TWMS J. App. and Eng. Math. V.10, N.4, 2020, pp. 987-1008

GENERALIZED LORENTZ GROUP OF SPACE-TIME TRANSFORMATIONS

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Abstract. We examine how Lorentz Symmetry (LS) breaks down in Yarman-Arik-Kholmetskii (YARK) theory of gravitation through an entirely different mechanism than that under metric theories of gravity. Said mechanism can be right away extended to all other fields of interaction under *Yarman's Approach* that forms the basis of YARK theory. The result is the disclosure of a new "Generalized Lorentz Group" of space-time transformations which contains an additional parameter denoting the interactional energy per unit mass. Hence, the core finding herein is that the Minkowskian metric for an empty space-time should, when one is in the presence of gravity or any other force field, be replaced by general equalities involving a novel coupling parameter for either attraction or repulsion..

Keywords: Lorentz Symmetry, Lorentz Transformations, Generalized Lorentz Group, Metric Theories of Gravity, YARK Theory of Gravitation

PACS: 11.30.Cp, 11.10.z

1. INTRODUCTION

Symmetry properties of time and space are principally correlated with conservation laws in physics, such as regarding energy, momentum and angular momentum, as was first proven

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§ Manuscript received: June 24, 2019; accepted: November 12, 2019.

by Cosserat brothers and Noether for non-dissipative scenarios that can be modelled on the action of a system from solely its Lagrangian [1, 2].

Another kind of fundamental symmetry assumed to hold in nature is "Lorentz Symmetry". In the recent past, it was suggested that Lorentz Covariance or Invariance — that is to say, *the independence of measured physical quantities from the uniform translational motion one's proper reference frame undergoes as viewed by a fixed outside observer* — should anyway point to Lorentz Symmetry (LS) being only an "approximate symmetry" of nature [3, 4, 5].

Case in point, it is well known that General Theory of Relativity (GTR) breaks LS [i].

The violation of Lorentz Symmetry in gravitation takes place in Yarman-Arik-Kholmetskii (YARK) theory, too — but differently than it does in GTR; fundamental relationships for space and time intervals in the presence of gravity are formulated in integral forms at the very outset in YARK theory (*which thusly cannot be classified as a purely metric theory, but rather subsumes the properties of both metric and dynamic theories*), with subsequent determination of differential relationships for space and time intervals whenever needed. Whereas GTR, for having been conceived as a purely metric theory, embarks on differential forms of "time" and "space" intervals at the onset, with yet no practical way to achieve their integral forms later on.

Let us briefly recall that Lorentz transformations in their familiar form were mathematically first framed by Poincaré [6] in 1905 to account for the anomalous result of the Michelson-Morley experiment [7]. Further on, Poincaré named them as "Lorentz transformations" because of the respect he held for his mentor.

We remind that Lorentz Symmetry was then first used by Einstein to construct his special theory of relativity (STR) [8, 9]. Einstein worked out the consequences of the LS, where he concluded that length and duration must be altered, yet *symmetrically*, when two observers move relative to each other. Lorentz transformations thus describe how measurements of space and time obtained by two different observers are related to the velocity of the uniform translational motion of one of them as gauged by the other.

All the same, Lorentz transformations, the way they had been originally forged, do not tell anything about space and time variations when the observers interact with each other. Concomitantly, it has been proven that *CPT (Charge-Parity-Time) violation* implies that LS breaks down [10, 11].

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[i] It is simple to understand how this happens. If, for the sake of demonstration, one may conceive that "Captain Kirk" is parked in gravitation while "Captain Picard" watches him from some remote distance, with both being at rest with respect to each other, then, based on GTR, they would both agree that Captain Kirk's watch runs slower, and his stickmeter (*along the direction of the pull of gravity*) is contracted; or the same, they would both agree that Captain Picard's watch runs faster and his stickmeter (*along yet the same direction according to GTR*) is elongated. Whereas, should either be in a state of uniform translational motion in free space with respect to the other, a perfect symmetry would reign in between them. In other words, in such a state, either party would measure the other's time as dilated and the other's stickmeter as contracted (*e.g.*, along the direction of motion). In case these parties came to rest, there would be just one time and one stickmeter for both. This is yet not the case when Captain Kirk is parked in gravity while Captain Picard is parked at a distance far removed from any gravity (or vice versa); or the same, when they are both embedded in gravity all the while still remaining at rest with respect to each other, but being at different altitudes from the source of gravity. Were they to move in the vicinity of this source of gravity, LS would not hold; for, such a symmetry is, for one thing, already broken when they were initially at rest at different altitudes to begin with.

Recall further that "CPT Symmetry" is what holds unchanged under the inversion of *charge*, *parity* and *time* simultaneously.

A good example of CPT violation is the one we can pick from the lepton sector; it is defined by the difference between the form factors of the electron and the positron [12, 13]:

$$(g_{e+} - g_e) / < g_e > = (-0.5 \pm 2.1) \times 10^{-12}.$$

Nowadays, new experimental techniques are tried to search for clues of the violation of Lorentz Symmetry at low energies, and they may well corroborate the expectation that Lorentz Invariance should indeed break down as such [14, 10].

Bound muon decay rate retardation is one interesting area to check if Lorentz Invariance breaks down in the atomistic world. Under *Yarman's Approach*, when a muon interacts with and is caught by a nucleus, the muon's rate of decay will slow down; in other words its decay half-life will be prolonged in contrast to its unbound siblings [15, 16, 17, 18]. This is one example where the electric field of the nucleus appears to affect space and time in just the same way as gravitation does [19]. As we shall soon see below, under the framework of Yarman's Approach — *that is extended to gravitation with YARK theory* — deformation of the field commensurately with the conversion of rest mass into the kinetic energy of the parent or of the ejected particle or both is indeed what is thought to transpire. The common feature in either the atomistic or the celestial scale, and in all plausible interactional cases, is that *binding through interaction* must, according to Yarman's Approach, alter space and time insofar as invalidating Lorentz Symmetry. The same occurs in the case of alpha particles richocheting from the "repulsive field" of gold atoms, where the "repulsive energy" is stored inside the alpha particle once more as per Yarman's Approach [20].

As had been shown at the outset (see below), what happens throughout binding is that the "rest mass" (*or the same, "rest energy" were the velocity of light taken as unity*) of the bound object is decreased, owing to the law of energy conservation embodying the mass and energy equivalence of STR, as much as the static binding energy the client object cedes.

When such "rest mass decrease" coming into play is inserted into the quantum mechanical description of the client object, the related total energy (*i.e., the eigenvalue*) is decreased as much — hence pointing to a "stretching of the period of time" associated with the internal dynamics the object at hand delineates, and conjointly to a "stretching of its size" just as much [21, 22]. Let us stress that Yarman's Approach is applicable to a repulsive field as well (*in which case, the rest mass of the ricocheted alpha particle would conversely increase*).

Notice that none of the available explanations given for bound muon decay rate retardation in the cited references [15, 16, 17, 18] were satisfactory enough to explain the phenomenon. Concurrently, the first co-author, already having predicted it theoretically, has been the first one who provided a simple explanation through his anticipation that any bound particle must undergo a "rest mass" (or the same — *taking the speed of light in vacuum as unity* — a "rest energy") decrease commensurate with the "static binding energy" the object transactions. This idea, along with the non-trivial modification of the field equations for an electromagnetic field generated by non-radiative bound quantum systems the way suggested by Kholmetskii et al. [23, 24, 25, 26, 27] allowed the elimination of a number of puzzling discrepancies between theory and experiment in precise atomic physics.

This already points to the fact that a free clock (*e.g.*, an unbound muon) and its twin immersed in an electric field (*e.g.*, a bound muon sitting in an isolated chamber where an electric field reigns) ought to run at different paces as per Yarman's Approach. In other words, the bound clock runs slower while a free clock runs faster.

Suppose we attach observers to these clocks / muons at hand when they are at rest with respect to each other; it is not that Lorentz Symmetry really breaks thereafter as they are put in motion, it is essentially that Lorentz Symmetry was never there to begin with.

Thence, the question we pose here is this: "How can we properly write space-time transformations related to two interacting objects?" We will provide an answer to this question within the framework of YARK — starting with gravitational interaction first, with yet no loss of generality.

Before we proceed, it would be useful to present a brief summary of YARK theory.

2. YARK THEORY OF GRAVITATION: BASIC CONCEPTS

In our previous papers [28, 29, 30, 31, 32], we have already gone over how YARK (Yarman-Arik-Kholmetskii) theory (as abbreviated from the initials of its chief developers) is based on the original "Universal Matter Architecture" and the subsequent "Yarman's Approach" framework developed by the first co-author [33, 34, 35, 36, 37, 38, 39, 40], and then advanced together with his colleagues [41, 42, 43, 44, 28, 29, 30, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63]. For the sake of convenience, we reproduce below some important points of this theory in order to stress its physical meaning.

The root Yarman's Approach postulate of YARK theory states that the overall energy of the object with the proper mass m, initially measured at an infinitely far away distance from all other masses in the presence of gravity, acquires the form [33, 34]

$$E = \gamma mc^2(1 - E_B/mc^2) = Constant, \tag{2.1}$$

where γ is the Lorentz factor associated with the motion of the test object, and E_B represents the "static binding energy" defined as the work one has to carry out in order to bring the object quasi-statically from infinity to the given location; it is set to a Constant in a closed system.

In fact, Eq.(2.1) states that the rest mass m of the object cannot be fixed, but is rather altered within the gravitational environment of concern (as a matter of fact, any force field it can interact with) by the value E_B/mc^2 owing to the law of energy conservation as assessed by the remote observer. Such an assertion also implies that the gravitational energy is localized inside interacting particles rather than getting distributed across the surrounding space.

Further, due to the thus-far unrecognized intrinsic quantum mechanical relationships the first co-author laid out between the quantities "mass", "energy", "frequency", "time" and "size", the variation of the rest mass of a test particle by the static binding energy (2.1) affects the time rate for the particle, and furnishes a corresponding transformation of spatial intervals in the presence of a force field and especially, for our purposes, gravity [33, 34, 35, 36, 37, 38, 39, 40].

Hence, the variation of the rest mass of a test particle by the static binding energy does, in effect, alter — just like in metric theories of gravity — *the metric of space-time in YARK* *theory.* In particular, in the radially-symmetric case, where r is the distance of the test object to the center of gravity at the given time t, we have [31, 32, 33, 34]

$$t = t_0 e^{\alpha}\,, \tag{2.2a}$$

$$r = r_0 e^{\alpha}\,, \tag{2.2b}$$

with t_0, r_0 standing for the corresponding quantities in the absence of gravity, and where, by the same token, they are proper quantities as measured by the observer attached to the test mass m. Let us also recall here that

$$\alpha = GM/rc^2\,, \tag{2.3}$$

where G is the gravitational constant, M the mass of the host body, r is the distance of the test mass to the center of the host body, and c is the speed of light in vacuum.

Note that the usual *squared space-time interval* ${s_0}^2$ in *empty space* is

$${s_0}^2 = c^2 {t_0}^2 - {r_0}^2\,. \tag{2.4}$$

Based on Eq.(2.2a), YARK's squared space-time interval s^2 in the presence of gravity thusly becomes

$$s^2 = {s_0}^2 e^{2\alpha}. \tag{2.5}$$

We will particularly elaborate on this *precious result* in order to achieve the goal of the current paper.

It should be emphasized that, unlike GTR, YARK's metric properties of space-time do not play a decisive role in the determination of the motion of objects in the gravitational environment. This statement can already be demonstrated by the fact that, for a test particle m moving in a gravitational field created by a considerably heavy host mass $M \gg m$ (*i.e.*, the one-body problem), the motional equation of the test particle can be derived straightforwardly via the differentiation of Eq.(2.1) — *as had been originally done by Yarman in refs.* [33, 34, 35, 36, 37] — independently from the metric properties of space-time. Indeed, due to the energy conservation law for the isolated system of interacting objects m and M at $M \gg m$, the time derivative of the right-hand-side of Eq.(2.1) should be equated to zero, which directly yields the motional equation of the test particle m without necessitating an *ad hoc* determination of the metric of space-time.

This indicates, in particular, that YARK, unlike GTR, is not a purely metric theory, but rather subsumes the properties of both dynamic theories and metric theories (see, *e.g.*, [33, 34, 40]).

The derivation of the motional equation of the particle m in the presence of gravity (*which can be straightforwardly extended to the interaction of many bodies*) can also be realized in YARK theory through the minimization of the action (see, *e.g.*, [30]) as usual; yet, only after the YARK outcome of Eq.(2.1) is known. For those who are accustomed to follow that line of reasoning, we have produced it in, *e.g.*, [30], where we could obtain the following expressions for the momentum and energy of the particle m in the presence of gravity [30]:

$$\boldsymbol{p} = \gamma m e^{-\alpha} \boldsymbol{v}, \tag{2.6}$$

$$E = \gamma m e^{-\alpha} c^2. \tag{2.7}$$

Thusly, as we had previously shown [47], under the framework of YARK theory, the independence of the motional equation of the particle from its rest mass in the presence of gravity remains in force in the general case of the many-body problem, too. This means that the weak equivalence principle (WEP) is perfectly fulfilled in YARK theory [26, 31, 32, 40]. In addition, it is important to emphasize that YARK theory is entirely compatible with the foundational premises of special theory of relativity (STR) [31, 32] and satisfies both local Lorentz invariance and local position invariance. Therefore, YARK theory is wholly compatible with the Einstein equivalence principle (EEP), too.

At the same time, the physical meaning of the EEP in YARK theory — *which combines the properties of dynamic and metric theories* — is different as referred to purely metric theories of gravity such as GTR. In particular, the dynamical side of YARK signifies that, in the case where the gravitational force experienced by a particle in a chosen frame of observation is not equal to zero, then, it does not disappear in any other frame — including the frame of free fall of the particle [51]. In the latter case, the gravitational force is "sensed" by the particle through the variation of its rest mass, even if it is exactly counterbalanced by a "*fictitious force*" existing in an accelerated frame of this particle. This means, in essence, that gravitational energy, contrary to what GTR delineates, can indeed be localized. Therefore, we see that the EEP does not, in general, make it requisite that only purely metric theories of gravity should be adopted; compliance to it in YARK theory is, as we have seen, assured by the existence of such a reference frame wherein, at each four-point, the force of gravity can be exactly counterbalanced by a fictitious force as experienced by the particle in this frame.

Next, we compare Eq.(2.7) with the known expression of GTR for the energy of the test particle in a gravitational field [64], *i.e.*,

$$E_{GTR} = \gamma m c^2 \sqrt{1 - 2\alpha}\ , \tag{2.8}$$

and find that the terms describing the effect of gravity in these equations coincide with each other up to the accuracy of c^{-3} $[me^{-\alpha} \cong m\sqrt{1-2\alpha} \cong m(1-\alpha)]$. Thus, with respect to many implementations, GTR and YARK do converge in the limit of a weak gravitational field, and, in particular, both provide successful explanations for gravitational redshift, gravitational lensing, Shapiro delay and precession of the perihelion of Mercury (see, *e.g.*, [33, 34, 36, 37, 44, 45, 65]. One should also mention that YARK theory also achieved considerable successes in the explanation of modern observations where the weak relativistic limit is abandoned (*e.g.*, derivation of the alternating sign for the accelerated expansion of the Universe without the need to involve a notion of "dark energy" [42]; presentation of the Hubble constant in an analytical form [42]; elimination of the information paradox for black holes of the YARK type [29]). What is more, YARK theory remains the only alternative to GTR which provides an adequate account of the GW150914 and GW151226 signals of LIGO beyond the hypothesis about gravitational waves [59, 60].

Besides these, we wish to spotlight two very recent experimental facts: The extra-energy shift between emission and absorption resonant lines in a rotating system [47, 48, 49], and the practically null bending of high-energy γ-quanta under Earth's gravity [66] — both of which have found a successful explanation under YARK theory [30], while they still remain as puzzles within the framework of GTR [28, 31, 50, 51, 52, 53, 54, 55, 56, 57, 58].

Finally, we stress that YARK theory of gravity is in natural symbiosis with quantum mechanics [40]; this fact definitely reflects advantages in combining metric and dynamical approaches in comparison with the purely metric approach of extended theories of gravity.

3. GENERALIZED LORENTZ GROUP OF SPACE-TIME TRANSFORMATIONS WHERE LORENTZ SYMMETRY BREAKS DOWN IN ALL INTERACTIONS

The violation of Lorentz Symmetry (LS), as we shall soon outline below, is not restricted to just gravitation, but can also be applied to any field the test object is embedded into. All the same, it would be helpful to specify the field we are dealing with; accordingly, we shall hereby pursue calculations in a gravitational field under the framework of YARK theory.

As we have mentioned above, lengths and periods of time are altered in the presence of gravity in just the same way in YARK theory [cf. Eq.(2.2a)]; that is to say, both are stretched, in effect, as much as the static binding enery coming into play. This precisely takes place commensurately with the rest mass decrease in gravity owing to the law of energy conservation embodying the mass and energy equivalence of special theory of relativity (STR) as framed by Yarman's Approach (*which, very unfortunately, is given up in GTR when the test object is assessed by the remote observer*).

And, when, for the location of concern, the "rest mass decrease" in, say, an H atom is inputted into its quantum mechanical description, then the client object's total energy (*i.e.*, eigenvalue) is decreased; as a result of which the related temporal rates at different energy states, as well as spatial dimensions at those states, are stretched by exactly the same amount.

Thereby, in YARK, size stretching is uniform; that is to say, spatial dimensions stretch by the same amount in all directions.

Before defining the related transformations under gravity, let us first state Lorentz transformations in their familiar form:

$$x_L = \gamma(x_0 + vt_0)\,, \tag{3.1a}$$

$$t_L = \gamma\left(t_0 + \frac{vx_0}{c^2}\right). \tag{3.1b}$$

Here, x_0 and t_0 represent the proper space and time coordinates of the moving object, while x_L and t_L represent the space and time coordinates of the moving object as assessed by a fixed local observer in gravity. The motion precisely occurs along the x direction no matter what this direction may be.

The conjoint reverse transformations are:

$$x_0 = \gamma(x_L - vt_L)\,, \tag{3.2a}$$

$$t_0 = \gamma\left(t_L - \frac{vx_L}{c^2}\right). \tag{3.2b}$$

As is known, the conventional relationship ${s_L}^2 = {s_0}^2$, *i.e.*,

$$ {x_L}^2 - c^2{t_L}^2 = {x_0}^2 - c^2{t_0}^2 , \tag{3.3} $$

is thereafter fulfilled.

Note further that these Lorentz transformations provide us with the following usual differential equations:

$$ dx_L = \gamma(dx_0 + vdt_0) , \tag{3.4a} $$

$$ dt_L = \gamma\left(dt_0 + \frac{vdx_0}{c^2}\right) . \tag{3.4b} $$

Their conjoint reverse transformations are:

$$ dx_0 = \gamma(dx_L - vdt_L) , \tag{3.5a} $$

$$ dt_0 = \gamma\left(dt_L - \frac{vdx_L}{c^2}\right) . \tag{3.5b} $$

Here, one lands at the conventional equality of squared differentials [ii],

$$ {dx_L}^2 - c^2{dt_L}^2 = {dx_0}^2 - c^2{dt_0}^2 . \tag{3.6} $$

We shall soon see that, not only is the equality ${s_L}^2 = {s_0}^2$ [cf. Eq.(3.3)] broken in interaction — *with yet the possibility remaining to redeem the cast at hand later on in the manner of YARK*, the equality of the squared differentials ${ds_L}^2 = {ds_0}^2$ [cf. Eq.(3.6)] is also broken in interaction — *with yet again the possibility remaining, in special cases, to redeem the latter cast afterwards in the manner of YARK.*

Extraordinarily enough, this revelation alone dismantles a whole century of mathematical progress on "curved spacetime" with its corresponding metric operations based on relationships involving just the "squared differentials".

Now, we are going to inject our gravitational interaction terms into the aforementioned transformations.

We hence expect in gravity, and in motion, that periods of time, when assessed by the distant observer, will dilate for two reasons:

— *Quantum mechanical stretching due to gravitational binding by the factor* e^{α};
— *Special relativistic stretching under uniform translational motion as much as the Lorentz coefficient* γ.

Let us recall that $\alpha = GM/rc^2$ [cf. Eq.(2.3)]. As for the lengths — *as assessed by the distant observer* — within the framework of YARK, they too are stretched in gravity; and this, in all directions, by the factor e^{α}. But, at the same time, they must get contracted by the factor γ along the direction of motion — *once again as assessed by the distant observer.* So, the factor γ and the factor e^{α} should, in that case, remarkably work against each other.

Consider, for simplicity, but without any loss of generality, that the x direction lies along constant gravity — thereby, on a plane perpendicular to the radial direction. To generalize

[ii] Eq.(3.6) is customarily written as ${ds_L}^2 = {ds_0}^2$; the differentials over here, as known, do not mean the differentials of ${s_L}^2$ or ${s_0}^2$, but simply demarcate ${ds_L}^2 = {dx_L}^2 - c^2{dt_L}^2$ and ${ds_0}^2 = {dx_0}^2 - c^2{dt_0}^2$.

what we will achieve below under the given constraint, one only needs to define the velocity v of the given motion as an instantaneous velocity.

Thus we have the climacteric "Generalized Lorentz Group":

$$x = \gamma(e^{\alpha} x_0 + e^{\alpha} v t_0)\,, \tag{3.7a}$$

$$t = \gamma\left(e^{\alpha} t_0 + e^{\alpha} \frac{v x_0}{c^2}\right). \tag{3.7b}$$

This is the standard writing pinned down of late by the first co-author where both time and space are stretched in gravity by the same e^{α} factor as seen from the reference frame of the distant observer.

Indeed, if v were 0, then we would be able to write

$$x = e^{\alpha} x_0\,, \tag{3.8a}$$

$$t = e^{\alpha} t_0\,. \tag{3.8b}$$

Thence, one gratifyingly has

$$x^2 - c^2 t^2 = e^{2\alpha}\left({x_0}^2 - c^2 {t_0}^2\right)\,; \tag{3.9a}$$

or the same,

$${x_0}^2 - c^2 {t_0}^2 = e^{-2\alpha}\left(x^2 - c^2 t^2\right)\,. \tag{3.9b}$$

So, one no longer has the proper Minkowskian mold ${x_0}^2 - c^2 {t_0}^2 = x^2 - c^2 t^2$ [cf. Eq.(3.3)] that one can anymore define in the absence of gravity.

In what follows, let us prescribe

$${s_0}^2 = {x_0}^2 - c^2 {t_0}^2\,, \tag{3.10a}$$

and

$$s^2 = x^2 - c^2 t^2\,. \tag{3.10b}$$

Therefore, instead of the accustomed ${s_0}^2 = s^2$, we now assert our novel "proper Minkowskian-Yarmanian" transformation:

$${s_0}^2 = s^2 e^{-2\alpha}\,. \tag{3.10c}$$

It is crucial to note that, via the present approach, one could — *in contradistinction to the manner in which it was exercised throughout the past century* — obtain a relationship between s_0 and s straightforwardly *in an integral form* at the outset — instead of in terms of the squares of differentials, whereby an integral result is recovered in metric theories only after much extensive labor.

The differential equation that comes out of Eqs.(3.10a) and (3.10b) would too have no correspondence with the original Minkowskian ${dx_L}^2 - c^2 {dt_L}^2 = {dx_0}^2 - c^2 {dt_0}^2$ resulting from "authentic" Lorentz transformations.

To show this, we will first embark on a simplified case without, once again, any loss of generality; thusly we will consider a motion in gravity along the radial direction r only, where the special relativistic displacement velocity v becomes the instantaneous velocity of the object in question.

If there was no gravity and the motion supposedly took place in the chosen radial direction r, we would normally write ${dr_L}^2 - c^2{dt_L}^2 = {dr_0}^2 - c^2{dt_0}^2$.

In the case of a spherically symmetric gravity, based on Eqs.(3.8a) and (3.8b), we can initially write [34] (cf. Appendix A)

$$dr = \frac{e^{\alpha}}{1+\alpha}\,dr_0\,, \tag{3.11a}$$

along with [cf. Eq.(2.2b)]

$$r = r_0 e^{\alpha}\,, \tag{3.11b}$$

and concurrently,

$$dt = \frac{e^{\alpha}}{1+\alpha}\,dt_0\,. \tag{3.11c}$$

As a consequence, from Eq.(3.10c), and for a fixed proper observer, we land at

$${ds_0}^2 = e^{-2\alpha}(1+\alpha)^2\,dr^2 - c^2 e^{-2\alpha}(1+\alpha)^2\,dt^2\,; \tag{3.12a}$$

or, for a fixed local observer, we get

$${dt_0}^2 = e^{-2\alpha}(1+\alpha)^2\left[dr^2 - c^2 dt^2\right]. \tag{3.12b}$$

This allows us, at the same time, to write

$${ds_0}^2 = e^{-2\alpha}(1+\alpha)^2\,ds^2 \tag{3.12c}$$

via positing

$${ds_0}^2 = {dr_0}^2 - c^2{dt_0}^2 \tag{3.13a}$$

and

$$ds^2 = dr^2 - c^2 dt^2\,. \tag{3.13b}$$

Eq.(3.12b) is the YARK equation relating ${ds_0}^2$ and ds^2 in the case of a radial motion in gravity.

Note that we do not even have to integrate Eq.(3.12b), for it is obtained through the differentation of our already available integral relationships; *i.e.*, the set of quantum mechanical Eqs.(3.8a) and (3.8b), as well as Eq.(3.11b) in the case of a radial motion in gravity.

Still, for a photon, one is to classically set to zero the left-hand-side of Eq.(3.12b), which expectedly leads to

$$c = dr\big/dt\,. \tag{3.14}$$

Recall that, right above, we dealt with a special case where the motion happened in the radial direction. If not, we would have had to write

$${ds_0}^2 = {dl_0}^2 - c^2{dt_0}^2 \tag{3.15a}$$

and

$$ds^2 = dl^2 - c^2 dt^2\,, \tag{3.15b}$$

where dl is the infinitely small distance crossed in gravity by the moving object as assessed by the distant observer, and dl_0 is the same distance, but as assessed by the fixed local observer.

Under these general circumstances where one relinquishes the radial path, Eq.(3.12b) does not hold of course; and while the construction of a similar equation will require further attention, it would in any event not bring in any new information other than the reassertion of the constancy of the speed of light c — thus being homologous to Eq.(3.14) in case Eq.(3.15b) were written for a photon. As we will not pursue the subject any further, we only briefly wish to state that no full metric description involving *squared differentials* in the manner of the Schwarzschild metric can be constructed upon the Minkowskian squared differential space-time line element ds^2, since such a route is neither allowed in nor is needed by YARK theory of gravity.

All the same, it is worth stressing how YARK theory, being in full symbiosis with quantum mechanics, allows the handling of either a resonant-bending quantal photon [46] or a projectile-like bending classical photon [30]; regarding which we have considered, particularly at the level of Eq.(3.14), just the latter case. To deal with a photon behaving according to the quantization of its frequency in resonance with gravity within the framework of YARK theory, one does not require anything more than the coupling of the customary relativistic energy

$$\frac{m_{00}c^2}{\sqrt{1 - v^2/c^2}} = m_{rel}c^2 = hf \tag{3.16}$$

of the photon (*where m_{rel} is the relativistic mass, h the Planck Constant, and f the frequency of the photon at hand*) to the root Eq.(2.1) so as to finally write

$$m_{rel}e^{-\alpha} = Constant \tag{3.17}$$

in a closed system. It is important to recall that, in YARK, only a photon behaving in line with quantized resonance would deflect in accordance with what is predicted by GTR in gravity, and this, primarily based on the latter equation.

Eq.(3.16) well says that the frequency of a quantal photon increases effectively whilst engaging a gravitational pool, wherefore its wavelength shortens. Eq.(3.17) hence intimates that there is a mass (precisely that of the quantal photon's relativistic mass) which increases in gravity. One thereby finds it amazing to notice that these are occurrences Einstein originally came up with based on his rotating disc gedanken experiment upon which he framed his Equivalence Principle [9]. Hence, GTR's end results — precisely those pertaining to what is delineated by a quantal photon under gravitational or accelerational influence — become, in a way, a subset of YARK's end results.

It is easy to notice that the foregoing derivation is valid for any interaction, inasmuch as yielding (for a test particle having the rest energy $m_{0\infty}c^2$)

$$r_0{}^2 - c^2t_0{}^2 = (1 - E_B/m_{0\infty}c^2)(r^2 - c^2t^2), \tag{3.18}$$

where E_B is the binding energy between the test particle and the host body.

Be that as it may, we leave pertinent details to be tackled outside this article.

4. CONCLUSION

In the present paper, and on the basis of *Yarman's Approach* that is the underlying framework of YARK (Yarman-Arik-Kholmetskii) theory of gravity, a new group of space-time transformations were formulated that we named the "Generalized Lorentz Group" which betokens the ubiquitous breaking down of Lorentz Symmetry (LS) under any force field. Thus, Lorentz Symmetry (LS) is naturally — but fundamentally differently compared to GTR — violated in YARK theory of gravity, too.

An interesting case other than an object moving in gravitation where LS is violated — *but where existing quantum electrodynamical explanations in the literature are not satisfactory* — is that of the bound muon when it is substituted in place of an electron around a given atomic nucleus, whose decay rate retardation can be explained under Yarman's Approach in a much more suitable and elegant way.

Under the given circumstances, note that an assymmetry is introduced already for resting muons bound to an electrical field as compared to those hanging in free space. So, it is not really the relative motion of the muon vis-à-vis the atomic nucleus which breaks the LS; it is that *no space-time symmetry existed between these interacting particles to start with when they were already at rest.*

In the present approach, one readily obtains a pivotal relationship between the local Minkowskian squared line element ${s_0}^2$ and the non-proper Minkowskian squared line element s^2 in a straightforward integral form ${s_0}^2 = s^2 e^{-2\alpha}$ [Eq.(3.10c)]. It should be reminded that YARK is not a purely metric theory, and Eq.(3.12b), or just the same, Eq.(3.12c), should anyway be considered along with the YARK root integral equation — *i.e.*, Eq.(2.1).

One can easily see that the derivation ${s_0}^2 = s^2 e^{-2\alpha}$ we framed remains valid for any kind of interaction, insofar as leading to the general form

$$ {s_0}^2 = {x_0}^2 - c^2 {t_0}^2 = \left(1 - E_B / m_{0\infty} c^2\right)\left(x^2 - c^2 t^2\right), \tag{4.1} $$

where E_B is the binding energy of the system comprised of a client object of mass $m_{0\infty}$ (measured at infinity) and a host body.

The above expression, remaining in full harmony with quantum mechanics and being fundamentally valid for all kinds of interactions, is moreover directly applicable to the many-body problem — just as well as being extensible to gravitation through YARK theory in a much simpler manner compared to what is available for metric theories of gravity.

✻ ✻ ✻

Acknowledgement The authors would like to extend their gratitude to especially Prof. Elman Hasanoglu, Prof. Garret Sobczyk, Prof. Sahin Kocak and Prof. Tekin Dereli for very many hours of creative discussions, and are furthermore indebted to the YARK e-mail group participants (under Googlegroups), and to Prof. Yilmaz Akyildiz in particular who created and manages said group.

Appendix A. YARK'S EXPRESSIONS FOR THE TOTAL DIFFERENTIAL OF THE RADIAL PATH AND THE TOTAL DIFFERENTIAL OF TIME

In this Appendix, we derive the expressions for the total differentials dr and dt in YARK and verify our results through a cross-check.

First, it is important to recall that while, due to the adoption of an infinitesimal rest mass for the photon, the divergence of the velocity v of light is, depending on its energy, actually indistinguishable from the utmost velocity c in YARK, this is still because of the conjectured indistinguishability of measuring distances via measuring locally just the period of time light takes to go forth to the edge of the distance we propose to measure and return from said edge after bouncing back.

In the case where the rest mass is set to exactly zero, the velocity v of light immediately becomes c. This is how we precisely wrote Eqs.(2.2a) and (2.2b) in the text; namely, $t = t_0\, exp(\alpha)$ and $r = r_0\, exp(\alpha)$ — *although, any aberration effect, if any, is, for the present, overlooked.*

The period of time t_0 is defined as locally registered. It becomes t as assessed by the remote observer. Likewise, r_0 is the locally measured distance of concern, and r the same distance as assessed by the remote observer.

The characteristic YARK streching factor $exp(\alpha)$ is furnished quantum mechanically on account of the decrease of the rest mass as described in the text. Under the given circumstances, were c taken unity, r_0 becomes t_0, and r thusly becomes t.

In applying such a measurement technique, it is worth recalling that YARK theory does not even have to bother with whether c varies throughout or not; as pointed out, it varies not in the limit where the photon rest mass is taken to be zero — which yields, a fortiori, $v = c$.

Recall further that $\alpha = GM/rc^2$ [cf. Eq.(2.3) of the text].

One can therefore write

$$c = \frac{r_0}{t_0} = \frac{r}{t}\,. \tag{A.1}$$

This being said, one can right away produce

$$ct_0 = r_0\,, \tag{A.2a}$$

and

$$ct = r; \tag{A.2b}$$

thusly,

$$cdt_0 = dr_0\,, \tag{A.2c}$$

and

$$cdt = dr\,. \tag{A.2d}$$

Therefore, we can extend Eq.(A1) to state further

$$c = \frac{r_0}{t_0} = \frac{r}{t} = \frac{dr_0}{dt_0} = \frac{dr}{dt}\,. \tag{A.3}$$

The differentials coming into play are *total differentials*. So, we already have arrived at a shortcut to answer the quest we posed as the title of this Appendix. Still, we propose to continue to offer further aspects of the present theory.

A.1. **The Total Differential of the Radial Path.**

The radial path's total differential can be straightforwardly grabbed out of Eq.(2.2b) as $[r = r_0\, exp(\alpha)]$ of the text:

$$dr = e^{\alpha} dr_0 + r_0 e^{\alpha} d\alpha\,. \tag{A.4}$$

Let us elaborate on this:

$$dr = e^{\alpha} dr_0 - \frac{GM}{c^2 {r_0}^2 e^{2\alpha}} dr\, r_0 e^{\alpha} = e^{\alpha} dr_0 - \frac{GM}{c^2 r_0 e^{\alpha}} dr = e^{\alpha} dr_0 - \frac{GM}{c^2 r} dr. \tag{A.5}$$

This leads to:

$$dr = e^{\alpha} dr_0 - \alpha dr. \tag{A.6}$$

After re-arranging it, we finally get

$$dr = \frac{e^{\alpha} dr_0}{1+\alpha}\,. \tag{A.7}$$

This is YARK's total differential for the radial path of concern.

A.2. **The Total Differential of Time.**

Now, we are going to write the total differential of time along the radial path in YARK, which requires a bit more elaboration.

What do we mean by the total differential of time along the radial path?

It is this: We have a clock at rest at an altitude r_0 as assessed by the local observer with respect to the center of the given host body, say, Earth; then r is larger than r_0 when assessed by the remote observer by as much as e^{α} [Eq.(2.2a)]. Here, the unit period of time of the clock of concern, when measured locally, is T_0; it becomes $T = T_0 e^{\alpha}$ when assessed by the distant observer. This is the same framework as that sketched by Eq.(2.2) of the text. Let us then write dT in full similarity with Eqs.(A4) and (A5):

$$dT = e^{\alpha} dT_0 + T_0 e^{\alpha} d\alpha\,. \tag{A.8}$$

It is important to thoroughly grasp this equation. Chiefly, it is essential to understand what is meant by dT and dT_0. We hence consider two twin clocks situated at r_0 and $r_0 + dr_0$ respectively as referred to by the local observer (situated at r_0). We had to define the unit period of time of the observer situated at r_0 as T_0. For further precision of denomination, we call it equivalently $T_{0_{(r_0)}}$.We likewise christen $T_{0_{(r_0+dr_0)}}$ as the unit period of time of the twin clock situated at $r_0 + dr_0$; this latter clock too is read by the local observer situated at r_0 as having a duration of $T_{0_{(r_0+dr_0)}}$, which is shorter than T_0 on account of being emburdened with a less intense gravity. We call dT_0 the difference between the two unit durations as assesed by the local observer of concern located at r_0:

$$dT_0 = T_{0_{(r_0+dr_0)}} - T_{0_{(r_0)}} . \tag{A.9}$$

This becomes dT when assessed by the remote observer:

$$dT = T_{(r+dr)} - T_{(r)} . \tag{A.10}$$

It is vital that one does not confuse T_0 or T with any arbitrary passage of time one may be inclined to assign over here; they merely represent *the unit periods of time, or unit durations, of the given clock* situated at r_0 as assessed by the local observer, or the same, at r as assessed by the remote observer.

We then re-arrange Eq.(A8) to land at:

$$\begin{aligned} dT &= e^{\alpha} dT_0 - \frac{GM}{c^2 {r_0}^2 e^{2\alpha}} dr\, T_0 e^{\alpha} = e^{\alpha} dT_0 - \frac{GM}{c^2 r_0 r_0 e^{\alpha}} dr T_0 \\ &= e^{\alpha} dT_0 - \frac{GM}{c^2 r} \frac{T_0}{r_0} dr = e^{\alpha} dT_0 - \frac{GM}{rc^2} \frac{T_0}{r_0} \frac{dT}{dT} dr \ . \end{aligned} \tag{A.11}$$

Let us further re-arrange it to obtain:

$$dT = e^{\alpha} dT_0 - \alpha \frac{T_0}{r_0} \frac{dr}{dT} dT. \tag{A.12}$$

We finally get:

$$dT = \frac{e^{\alpha} dT_0}{\alpha \dfrac{T_0}{r_0} \dfrac{dr}{dT} + 1} . \tag{A.13}$$

At first strike, this looks awkward. All the same, we have to thoroughly understand what we are desiring to achieve over here: We aim to formulate — *under a strictly static framework where we have no motion whatsoever* — how the *local unit period of time* T_0 associated with the local observer's clock would vary when we transition from r_0 to $r_0 + dr_0$ as assessed by the distant observer. Recall that dT_0 is the infinitely short algebraic increment (difference of unit durations) in T_0 as measured by the local observer situated at r_0 if a clock identical to his were re-located to $r_0 + dr_0$. Meanwhile, dT is the same difference of unit durations, but now as assessed by the remote observer.

It will be useful to reformulate our above statement:

Suppose, in effect, we have a pair of twin clocks — one is situated at r_0, and the other at $r_0 + dr_0$; and we like to determine the infinitely small increment dT as delineated by the unit duration of these clocks as assessed by the distant observer.

While the ratios T_0 / r_0 and dr/ dT taking place in Eq.(A13) are at first disturbing, we can right away notice that the local observer counts k number of beats T_0 through a light pulse's round trip in going forth from and coming back to the center of the host body.

Therefore, we can well establish

$$ckT_0 = r_0 , \tag{A.14a}$$

and similarly,

$$ckT = r . \tag{A.14b}$$

Let us now take the differentials respectively. We find:

$$cdt_0 = dr_0 , \tag{A.15a}$$

and

$$ckdT = dr . \tag{A.15b}$$

Hence, we have the quintessential YARK light velocity expressions:

$$c = \frac{r_0}{t_0} = \frac{dr_0}{dt_0} = \frac{e^{\alpha} r_0}{e^{\alpha} t_0} = \frac{r}{t} = \frac{dr}{dt} . \tag{A.16}$$

This is fortunately the same result as that presented at the level of Eq.(A3).

Recall that, in YARK, the velocity of light is an invariant; *i.e.*, the local observer and a distant observer will measure the same value for it. Not only this, but as a matter of fact, all velocities are invariant under YARK. Note that a velocity in, for instance, a free fall or in an elliptical orbit would naturally vary; but what is meant over here is that no matter who — the local observer or the remote observer — assesses it, they will both come out with the same instantaneous value for it.

We have to note that one could not be able to tap such an aesthetically pleasing set of equations presented in Eq.(A16) unless c is both a universal constant and a YARK invariant.

We now go back to Eq.(A13) to re-state

$$dT = \frac{e^{\alpha} dT_0}{\alpha \frac{kT_0}{r_0} \frac{dr}{kdT} + 1} = \frac{e^{\alpha} dT_0}{\alpha \frac{1}{c} c + 1} = \frac{e^{\alpha} dT_0}{\alpha + 1} . \tag{A.17a}$$

or, in short,

$$dT = dT_0 \frac{e^{\alpha}}{1 + \alpha} . \tag{A.17b}$$

As expected, the right-hand-side depends only on dT_0 and not on any other arbitrary "passage of time" that one might imagine. It moreover certainly does not depend either on the unit choice of T_0 (at any rate, this latter unit period of time, or unit duration, cannot ever be set to zero come what may, for it is merely a unit duration and not an arbitrary time lapse in space-time). All this amounts to saying that the factor $e^{\alpha} / (1 + \alpha)$ that comes to multiply dT_0 at the right-hand-side of Eq.(A17b) is strictly identical to what comes to multiply dr_0 in the right-hand-side of Eq.(A7).

Eq.(A17b) says what the infinitely short incremental unit period of time dT the clock of the local observer sitting at $r_0 + dr_0$ must delineate as compared to its twin sitting at r_0, but as assessed by the distant observer. Remember that dT_0 is the same increment, but as evaluated by the local observer sitting at r_0.

Having tapped Eq.(A17b), one can immediately write the infinitesimal period of time dT the clock of the local observer sitting at $r_0 + dr_0$ would tally as compared to its twin sitting at r_0 as assessed by the distant observer:

$$dt = dt_0 \frac{e^{\alpha}}{1+\alpha} . \tag{A.18}$$

Let us then notice that dt_0 is the same increment, but as evaluated by the local observer sitting at r_0. It is finally important to notice that the factor $e^{\alpha}/(1+\alpha)$ that comes to multiply dr_0 at the right-hand-side of Eq.(A7) now came to multiply over here, and in its entirety, dt_0. This is, in effect, how we ultimately landed at Eqs.(3.12a) and (3.12b) of the text.

A.3. **Cross-checking Through the Rest Mass Variation.**

Finally, we propose to write down the YARK rest mass difference for the two twin clocks we have operated with right above, where one of them was sitting at r and the other one was sitting at $r + dr$.

The infinitesimal YARK rest mass incremental change the clock at r would delineate from r to $r + dr$ as assesed by the distant observer is given, for $c = 1$, by the root YARK differential equation [33, 34, 37]

$$dm(r) = \frac{GMm(r)dr}{r^2} . \tag{A.19}$$

Owing to the YARK relationship [44]

$$\frac{G}{r^2} = \frac{G_0}{{r_0}^2} , \tag{A.20}$$

Eq.(A19) will be transformed to

$$\begin{aligned} dm(r) &= \frac{G_0 Mm(r)dr}{{r_0}^2} = \frac{G_0 Mm(r_0)dr}{{r_0}^2}\frac{dr_0}{dr_0} \\ &= dm(r_0)\frac{dr}{dr_0} = dm(r_0)\frac{e^{\alpha}}{1+\alpha} . \end{aligned} \tag{A.21a}$$

In short, we have:

$$dm(r) = dm(r_0)\frac{e^{\alpha}}{1+\alpha} \quad (qed). \tag{A.21b}$$

This means, one has to insert into the quantum mechanical description of the object at hand a mass variation by a factor of $e^{\alpha}/(1+\alpha)$ to sort out how r_0 and t_0 would vary in terms of dr_0 and dt_0, and this is precisely what we found out above.

In summary, we have, within the framework of YARK Theory, the following set of fundamental relationships:

$$t = e^{\alpha} t_0 \ , \tag{A.22a}$$

$$r = e^{\alpha} r_0 \ , \tag{A.22b}$$

$$c = \frac{r_0}{t_0} = \frac{dr_0}{dt_0} = \frac{e^{\alpha} r_0}{e^{\alpha} t_0} = \frac{r}{t} = \frac{dr}{dt} \ , \tag{A.22c}$$

$$dr = dr_0 \frac{e^{\alpha}}{1+\alpha} \ , \tag{A.22d}$$

$$dt = dt_0 \frac{e^{\alpha}}{1+\alpha} \ . \tag{A.22e}$$

While YARK allows us to work with either a resonant bending photon (whose frequency gets quantized in resonance with gravity insofar as providing the expected deflection amount), or just a projectile-like behaving photon (which can be expressed classically and yet exhibits anomalous deflection), we have not considered over here the particularities of the former case. Nonetheless, its consideration would not in any way alter the institution of the abovementioned fundamental YARK relationships — except that they should be re-dealt with in the light of Eq.(3.16) of the text.

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