ldots, (T^+_{E_{2p}}, I^+_{E_{2p}}, F^+_{E_{2p}}, T^-_{E_{2p}}, I^-_{E_{2p}}, F^-_{E_{2p}})\}, \end{aligned}$$

where

$$\begin{aligned} T^+_{E_{1i}}, I^+_{E_{1i}}, F^+_{E_{1i}} : Z_1 \to [0,1],\ T^-_{E_{1i}}, I^-_{E_{1i}}, F^-_{E_{1i}} : Z_1 \to [-1,0], \\ T^+_{E_{2j}}, I^+_{E_{2j}}, F^+_{E_{2j}} : Z_2 \to [0,1],\ T^-_{E_{2j}}, I^-_{E_{2j}}, F^-_{E_{2j}} : Z_2 \to [-1,0], \end{aligned}$$

$\forall i = 1, 2, 3, \ldots, k$ and $j = 1, 2, 3, \ldots, p$. Then the cartesian product $H_1 \times H_2$ of H_1 and H_2 is defined as an ordered pair $H_1 \times H_2 = (Z_1 \times Z_2, E_1 \times E_2)$, where

$$\begin{aligned} (A_1^+ \times A_2^+)(\xi, \eta) = \min(A_1^+(\xi), A_2^+(\eta)),\ (A_1^- \times A_2^-)(\xi, \eta) = \max(A_1^-(\xi), A_2^-(\eta)) \\ (C_1^+ \times C_2^+)(\xi, \eta) = \max(C_1^+(\xi), C_2^+(\eta)),\ (C_1^- \times C_2^-)(\xi, \eta) = \min(C_1^-(\xi), C_2^-(\eta)) \\ (B_1^+ \times B_2^+)(\xi, \eta) = \max(B_1^+(\xi), B_2^+(\eta)),\ (B_1^- \times B_2^-)(\xi, \eta) = \min(B_1^-(\xi), B_2^-(\eta)) \end{aligned}$$

$E_1 \times E_2 = \{((T^+_{E_{11}} \times T^+_{E_{21}}), (I^+_{E_{11}} \times I^+_{E_{21}}), (F^+_{E_{11}} \times F^+_{E_{21}}), (T^-_{E_{11}} \times T^-_{E_{21}}), (I^-_{E_{11}} \times I^-_{E_{21}}), (F^-_{E_{11}} \times F^-_{E_{21}})), \ldots, ((T^+_{E_{11}} \times T^+_{E_{2p}}), (I^+_{E_{11}} \times I^+_{E_{2p}}), (F^+_{E_{11}} \times F^+_{E_{2p}}), (T^-_{E_{11}} \times T^-_{E_{2p}}), (I^-_{E_{11}} \times I^-_{E_{2p}}), (F^-_{E_{11}} \times F^-_{E_{2p}})), \ldots, ((T^+_{E_{1k}} \times T^+_{E_{2p}}), (I^+_{E_{1k}} \times I^+_{E_{2p}}), (F^+_{E_{1k}} \times F^+_{E_{2p}}), (T^-_{E_{1k}} \times T^-_{E_{2p}}), (I^-_{E_{1k}} \times I^-_{E_{2p}}), (F^-_{E_{1k}} \times F^-_{E_{2p}}))\}$, which are defined by

$$\begin{aligned} (I^+_{E_{1i}} \times I^+_{E_{2j}})(\xi, \eta) = \max(I^+_{E_{1i}}(\xi), I^+_{E_{2j}}(\eta)),\ (I^-_{E_{1i}} \times I^-_{E_{2j}})(\xi, \eta) = \min(I^-_{E_{1i}}(\xi), I^-_{E_{2j}}(\eta)) \\ (T^+_{E_{1i}} \times T^+_{E_{2j}})(\xi, \eta) = \min(T^+_{E_{1i}}(\xi), T^+_{E_{2j}}(\eta)),\ (T^-_{E_{1i}} \times T^-_{E_{2j}})(\xi, \eta) = \max(T^-_{E_{1i}}(\xi), T^-_{E_{2j}}(\eta)) \\ (F^+_{E_{1i}} \times F^+_{E_{2j}})(\xi, \eta) = \max(F^+_{E_{1i}}(\xi), F^+_{E_{2j}}(\eta)),\ (F^-_{E_{1i}} \times F^-_{E_{2j}})(\xi, \eta) = \min(F^-_{E_{1i}}(\xi), F^-_{E_{2j}}(\eta)) \end{aligned}$$

$\forall \xi \in Z_1$ and $\eta \in Z_2$, $\forall i = 1, 2, 3, \ldots, k$ and $\forall j = 1, 2, 3, \ldots, p$.

Remark 4.4. *If both H_1 and H_2 are not GSBSVNHGs, then $H_1 \times H_2$ may or may not be GSBSVNHG.*

Example 4.4. *Consider the GBSVNHGs $H_1 = (X_1, E_1)$ and $H_2 = (X_2, E_2)$ where $X_1 = \{a, b\}$, $X_2 = \{p, q\}$, $E_1 = \{P, Q\}$, $E_2 = \{P', Q'\}$. Also $A_1^+, B_1^+, C_1^+ : X_1 \to [0,1]$ defined by $A_1^+(a) = .3$, $A_1^+(b) = .5$, $B_1^+(a) = .2$, $B_1^+(b) = .4$, $C_1^+(a) = .5$, $C_1^+(b) = .5$, $A_2^+, B_2^+, C_2^+ : X_2 \to [0,1]$ defined by $A_2^+(p) = .5$, $A_2^+(q) = .9$, $B_2^+(p) = .1$, $B_2^+(q) = .5$, $C_2^+(p) = .5$, $C_2^+(q) = .5$, $A_1^-, B_1^-, C_1^- : X_1 \to [-1,0]$ defined by $A_1^-(a) = -.1$, $A_1^-(b) = -.1$, $B_1^-(a) = -.2$, $B_1^-(b) = -.2$, $C_1^-(a) = -.3$, $C_1^-(b) = -.3$, $A_2^-, B_2^-, C_2^- : X_2 \to [0,1]$ defined by $A_2^-(p) = -.1$, $A_2^-(q) = -.1$, $B_2^-(p) = -.2$, $B_2^-(q) = -.2$, $C_2^-(p) = -.3$, $C_2^-(q) = -.3$,*

$$\begin{aligned} P &= \{(a, .1, .2, .5, -.1, -.2, -.3), (b, .5, .4, .5, -.1, -.2, -.3)\}, \\ Q &= \{(a, .3, .4, .5, -.1, -.2, -.3), (b, .4, .6, .5, -.1, -.2, -.3)\}, \\ P' &= \{(p, .5, .3, .5, -.1, -.2, -.3), (q, .8, .5, .5, -.1, -.2, -.3)\}, \\ Q' &= \{(p, .4, .6, .5, -.1, -.2, -.3), (q, .1, .5, .5, -.1, -.2, -.3)\}. \end{aligned}$$

Then by routine calculations H_1 *is GSBSVNHG and* H_2 *is GBSVNHG. Let* $H = (X_1 \times X_2, E_1 \times E_2)$, $A = A_1 \times A_2$, $B = B_1 \times B_2$, $C = C_1 \times C_2$. *Then* $A^+((a,p)) = .3$, $A^+((a,q)) = .3$, $A^+((b,p)) = .5$, $A^+((b,q)) = .5$, $B^+((a,p)) = .2$, $B^+((a,q)) = .5$, $B^+((b,p)) = .4$, $B^+((b,q)) = .5$, $C^+((a,p)) = .5$, $C^+((a,q)) = .5$, $C^+((b,p)) = .5$, $C^+((b,q)) = .5$, $A^-((a,p)) = -.1$, $A^-((a,q)) = -.1$, $A^-((b,p)) = -.1$, $A^-((b,q)) = -.1$, $B^-((a,p)) = -.2$, $B^-((a,q)) = -.2$, $B^-((b,p)) = -.2$, $B^-((b,q)) = -.2$, $C^-((a,p)) = -.3$, $C^-((a,q)) = -.3$, $C^-((b,p)) = -.3$, $C^-((b,q)) = -.3$,

$$\begin{aligned}
P \times P' &= \{((a,p), .1, .3, .5, -.1, -.2, -.3), ((a,q), .1, .5, .5, -.1, -.2, -.3),\\
&\quad ((b,p), .5, .4, .5, -.1, -.2, -.3), ((b,q), .5, .5, .5, -.1, -.2, -.3)\},\\
P \times Q' &= \{((a,p), .1, .6, .5, -.1, -.2, -.3), ((a,q), .1, .5, .5, -.1, -.2, -.3),\\
&\quad ((b,p), .4, .6, .5, -.1, -.2, -.3), ((b,q), .1, .5, .5, -.1, -.2, -.3)\},\\
Q \times P' &= \{((a,p), .3, .4, .5, -.1, -.2, -.3), ((a,q), .3, .5, .5, -.1, -.2, -.3),\\
&\quad ((b,p), .4, .6, .5, -.1, -.2, -.3), ((b,q), .4, .6, .5, -.1, -.2, -.3)\},\\
Q \times Q' &= \{((a,p), .3, .6, .5, -.1, -.2, -.3), ((a,q), .1, .5, .5, -.1, -.2, -.3),\\
&\quad ((b,p), .4, .6, .5, -.1, -.2, -.3), ((b,q), .1, .6, .5, -.1, -.2, -.3)\}.
\end{aligned}$$

By calculations H *is not GSBSVNHG.*

Example 4.5. *Consider the GBSVNHGs* $H_1 = (X_1, E_1)$ *and* $H_2 = (X_2, E_2)$ *where* $X_1 = \{a, b\}$, $X_2 = \{p, q\}$, $E_1 = \{P, Q\}$, $E_2 = \{P', Q'\}$. *Also* $A_1^+, B_1^+, C_1^+ : X_1 \to [0,1]$ *defined by* $A_1^+(a) = .3$, $A_1^+(b) = .5$, $B_1^+(a) = .3$, $B_1^+(b) = .4$, $C_1^+(a) = .5$, $C_1^+(b) = .5$, $A_2^+, B_2^+, C_2^+ : X_2 \to [0,1]$ *defined by* $A_2^+(p) = .5$, $A_2^+(q) = .9$, $B_2^+(p) = .1$, $B_2^+(q) = .5$, $C_2^+(p) = .5$, $C_2^+(q) = .5$, $A_1^-, B_1^-, C_1^- : X_1 \to [-1,0]$ *defined by* $A_1^-(a) = -.5$, $A_1^-(b) = -.5$, $B_1^-(a) = -.6$, $B_1^-(b) = -.6$, $C_1^-(a) = -.7$, $C_1^-(b) = -.7$, $A_2^-, B_2^-, C_2^- : X_2 \to [0,1]$ *defined by* $A_2^-(p) = -.5$, $A_2^-(q) = -.5$, $B_2^-(p) = -.6$, $B_2^-(q) = -.6$, $C_2^-(p) = -.7$, $C_2^-(q) = -.7$,

$$\begin{aligned}
P &= \{(a, .1, .3, .5, -.5, -.6, -.7), (b, .5, .4, .5, -.5, -.6, -.7)\},\\
Q &= \{(a, .3, .4, .5, -.5, -.6, -.7), (b, .4, .6, .5, -.5, -.6, -.7)\},\\
P' &= \{(p, .5, .3, .5, -.5, -.6, -.7), (q, .8, .5, .5, -.5, -.6, -.7)\},\\
Q' &= \{(p, .4, .6, .5, -.5, -.6, -.7), (q, .1, .5, .5, -.5, -.6, -.7)\}.
\end{aligned}$$

Then by routine calculations H_1 *is GSBSVNHG and* H_2 *is GBSVNHG.*
Let $H = (X_1 \times X_2, E_1 \times E_2)$, $A = A_1 \times A_2$, $B = B_1 \times B_2$, $C = C_1 \times C_2$. *Then* $A^+((a,p)) = .3$, $A^+((a,q)) = .3$, $A^+((b,p)) = .5$, $A^+((b,q)) = .5$, $B^+((a,p)) = .3$, $B^+((a,q)) = .5$, $B^+((b,p)) = .4$, $B^+((b,q)) = .5$, $C^+((a,p)) = .5$, $C^+((a,q)) = .5$, $C^+((b,p)) = .5$, $C^+((b,q)) = .5$, $A^-((a,p)) = -.5$, $A^-((a,q)) = -.5$, $A^-((b,p)) = -.5$, $A^-((b,q)) = -.5$, $B^-((a,p)) = -.6$, $B^-((a,q)) = -.6$, $B^-((b,p)) = -.6$, $B^-((b,q)) = -.6$,

$C^-((a,p)) = -.7$, $C^-((a,q)) = -.7$, $C^-((b,p)) = -.7$, $C^-((b,q)) = -.7$,

$$\begin{aligned} P\times P^{'} &= \{((a,p),.1,.3,.5,-.5,-.6,-.7),((a,q),.1,.5,.5,-.5,-.6,-.7),\\ &\quad ((b,p),.5,.4,.5,-.5,-.6,-.7),((b,q),.5,.5,.5,-.5,-.6,-.7)\},\\ P\times Q^{'} &= \{((a,p),.1,.6,.5,-.5,-.6,-.7),((a,q),.1,.5,.5,-.5,-.6,-.7),\\ &\quad ((b,p),.4,.6,.5,-.5,-.6,-.7),((b,q),.1,.5,.5,-.5,-.6,-.7)\},\\ Q\times P^{'} &= \{((a,p),.3,.4,.5,-.5,-.6,-.7),((a,q),.3,.5,.5,-.5,-.6,-.7),\\ &\quad ((b,p),.4,.6,.5,-.5,-.6,-.7),((b,q),.4,.6,.5,-.5,-.6,-.7)\},\\ Q\times Q^{'} &= \{((a,p),.3,.6,.5,-.5,-.6,-.7),((a,q),.1,.5,.5,-.5,-.6,-.7),\\ &\quad ((b,p),.4,.6,.5,-.5,-.6,-.7),((b,q),.1,.6,.5,-.5,-.6,-.7)\}. \end{aligned}$$

By calculations H is GSBSVNHG.

Proposition 4.1. *If both H_1 and H_2 are GBSVNHGs, then $H_1 \times H_2$ is also GBSVNHG.*

Proof. Let $H_1 = (Z_1, E_1)$ and $H_2 = (Z_2, E_2)$ be two GBSVNHGs, where $Z_1 = \{x_1, x_2, \ldots, x_n\}$, $Z_2 = \{y_1, y_2, \ldots, y_n\}$, $A_1^+, B_1^+, C_1^+ : Z_1 \to [0,1]$, $A_1^-, B_1^-, C_1^- : Z_1 \to [-1,0]$, $A_2^+, B_2^+, C_2^+ : Z_2 \to [0,1]$, $A_2^-, B_2^-, C_2^- : Z_2 \to [-1,0]$ and

$$\begin{aligned} E_1 &= \{(T^+_{E_{11}}, I^+_{E_{11}}, F^+_{E_{11}}, T^-_{E_{11}}, I^-_{E_{11}}, F^-_{E_{11}}), \ldots, (T^+_{E_{1k}}, I^+_{E_{1k}}, F^+_{E_{1k}}, T^-_{E_{1k}}, I^-_{E_{1k}}, F^-_{E_{1k}})\}\\ E_2 &= \{(T^+_{E_{21}}, I^+_{E_{21}}, F^+_{E_{21}}, T^-_{E_{21}}, I^-_{E_{21}}, F^-_{E_{21}}), \ldots, (T^+_{E_{2p}}, I^+_{E_{2p}}, F^+_{E_{2p}}, T^-_{E_{2p}}, I^-_{E_{2p}}, F^-_{E_{2p}})\} \end{aligned}$$

where

$$\begin{aligned} T^+_{E_{1i}}, I^+_{E_{1i}}, F^+_{E_{1i}} : Z_1 \to [0,1],\ T^-_{E_{1i}}, I^-_{E_{1i}}, F^-_{E_{1i}} : Z_1 \to [-1,0]\\ T^+_{E_{2j}}, I^+_{E_{2j}}, F^+_{E_{2j}} : Z_2 \to [0,1],\ T^-_{E_{2j}}, I^-_{E_{2j}}, F^-_{E_{2j}} : Z_2 \to [-1,0] \end{aligned}$$

$\forall i = 1,2,3,\ldots,k$ and $j = 1,2,3,\ldots,p$. Then the cartesian product $H_1 \times H_2 = (Z_1 \times Z_2, E_1 \times E_2)$ where $E_1 \times E_2 = \{((T^+_{E_{11}} \times T^+_{E_{21}}), (I^+_{E_{11}} \times I^+_{E_{21}}), (F^+_{E_{11}} \times F^+_{E_{21}}), (T^-_{E_{11}} \times T^-_{E_{21}}), (I^-_{E_{11}} \times I^-_{E_{21}}), (F^-_{E_{11}} \times F^-_{E_{21}})), \ldots, ((T^+_{E_{11}} \times T^+_{E_{2p}}), (I^+_{E_{11}} \times I^+_{E_{2p}}), (F^+_{E_{11}} \times F^+_{E_{2p}}), (T^-_{E_{11}} \times T^-_{E_{2p}}), (I^-_{E_{11}} \times I^-_{E_{2p}}), (F^-_{E_{11}} \times F^-_{E_{2p}})), \ldots, ((T^+_{E_{1k}} \times T^+_{E_{2p}}), (I^+_{E_{1k}} \times I^+_{E_{2p}}), (F^+_{E_{1k}} \times F^+_{E_{2p}}), (T^-_{E_{1k}} \times T^-_{E_{2p}}), (I^-_{E_{1k}} \times I^-_{E_{2p}}), (F^-_{E_{1k}} \times F^-_{E_{2p}}))\}$, which satisfies

$$\begin{aligned} &\bigvee_{r=1}^{k} T^+_{E_{1r}}(x_i) \le A_1^+(x_i),\ \bigvee_{s=1}^{p} T^+_{E_{2s}}(y_j) \le A_2^+(y_j),\ \bigwedge_{r=1}^{k} I^+_{E_{1r}}(x_i) \ge B_1^+(x_i)\\ &\bigwedge_{s=1}^{p} I^+_{E_{2s}}(y_j), \ge B_2^+(y_j),\ \bigwedge_{r=1}^{k} F^+_{E_{1r}}(x_i) \ge C_1^+(x_i),\ \bigwedge_{s=1}^{p} F^+_{E_{2s}}(y_j) \ge C_2^+(y_j)\\ &\bigwedge_{r=1}^{k} T^-_{E_{1r}}(x_i) \ge A_1^-(x_i),\ \bigwedge_{s=1}^{p} T^-_{E_{2s}}(y_j) \ge A_2^-(y_j),\ \bigvee_{r=1}^{k} I^-_{E_{1r}}(x_i) \le B_1^-(x_i)\\ &\bigvee_{s=1}^{p} I^-_{E_{2s}}(y_j) \le B_2^-(y_j),\ \bigvee_{r=1}^{k} F^-_{E_{1r}}(x_i) \le C_1^-(x_i),\ \bigvee_{s=1}^{p} F^-_{E_{2s}}(y_j) \le C_2^-(y_j) \end{aligned}$$

$\forall i = 1, 2, 3, \ldots, n$ and $\forall j = 1, 2, 3, \ldots, m$. Now consider

$$\begin{aligned}\bigvee_{s=1}^{p}\bigvee_{r=1}^{k}(T^{+}_{E_{1r}} \times T^{+}_{E_{2s}})(x_i, y_j) &= \bigvee_{s=1}^{p}\bigvee_{r=1}^{k}(T^{+}_{E_{1r}}(x_i), T^{+}_{E_{2s}}(y_j)) \\ &= (\bigvee_{r=1}^{k} T^{+}_{E_{1r}}(x_i)) \wedge (\bigvee_{s=1}^{p} T^{+}_{E_{2s}}(y_j)) \\ &\leq A^{+}_1(x_i) \wedge A^{+}_2(y_j) = (A^{+}_1 \times A^{+}_2)(x_i, y_j)\end{aligned}$$

$\forall i$ and $\forall j$. Similarly

$$\bigvee_{s=1}^{p}\bigvee_{r=1}^{k}(T^{-}_{E_{1r}} \times T^{-}_{E_{2s}})(x_i, y_j) \geq (A^{-}_1 \times A^{-}_2)(x_i, y_j)$$
$$\bigwedge_{s=1}^{p}\bigwedge_{r=1}^{k}(I^{+}_{E_{1r}} \times I^{+}_{E_{2s}})(x_i, y_j) \geq (B^{+}_1 \times B^{+}_2)(x_i, y_j)$$
$$\bigwedge_{s=1}^{p}\bigwedge_{r=1}^{k}(F^{+}_{E_{1r}} \times F^{+}_{E_{2s}})(x_i, y_j) \geq (C^{+}_1 \times C^{+}_2)(x_i, y_j)$$
$$\bigvee_{s=1}^{p}\bigwedge_{r=1}^{k}(I^{-}_{E_{1r}} \times I^{-}_{E_{2s}})(x_i, y_j) \leq (B^{-}_1 \times B^{-}_2)(x_i, y_j)$$
$$\bigvee_{s=1}^{p}\bigwedge_{r=1}^{k}(F^{-}_{E_{1r}} \times F^{+}_{E_{2s}})(x_i, y_j) \leq (C^{-}_1 \times C^{-}_2)(x_i, y_j)$$

$\forall i$ and $\forall j$. Thus $H_1 \times H_2$ is the GBSVNHG. □

Proposition 4.2. *If both H_1 and H_2 are GSBSVNHGs, then $H_1 \times H_2$ is also GSBSVNHG.*

Proposition 4.3. *If $H_1 \times H_2$ be GSSVNHG, then at least H_1 or H_2 must be GSSVNHG.*

Proof. Let $H_1 = (Z_1, E_1)$ and $H_2 = (Z_2, E_2)$ be two GBSVNHGs, where $Z_1 = \{x_1, x_2, \ldots, x_n\}$, $Z_2 = \{y_1, y_2, \ldots, y_n\}$, $A^{+}_1, B^{+}_1, C^{+}_1 : Z_1 \to [0, 1]$, $A^{-}_1, B^{-}_1, C^{-}_1 : Z_1 \to [-1, 0]$, $A^{+}_2, B^{+}_2, C^{+}_2 : Z_2 \to [0, 1]$, $A^{-}_2, B^{-}_2, C^{-}_2 : Z_2 \to [-1, 0]$,

$$\begin{aligned}E_1 &= \{(T^{+}_{E_{11}}, I^{+}_{E_{11}}, F^{+}_{E_{11}}, T^{-}_{E_{11}}, I^{-}_{E_{11}}, F^{-}_{E_{11}}), \ldots, (T^{+}_{E_{1k}}, I^{+}_{E_{1k}}, F^{+}_{E_{1k}}, T^{-}_{E_{1k}}, I^{-}_{E_{1k}}, F^{-}_{E_{1k}})\}, \\ E_2 &= \{(T^{+}_{E_{21}}, I^{+}_{E_{21}}, F^{+}_{E_{21}}, T^{-}_{E_{21}}, I^{-}_{E_{21}}, F^{-}_{E_{21}}), \ldots, (T^{+}_{E_{2p}}, I^{+}_{E_{2p}}, F^{+}_{E_{2p}}, T^{-}_{E_{2p}}, I^{-}_{E_{2p}}, F^{-}_{E_{2p}})\},\end{aligned}$$

where

$$T^{+}_{E_{1i}}, I^{+}_{E_{1i}}, F^{+}_{E_{1i}} : Z_1 \to [0, 1],\ T^{-}_{E_{1i}}, I^{-}_{E_{1i}}, F^{-}_{E_{1i}} : Z_1 \to [-1, 0],$$
$$T^{+}_{E_{2j}}, I^{+}_{E_{2j}}, F^{+}_{E_{2j}} : Z_2 \to [0, 1],\ T^{-}_{E_{2j}}, I^{-}_{E_{2j}}, F^{-}_{E_{2j}} : Z_2 \to [-1, 0],$$

$\forall i = 1, 2, 3, \ldots, k$ and $j = 1, 2, 3, \ldots, p$. Then the cartesian product $H_1 \times H_2 = (Z_1 \times Z_2, E_1 \times E_2)$ where $E_1 \times E_2 = \{((T^{+}_{E_{11}} \times T^{+}_{E_{21}}), (I^{+}_{E_{11}} \times I^{+}_{E_{21}}), (F^{+}_{E_{11}} \times F^{+}_{E_{21}}), (T^{-}_{E_{11}} \times T^{-}_{E_{21}}), (I^{-}_{E_{11}} \times I^{-}_{E_{21}}), (F^{-}_{E_{11}} \times F^{-}_{E_{21}})), \ldots, ((T^{+}_{E_{11}} \times T^{+}_{E_{2p}}), (I^{+}_{E_{11}} \times I^{+}_{E_{2p}}), (F^{+}_{E_{11}} \times F^{+}_{E_{2p}}), (T^{-}_{E_{11}} \times T^{-}_{E_{2p}}), (I^{-}_{E_{11}} \times I^{-}_{E_{2p}}), (F^{-}_{E_{11}} \times F^{-}_{E_{2p}})), \ldots, ((T^{+}_{E_{1k}} \times T^{+}_{E_{2p}}), (I^{+}_{E_{1k}} \times I^{+}_{E_{2p}}), (F^{+}_{E_{1k}} \times F^{+}_{E_{2p}}), (T^{-}_{E_{1k}} \times T^{-}_{E_{2p}}), (I^{-}_{E_{1k}} \times I^{-}_{E_{2p}}), (F^{-}_{E_{1k}} \times F^{-}_{E_{2p}}))\}$. Suppose that $H_1 \times H_2$ is GSBSVNHG, but H_1 and

H_2 are not GSBSVNHGs then by definition we have

$$\bigvee_{r=1}^{k} T^+_{E_{1r}}(x_i) < A^+_1(x_i),\ \bigvee_{s=1}^{p} T^+_{E_{2s}}(y_j) < A^+_2(y_j),\ \bigwedge_{r=1}^{k} I^+_{E_{1r}}(x_i) > B^+_1(x_i)$$
$$\bigwedge_{s=1}^{p} I^+_{E_{2s}}(y_j) > B^+_2(y_j),\ \bigwedge_{r=1}^{k} F^+_{E_{1r}}(x_i) > C^+_1(x_i),\ \bigwedge_{s=1}^{p} F^+_{E_{2s}}(y_j) > C^+_2(y_j)$$
$$\bigwedge_{r=1}^{k} T^-_{E_{1r}}(x_i) > A^-_1(x_i),\ \bigwedge_{s=1}^{p} T^-_{E_{2s}}(y_j) > A^-_2(y_j),\ \bigvee_{r=1}^{k} I^-_{E_{1r}}(x_i) < B^-_1(x_i)$$
$$\bigvee_{s=1}^{p} I^-_{E_{2s}}(y_j) < B^-_2(y_j),\ \bigvee_{r=1}^{k} F^-_{E_{1r}}(x_i) < C^-_1(x_i),\ \bigvee_{s=1}^{p} F^-_{E_{2s}}(y_j) < C^-_2(y_j)$$

$\forall i = 1, 2, 3, \ldots, n$ and $\forall j = 1, 2, 3, \ldots, m$. Therefore

$$\begin{aligned}
\bigvee_{s=1}^{p}\bigvee_{r=1}^{k}(T^+_{E_{1r}} \times T^+_{E_{2s}})(x_i, y_j) &= \bigvee_{s=1}^{p}\bigvee_{r=1}^{k}(T^+_{E_{1r}}(x_i), T^+_{E_{2s}}(y_j)) \\
&= (\bigvee_{r=1}^{k} T^+_{E_{1r}}(x_i)) \wedge (\bigvee_{s=1}^{p} T^+_{E_{2s}}(y_j)) \\
&< A^+_1(x_i) \wedge A^+_2(y_j) = (A^+_1 \times A^+_2)(x_i, y_j)
\end{aligned}$$

$\forall i$ and $\forall j$. Similarly

$$\bigwedge_{s=1}^{p}\bigwedge_{r=1}^{k}(T^-_{E_{1r}} \times T^-_{E_{2s}})(x_i, y_j) > (A^-_1 \times A^-_2)(x_i, y_j)$$
$$\bigwedge_{s=1}^{p}\bigwedge_{r=1}^{k}(I^+_{E_{1r}} \times I^+_{E_{2s}})(x_i, y_j) > (B^+_1 \times B^+_2)(x_i, y_j)$$
$$\bigwedge_{s=1}^{p}\bigwedge_{r=1}^{k}(F^+_{E_{1r}} \times F^+_{E_{2s}})(x_i, y_j) > (C^+_1 \times C^+_2)(x_i, y_j)$$
$$\bigvee_{s=1}^{p}\bigvee_{r=1}^{k}(I^-_{E_{1r}} \times I^-_{E_{2s}})(x_i, y_j) < (B^-_1 \times B^-_2)(x_i, y_j)$$
$$\bigvee_{s=1}^{p}\bigvee_{r=1}^{k}(F^-_{E_{1r}} \times F^-_{E_{2s}})(x_i, y_j) < (C^-_1 \times C^-_2)(x_i, y_j)$$

$\forall i$ and $\forall j$. Therefore $H_1 \times H_2$ is not GSBSVNHG, which is contradiction, hence at least one of H_1 or H_2 must be GSBSVNHG. □

5. Conclusion

In this paper, the concept of single valued neutrosophic hypergraph and bipolar single valued neutrosophic hypergraph has been generalized by considering single valued neutrosophic vertex set and bipolar single valued neutrosophic vertex instead of crisp vertex set and also considering interrelation between single valued neutrosophic vertices and bipolar single valued neutrosophic vertices with and family of single valued neutrosophic edges and bipolar single valued neutrosophic edges.

References

[1] Broumi, S., Talea, M., Bakali, A., Smarandache, F., Vladareanu, L., (2016), An introduction to bipolar single valued neutrosophic graph theory, Applied Mechanics and Materials, 841, pp. 184-191.

[2] Broumi, S., Talea, M., Bakali, A., Smarandache, F. and Ali, M., (2016), Shortest path problem under bipolar neutrosphic setting, Applied Mechanics and Materials, 859, pp. 59-66.

[3] De. R. K., Biswas, R. and Roy, R.A., (2001), An application of intuitionistic Fuzzy set in medical diagnosis, Fuzzy Sets and Systems, 4, pp. 58-64.

[4] Hassan, A. and Malik, M. A., (2018), Single valued neutrosophic trees, TWMS Journal of Applied and Engineering Mathematics, 8(1a), pp. 255-266.

[5] Hassan, A. and Malik, M. A., (2018), Special types of single valued neutrosophic graphs, TWMS Journal of Applied and Engineering Mathematics, 8(2), pp. 341-352.

[6] Hassan, A. and Malik, M. A., (2020), The classes of bipolar single valued neutrosophic graphs, TWMS Journal of Applied and Engineering Mathematics, 10(3), pp. 547-567.

[7] Mordeson, J. N. and Nair, P. S., (1998), Fuzzy graphs and fuzzy hypergraphs, Physica Verlag, Heidelberg, Second edition 2001.

[8] Nasir, S., (2016), Some studies in neutrosophic graphs, Neutrosophic Sets and Systems, 12, pp. 54-64.

[9] Samanta, T. K. and Mohita, S., (2014), Generalized strong intuitionstic fuzzy hypergraphs, Mathematica Moravica, 18(1), pp. 55-65.

[10] Wang, H., Smarandache, F., Zhang, Y. and Sunderraman, R., (2010), Single valued neutrosophic Sets, Multisspace and Multistructure, 4, pp. 410-413.

[11] Yang, H. L., Guo, Z. L., She, Y. and Liao. X., (2016), On single valued neutrosophic relations, Journal of Intelligent and Fuzzy Systems, 30(2), pp. 1045-1056

Ali Hassan is currently working as a research student in the Department of Mathematics in Karachi University. He received his master degree from University of the Punjab, Lahore, Pakistan. His area of interest includes Fuzzy group theory and fuzzy graph theory.

Muhammad Aslam Malik is currently working as an associate professor in the Department of Mathematics University of the Punjab, Lahore. He received his Ph.D degree from University of the Punjab, Lahore. He did Post Doc in Graph theory from Birmingham University, UK. His area of interest includes Graph theory, Fuzzy group theory and fuzzy graph theory.