TWMS J. App. and Eng. Math. V.10, N.4, 2020, pp. 1105-1115

FUZZY PARAMETERIZED FUZZY SOFT MATRICES AND THEIR APPLICATION IN DECISION-MAKING

SERDAR ENGİNOĞLU[1], NAİM ÇAĞMAN[2], §

ABSTRACT. In this study, we define the concept of fuzzy parameterized fuzzy soft matrices (*fpfs*-matrices) and present some of their basic properties. By using *fpfs*-matrices, we then suggest a new algorithm, i.e. Prevalence Effect Method (PEM), and apply this method to a performance-based value assignment, so that we can order noise removal filters regarding performance. The results show that PEM has a potential for several areas, such as machine learning and image processing. Finally, we discuss *fpfs*-matrices and PEM for further research.

Keywords: Fuzzy sets, soft sets, soft matrices, *fpfs*-matrices, prevalence effect method
AMS Subject Classification: 03E72, 03E75, 15B15

1. Introduction

The concept of soft sets, the graphic of a function from a parameter set to the power set of a universal set, was defined by Molodtsov [1] in 1999 to deal with many problems containing uncertainties and so far a broad range of theoretical and applied studies from algebra to decision-making have been conducted on this concept [2–43].

In 2010, Çağman et al. [6] introduced the fuzzy parameterized fuzzy soft sets (*fpfs*-sets) because back then a more general form was needed for mathematical modelling of some problems in the event of parameters or objects with uncertainties - today such a need still exists. However, in the case that a large body of data is processed, computer mathematics should be employed. To deal with this problem, in Section 2, we propose the concept of fuzzy parameterized fuzzy soft matrices (*fpfs*-matrices) and investigate some of their basic properties. This concept is first mentioned in the first author's PhD dissertation. In Section 3, by using *fpfs*-matrices we describe a fast and simple algorithm named Prevalence Effect Method (PEM). In Section 4, we apply this method to a performance-based value assignment problem. Finally, we discuss *fpfs*-matrices and PEM for further research.

[1] Department of Mathematics, Faculty of Arts and Sciences, Çanakkale Onsekiz Mart University, Çanakkale, Turkey (Corresponding Author).
e-mail: serdarenginoglu@gmail.com; ORCID: https://orcid.org/0000-0002-7188-9893.
[2] Department of Mathematics, Faculty of Arts and Sciences, Tokat Gaziosmanpaşa University, Tokat, Turkey.
e-mail: naim.cagman@gop.edu.tr; ORCID: https://orcid.org/0000-0003-3037-1868.
§ Manuscript received: November 14, 2018; accepted: January 17, 2019.
TWMS Journal of Applied and Engineering Mathematics, Vol.10, No.4 © Işık University, Department of Mathematics, 2020; all rights reserved.

2. Fuzzy Parameterized Fuzzy Soft Matrices

In this section, we introduce the concept of *fpfs*-matrices and their basic properties that are first given in the first author's PhD dissertation [17].

Throughout this paper, let E be a parameter set, $F(E)$ be the set of all fuzzy sets over E, and $\mu \in F(E)$. Here, a fuzzy set is denoted by $\{^{\mu(x)}x : x \in E\}$ instead of $\{(x, \mu(x)) : x \in E\}$.

Definition 2.1. [6, 17] *Let U be a universal set, $\mu \in F(E)$, and α be a function from μ to $F(U)$. Then, the set $\{(^{\mu(x)}x, \alpha(^{\mu(x)}x)) : x \in E\}$ being the graphic of α is called a fuzzy parameterized fuzzy soft set (fpfs-set) parameterized via E over U (or briefly over U).*

In the present paper, the set of all *fpfs*-sets over U is denoted by $FPFS_E(U)$. In $FPFS_E(U)$, since the $graph(\alpha)$ and α generate each other uniquely, the notations are interchangeable. Therefore, unless it causes any confusion, we denote an *fpfs*-set $graph(\alpha)$ by α.

Example 2.1. *Let $E = \{x_1, x_2, x_3, x_4\}$ and $U = \{u_1, u_2, u_3, u_4, u_5\}$. Then,*

$$\alpha = \{(^{0.8}x_1, \{^{0.9}u_1, {}^{0.5}u_4\}), (^{0}x_2, \{^{0.3}u_2, {}^{0.5}u_3\}), (^{0.1}x_3, \{^{0.7}u_1, {}^{0.8}u_3, {}^{0.6}u_4\}), (x_4, \{u_3, {}^{0.9}u_5\})\}$$

is an fpfs-set over U.

Definition 2.2. [17] *Let $\alpha \in FPFS_E(U)$. Then, $[a_{ij}]$ is called the matrix representation of α (or briefly fpfs-matrix of α) and is defined by*

$$[a_{ij}] := \begin{bmatrix} a_{01} & a_{02} & a_{03} & \cdots & a_{0n} & \cdots \\ a_{11} & a_{12} & a_{13} & \cdots & a_{1n} & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \ddots \end{bmatrix}$$

such that for $i \in \{0, 1, 2, \cdots\}$ and $j \in \{1, 2, \cdots\}$,

$$a_{ij} := \begin{cases} \mu(x_j), & i = 0 \\ \alpha(^{\mu(x_j)}x_j)(u_i), & i \neq 0 \end{cases}$$

Here, if $|U| = m - 1$ and $|E| = n$, then $[a_{ij}]$ has order $m \times n$.

From now on, the set of all *fpfs*-matrices parameterized via E over U is denoted by $FPFS_E[U]$.

Example 2.2. *The fpfs-matrix of α provided in Example 2.1 is as follows:*

$$[a_{ij}] = \begin{bmatrix} 0.8 & 0 & 0.1 & 1 \\ 0.9 & 0 & 0.7 & 0 \\ 0 & 0.3 & 0 & 0 \\ 0 & 0.5 & 0.8 & 1 \\ 0.5 & 0 & 0.6 & 0 \\ 0 & 0 & 0 & 0.9 \end{bmatrix}$$

Definition 2.3. *Let $[a_{ij}] \in FPFS_E[U]$. For all i and j, if $a_{ij} = \lambda$, then $[a_{ij}]$ is called λ-fpfs-matrix and is denoted by $[\lambda]$. Here, $[0]$ is called empty fpfs-matrix and $[1]$ is called universal fpfs-matrix.*

Definition 2.4. *Let* $[a_{ij}], [b_{ij}], [c_{ij}] \in FPFS_E[U]$, $I_E := \{j : x_j \in E\}$, *and* $R \subseteq I_E$. *If*

$$c_{ij} := \begin{cases} a_{ij}, & j \in R \\ b_{ij}, & j \in I_E \setminus R \end{cases}$$

then $[c_{ij}]$ *is called Rb-restriction of* $[a_{ij}]$ *and is denoted by* $[(a_{Rb})_{ij}]$. *Briefly, if* $[b_{ij}] = [0]$, *then* $[(a_R)_{ij}]$ *can be used instead of* $[(a_{R0})_{ij}]$. *It is clear that*

$$(a_R)_{ij} = \begin{cases} a_{ij}, & j \in R \\ 0, & j \in I_E \setminus R \end{cases}$$

Example 2.3. *Let* $R = \{1, 3, 4\}$ *and* $S = \{2, 4\}$. *Then, the* $R1$*- and* S*-restriction of* $[a_{ij}]$ *provided in Example 2.2 are as follows:*

$$[(a_{R1})_{ij}] = \begin{bmatrix} 0.8 & 1 & 0.1 & 1 \\ 0.9 & 1 & 0.7 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0.8 & 1 \\ 0.5 & 1 & 0.6 & 0 \\ 0 & 1 & 0 & 0.9 \end{bmatrix} \quad \text{and} \quad [(a_S)_{ij}] = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0.3 & 0 & 0 \\ 0 & 0.5 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.9 \end{bmatrix}$$

Definition 2.5. [17] *Let* $[a_{ij}], [b_{ij}] \in FPFS_E[U]$. *For all* i *and* j, *if* $a_{ij} \leq b_{ij}$, *then* $[a_{ij}]$ *is called a submatrix of* $[b_{ij}]$ *and is denoted by* $[a_{ij}]\tilde{\subseteq}[b_{ij}]$.

Proposition 2.1. [17] *Let* $[a_{ij}], [b_{ij}], [c_{ij}] \in FPFS_E[U]$. *Then,*

i. $[a_{ij}]\tilde{\subseteq}[1]$
ii. $[0]\tilde{\subseteq}[a_{ij}]$
iii. $[a_{ij}]\tilde{\subseteq}[a_{ij}]$
iv. $\big([a_{ij}]\tilde{\subseteq}[b_{ij}] \wedge [b_{ij}]\tilde{\subseteq}[c_{ij}]\big) \Rightarrow [a_{ij}]\tilde{\subseteq}[c_{ij}]$

Definition 2.6. [17] *Let* $[a_{ij}], [b_{ij}] \in FPFS_E[U]$. *For all* i *and* j, *if* $a_{ij} = b_{ij}$, *then* $[a_{ij}]$ *and* $[b_{ij}]$ *are called equal fpfs-matrices and is denoted by* $[a_{ij}] = [b_{ij}]$.

Definition 2.7. [17] *Let* $[a_{ij}], [b_{ij}] \in FPFS_E[U]$. *If* $[a_{ij}]\tilde{\subseteq}[b_{ij}]$ *and* $[a_{ij}] \neq [b_{ij}]$, *then* $[a_{ij}]$ *is called a proper submatrix of* $[b_{ij}]$ *and is denoted by* $[a_{ij}]\tilde{\subsetneq}[b_{ij}]$.

Proposition 2.2. [17] *Let* $[a_{ij}], [b_{ij}], [c_{ij}] \in FPFS_E[U]$. *Then,*

i. $([a_{ij}] = [b_{ij}] \wedge [b_{ij}] = [c_{ij}]) \Rightarrow [a_{ij}] = [c_{ij}]$
ii. $\big([a_{ij}]\tilde{\subseteq}[b_{ij}] \wedge [b_{ij}]\tilde{\subseteq}[a_{ij}]\big) \Leftrightarrow [a_{ij}] = [b_{ij}]$

Definition 2.8. [17] *Let* $[a_{ij}], [b_{ij}], [c_{ij}] \in FPFS_E[U]$. *For all* i *and* j, *if* $c_{ij} := \max\{a_{ij}, b_{ij}\}$, *then* $[c_{ij}]$ *is called union of* $[a_{ij}]$ *and* $[b_{ij}]$ *and is denoted by* $[a_{ij}]\tilde{\cup}[b_{ij}]$.

Definition 2.9. [17] *Let* $[a_{ij}], [b_{ij}], [c_{ij}] \in FPFS_E[U]$. *For all* i *and* j, *if* $c_{ij} := \min\{a_{ij}, b_{ij}\}$, *then* $[c_{ij}]$ *is called intersection of* $[a_{ij}]$ *and* $[b_{ij}]$ *and is denoted by* $[a_{ij}]\tilde{\cap}[b_{ij}]$.

Example 2.4. *Assume that two fpfs-matrices* $[a_{ij}]$ *and* $[b_{ij}]$ *are as follows:*

$$[a_{ij}] = \begin{bmatrix} 0.3 & 0.7 & 0.1 & 1 \\ 0.5 & 0 & 0.8 & 0.6 \\ 1 & 0.1 & 0.6 & 0 \\ 0 & 0.5 & 0.1 & 1 \\ 0 & 0.2 & 0.7 & 0.9 \\ 0.4 & 1 & 0 & 0.4 \end{bmatrix} \quad \text{and} \quad [b_{ij}] = \begin{bmatrix} 0.9 & 0.5 & 0.7 & 0.8 \\ 1 & 0 & 0.6 & 0.7 \\ 0.8 & 0.5 & 0.5 & 0.2 \\ 0.4 & 0.7 & 0.6 & 1 \\ 0.6 & 0.2 & 0.4 & 0.5 \\ 0.3 & 0.9 & 0.8 & 0.9 \end{bmatrix}$$

Then,

$$[a_{ij}]\tilde{\cup}[b_{ij}] = \begin{bmatrix} 0.9 & 0.7 & 0.7 & 1 \\ 1 & 0 & 0.8 & 0.7 \\ 1 & 0.5 & 0.6 & 0.2 \\ 0.4 & 0.7 & 0.6 & 1 \\ 0.6 & 0.2 & 0.7 & 0.9 \\ 0.4 & 1 & 0.8 & 0.9 \end{bmatrix} \quad \text{and} \quad [a_{ij}]\tilde{\cap}[b_{ij}] = \begin{bmatrix} 0.3 & 0.5 & 0.1 & 0.8 \\ 0.5 & 0 & 0.6 & 0.6 \\ 0.8 & 0.1 & 0.5 & 0 \\ 0 & 0.5 & 0.1 & 1 \\ 0 & 0.2 & 0.4 & 0.5 \\ 0.3 & 0.9 & 0 & 0.4 \end{bmatrix}$$

Proposition 2.3. [17] *Let* $[a_{ij}], [b_{ij}], [c_{ij}] \in FPFS_E[U]$. *Then,*

i. $[a_{ij}]\tilde{\cup}[a_{ij}] = [a_{ij}]$ *and* $[a_{ij}]\tilde{\cap}[a_{ij}] = [a_{ij}]$
ii. $[a_{ij}]\tilde{\cup}[0] = [a_{ij}]$ *and* $[a_{ij}]\tilde{\cap}[0] = [0]$
iii. $[a_{ij}]\tilde{\cup}[1] = [1]$ *and* $[a_{ij}]\tilde{\cap}[1] = [a_{ij}]$
iv. $[a_{ij}]\tilde{\cup}[b_{ij}] = [b_{ij}]\tilde{\cup}[a_{ij}]$ *and* $[a_{ij}]\tilde{\cap}[b_{ij}] = [b_{ij}]\tilde{\cap}[a_{ij}]$
v. $([a_{ij}]\tilde{\cup}[b_{ij}])\tilde{\cup}[c_{ij}] = [a_{ij}]\tilde{\cup}([b_{ij}]\tilde{\cup}[c_{ij}])$ *and* $([a_{ij}]\tilde{\cap}[b_{ij}])\tilde{\cap}[c_{ij}] = [a_{ij}]\tilde{\cap}([b_{ij}]\tilde{\cap}[c_{ij}])$
vi. $[a_{ij}]\tilde{\cup}([b_{ij}]\tilde{\cap}[c_{ij}]) = ([a_{ij}]\tilde{\cup}[b_{ij}])\tilde{\cap}([a_{ij}]\tilde{\cup}[c_{ij}])$ *and* $[a_{ij}]\tilde{\cap}([b_{ij}]\tilde{\cup}[c_{ij}]) = ([a_{ij}]\tilde{\cap}[b_{ij}])\tilde{\cup}([a_{ij}]\tilde{\cap}[c_{ij}])$

Proof.

$$\begin{aligned} [a_{ij}]\tilde{\cup}([b_{ij}]\tilde{\cap}[c_{ij}]) &= [a_{ij}]\tilde{\cup}[\min\{b_{ij}, c_{ij}\}] \\ &= [\max\{a_{ij}, \min\{b_{ij}, c_{ij}\}\}] \\ &= [\min\{\max\{a_{ij}, b_{ij}\}, \max\{a_{ij}, c_{ij}\}\}] \\ &= [\max\{a_{ij}, b_{ij}\}]\tilde{\cap}[\max\{a_{ij}, c_{ij}\}] \\ &= ([a_{ij}]\tilde{\cup}[b_{ij}])\tilde{\cap}([a_{ij}]\tilde{\cup}[c_{ij}]) \end{aligned}$$

□

The proofs of the others can be performed similarly.

Definition 2.10. [17] *Let* $[a_{ij}], [b_{ij}], [c_{ij}] \in FPFS_E[U]$. *For all* i *and* j, *if* $c_{ij} := \max\{0, a_{ij} - b_{ij}\}$, *then* $[c_{ij}]$ *is called difference between* $[a_{ij}]$ *and* $[b_{ij}]$ *and is denoted by* $[a_{ij}]\tilde{\setminus}[b_{ij}]$.

Proposition 2.4. [17] *Let* $[a_{ij}], [b_{ij}] \in FPFS_E[U]$. *Then,*

i. $[a_{ij}]\tilde{\setminus}[a_{ij}] = [0]$
ii. $[a_{ij}]\tilde{\setminus}[0] = [a_{ij}]$
iii. $[a_{ij}]\tilde{\setminus}[1] = [0]$
iv. $([a_{ij}]\tilde{\setminus}[b_{ij}] = [0]) \Rightarrow [a_{ij}]\tilde{\subseteq}[b_{ij}]$

Remark 2.1. *It must be noted that the difference operation is non-commutative and non-associative.*

Definition 2.11. [17] *Let* $[a_{ij}], [b_{ij}] \in FPFS_E[U]$. *For all* i *and* j, *if* $b_{ij} := 1 - a_{ij}$, *then* $[b_{ij}]$ *is complement of* $[a_{ij}]$ *and is denoted by* $[a_{ij}]^{\tilde{c}}$ *or* $[a_{ij}^{\tilde{c}}]$.

Proposition 2.5. [17] *Let* $[a_{ij}] \in FPFS_E[U]$. *Then,*

i. $([a_{ij}]^{\tilde{c}})^{\tilde{c}} = [a_{ij}]$
ii. $[0]^{\tilde{c}} = [1]$

Proposition 2.6. [17] *Let* $[a_{ij}], [b_{ij}] \in FPFS_E[U]$. *Then, the following De Morgan's laws are valid.*

i. $([a_{ij}]\tilde{\cup}[b_{ij}])^{\tilde{c}} = [a_{ij}]^{\tilde{c}}\tilde{\cap}[b_{ij}]^{\tilde{c}}$
ii. $([a_{ij}]\tilde{\cap}[b_{ij}])^{\tilde{c}} = [a_{ij}]^{\tilde{c}}\tilde{\cup}[b_{ij}]^{\tilde{c}}$

Proof.

$$\begin{aligned}([a_{ij}]\tilde{\cup}[b_{ij}])^{\tilde{c}} &= [\max\{a_{ij}, b_{ij}\}]^{\tilde{c}} \\ &= [1-\max\{a_{ij}, b_{ij}\}] \\ &= [\min\{1-a_{ij}, 1-b_{ij}\}] \\ &= [a_{ij}]^{\tilde{c}}\tilde{\cap}[b_{ij}]^{\tilde{c}}\end{aligned}$$

□

The proof of *ii.* can be performed similarly.

Definition 2.12. [17] *Let* $[a_{ij}], [b_{ij}], [c_{ij}] \in FPFS_E[U]$*. For all* i *and* j*, if* $c_{ij} := |a_{ij} - b_{ij}|$*, then* $[c_{ij}]$ *is called symmetric difference between* $[a_{ij}]$ *and* $[b_{ij}]$ *and is denoted by* $[a_{ij}]\tilde{\triangle}[b_{ij}]$*.*

Definition 2.13. [17] *Let* $[a_{ij}], [b_{ij}] \in FPFS_E[U]$*. If* $[a_{ij}]\tilde{\cap}[b_{ij}] = [0]$*, then* $[a_{ij}]$ *and* $[b_{ij}]$ *are called disjoint.*

Example 2.5. *Let us consider the fpfs-matrices* $[a_{ij}]$ *and* $[b_{ij}]$ *provided in Example 2.4. Then,*

$$[a_{ij}]\tilde{\setminus}[b_{ij}] = \begin{bmatrix} 0 & 0.2 & 0 & 0.2 \\ 0 & 0 & 0.2 & 0 \\ 0.2 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0.3 & 0.4 \\ 0.1 & 0.1 & 0 & 0 \end{bmatrix} \quad \text{and} \quad [a_{ij}]\tilde{\triangle}[b_{ij}] = \begin{bmatrix} 0.6 & 0.2 & 0.6 & 0.2 \\ 0.5 & 0 & 0.2 & 0.1 \\ 0.2 & 0.4 & 0.1 & 0.2 \\ 0.4 & 0.2 & 0.5 & 0 \\ 0.6 & 0 & 0.3 & 0.4 \\ 0.1 & 0.1 & 0.8 & 0.5 \end{bmatrix}$$

Definition 2.14. [17] *Let* $[a_{ij}], [b_{ij}], [c_{ij}] \in FPFS_E[U]$*,* $I_E := \{j : x_j \in E\}$*, and* $R \subseteq I_E$*. If*

$$c_{ij} := \begin{cases} \max\{a_{ij}, \min_{k\in R}\{b_{ik}\}\}, & j \in R \\ a_{ij}, & j \in I_E \setminus R \end{cases}$$

then $[c_{ij}]$ *is called* R*-relative union of* $[a_{ij}]$ *and* $[b_{ij}]$ *and is denoted by* $[a_{ij}]\tilde{\cup}^r_R[b_{ij}]$*. Here, for the brevity, "relative union" can be used instead of "*I_E*-relative union" and can be denoted by* $[a_{ij}]\tilde{\cup}^r[b_{ij}]$*.*

Definition 2.15. [17] *Let* $[a_{ij}], [b_{ij}], [c_{ij}] \in FPFS_E[U]$*,* $I_E := \{j : x_j \in E\}$*, and* $R \subseteq I_E$*. If*

$$c_{ij} := \begin{cases} \min\{a_{ij}, \max_{k\in R}\{b_{ik}\}\}, & j \in R \\ a_{ij}, & j \in I_E \setminus R \end{cases}$$

then $[c_{ij}]$ *is called* R*-relative intersection of* $[a_{ij}]$ *and* $[b_{ij}]$ *and is denoted by* $[a_{ij}]\tilde{\cap}^r_R[b_{ij}]$*. Here, for brevity, "relative intersection" can be used instead of "*I_E*-relative intersection" and can be denoted by* $[a_{ij}]\tilde{\cap}^r[b_{ij}]$*.*

Definition 2.16. [17] *Let* $[a_{ij}], [b_{ij}], [c_{ij}] \in FPFS_E[U]$*,* $I_E := \{j : x_j \in E\}$*, and* $R \subseteq I_E$*. If*

$$c_{ij} := \begin{cases} \max\{0, a_{ij} - \min_{k\in R}\{b_{ik}\}\}, & j \in R \\ a_{ij}, & j \in I_E \setminus R \end{cases}$$

then $[c_{ij}]$ *is called* R*-relative difference between* $[a_{ij}]$ *and* $[b_{ij}]$ *and is denoted by* $[a_{ij}]\tilde{\setminus}^r_R[b_{ij}]$*. Here, for brevity, "relative difference" can be used instead of "*I_E*-relative difference" and can be denoted by* $[a_{ij}]\tilde{\setminus}^r[b_{ij}]$*.*

Example 2.6. *Let us consider the fpfs-matrices $[a_{ij}]$ and $[b_{ij}]$ provided in Example 2.4 and let $R = \{2,3,4\}$. Then,*

$$[a_{ij}]\tilde{\cup}^r[b_{ij}] = \begin{bmatrix} 0.5 & 0.7 & 0.5 & 1 \\ 0.5 & 0 & 0.8 & 0.6 \\ 1 & 0.2 & 0.6 & 0.2 \\ 0.4 & 0.5 & 0.4 & 1 \\ 0.2 & 0.2 & 0.7 & 0.9 \\ 0.4 & 1 & 0.3 & 0.4 \end{bmatrix} \quad \text{and} \quad [a_{ij}]\tilde{\cap}^r_R[b_{ij}] = \begin{bmatrix} 0.3 & 0.7 & 0.1 & 0.8 \\ 0.5 & 0 & 0.7 & 0.6 \\ 1 & 0.1 & 0.5 & 0 \\ 0 & 0.5 & 0.1 & 1 \\ 0 & 0.2 & 0.5 & 0.5 \\ 0.4 & 0.9 & 0 & 0.4 \end{bmatrix}$$

Proposition 2.7. [17] *Let $[a_{ij}], [b_{ij}], [c_{ij}] \in FPFS_E[U]$. Then,*

i. $[a_{ij}]\tilde{\cup}^r_R[0] = [a_{ij}]$ *and* $[0]\tilde{\cap}^r_R[a_{ij}] = [0]$
ii. $[1]\tilde{\cup}^r_R[a_{ij}] = [1]$ *and* $[a_{ij}]\tilde{\cap}^r_R[1] = [a_{ij}]$
iii. $([a_{ij}]\tilde{\cup}^r_R[b_{ij}])\tilde{\cup}^r_R[c_{ij}] = [a_{ij}]\tilde{\cup}^r_R([b_{ij}]\tilde{\cup}^r_R[c_{ij}])$ *and* $([a_{ij}]\tilde{\cap}^r_R[b_{ij}])\tilde{\cap}^r_R[c_{ij}] = [a_{ij}]\tilde{\cap}^r_R([b_{ij}]\tilde{\cap}^r_R[c_{ij}])$

Remark 2.2. *It must be noted that the relative intersection and relative union of fpfs-matrices are non-commutative and non-distributive.*

Proposition 2.8. [17] *Let $[a_{ij}], [b_{ij}] \in FPFS_E[U]$. Then, the following De Morgan's laws are valid.*

i. $([a_{ij}]\tilde{\cup}^r_R[b_{ij}])^{\tilde{c}} = [a_{ij}]^{\tilde{c}}\tilde{\cap}^r_R[b_{ij}]^{\tilde{c}}$
ii. $([a_{ij}]\tilde{\cap}^r_R[b_{ij}])^{\tilde{c}} = [a_{ij}]^{\tilde{c}}\tilde{\cup}^r_R[b_{ij}]^{\tilde{c}}$

Proof. If $j \in R \subseteq I_E$, then

$$\begin{aligned} ([a_{ij}]\tilde{\cup}^r_R[b_{ij}])^{\tilde{c}} &= [\max\{a_{ij}, \min_{k\in R}\{b_{ik}\}\}]^{\tilde{c}} \\ &= [1 - \max\{a_{ij}, \min_{k\in R}\{b_{ik}\}\}] \\ &= [\min\{1 - a_{ij}, 1 - \min_{k\in R}\{b_{ik}\}\}] \\ &= [\min\{1 - a_{ij}, \max_{k\in R}\{1 - b_{ik}\}\}] \\ &= [a_{ij}]^{\tilde{c}}\tilde{\cap}^r_R[b_{ij}]^{\tilde{c}} \end{aligned}$$

and if $j \in I_E \setminus R$, then

$$\begin{aligned} ([a_{ij}]\tilde{\cup}^r_R[b_{ij}])^{\tilde{c}} &= [a_{ij}]^{\tilde{c}} \\ &= [1 - a_{ij}] \\ &= [a_{ij}]^{\tilde{c}}\tilde{\cap}^r_R[b_{ij}]^{\tilde{c}} \end{aligned}$$

□

The proof of *ii.* can be made in a similar way.

Definition 2.17. *Let $[a_{ij}]_{m\times n_1} \in FPFS_{E_1}[U]$, $[b_{ik}]_{m\times n_2} \in FPFS_{E_2}[U]$, and $[c_{ip}]_{m\times n_1 n_2} \in FPFS_{E_1\times E_2}[U]$ such that $p = n_2(j-1)+k$. For all i and p, if $c_{ip} := \min\{a_{ij}, b_{ik}\}$, then $[c_{ip}]$ is called and-product of $[a_{ij}]$ and $[b_{ik}]$ and is denoted by $[a_{ij}]\wedge[b_{ik}]$.*

Definition 2.18. *Let $[a_{ij}]_{m\times n_1} \in FPFS_{E_1}[U]$, $[b_{ik}]_{m\times n_2} \in FPFS_{E_2}[U]$, and $[c_{ip}]_{m\times n_1 n_2} \in FPFS_{E_1\times E_2}[U]$ such that $p = n_2(j-1)+k$. For all i and p, if $c_{ip} := \max\{a_{ij}, b_{ik}\}$, then $[c_{ip}]$ is called or-product of $[a_{ij}]$ and $[b_{ik}]$ and is denoted by $[a_{ij}]\vee[b_{ik}]$.*

Definition 2.19. *Let $[a_{ij}]_{m\times n_1} \in FPFS_{E_1}[U]$, $[b_{ik}]_{m\times n_2} \in FPFS_{E_2}[U]$, and $[c_{ip}]_{m\times n_1 n_2} \in FPFS_{E_1\times E_2}[U]$ such that $p = n_2(j-1)+k$. For all i and p, if $c_{ip} := \min\{a_{ij}, 1-b_{ik}\}$, then $[c_{ip}]$ is called andnot-product of $[a_{ij}]$ and $[b_{ik}]$ and is denoted by $[a_{ij}]\overline{\wedge}[b_{ik}]$.*

Definition 2.20. *Let $[a_{ij}]_{m\times n_1} \in FPFS_{E_1}[U]$, $[b_{ik}]_{m\times n_2} \in FPFS_{E_2}[U]$, and $[c_{ip}]_{m\times n_1 n_2} \in FPFS_{E_1\times E_2}[U]$ such that $p = n_2(j-1)+k$. For all i and p, if $c_{ip} := \max\{a_{ij}, 1-b_{ik}\}$, then $[c_{ip}]$ is called ornot-product of $[a_{ij}]$ and $[b_{ik}]$ and is denoted by $[a_{ij}]\underline{\vee}[b_{ik}]$.*

Example 2.7. *Let us consider the fpfs-matrices* $[a_{ij}]$ *and* $[b_{ik}]$ *provided in Example 2.4. Then,* $[a_{ij}]\underline{\vee}[b_{ik}]$ *is as follows:*

$$\begin{bmatrix} 0.3 & 0.5 & 0.3 & 0.3 & 0.7 & 0.7 & 0.7 & 0.7 & 0.1 & 0.5 & 0.3 & 0.2 & 1 & 1 & 1 & 1 \\ 0.5 & 1 & 0.5 & 0.5 & 0 & 1 & 0.4 & 0.3 & 0.8 & 1 & 0.8 & 0.8 & 0.6 & 1 & 0.6 & 0.6 \\ 1 & 1 & 1 & 1 & 0.2 & 0.5 & 0.5 & 0.8 & 0.6 & 0.6 & 0.6 & 0.8 & 0.2 & 0.5 & 0.5 & 0.8 \\ 0.6 & 0.3 & 0.4 & 0 & 0.6 & 0.5 & 0.5 & 0.5 & 0.6 & 0.3 & 0.4 & 0.1 & 1 & 1 & 1 & 1 \\ 0.4 & 0.8 & 0.6 & 0.5 & 0.4 & 0.8 & 0.6 & 0.5 & 0.7 & 0.8 & 0.7 & 0.7 & 0.9 & 0.9 & 0.9 & 0.9 \\ 0.7 & 0.4 & 0.4 & 0.4 & 1 & 1 & 1 & 1 & 0.7 & 0.1 & 0.2 & 0.1 & 0.7 & 0.4 & 0.4 & 0.4 \end{bmatrix}$$

Proposition 2.9. *Let* $[a_{ij}], [b_{ik}], [c_{il}]$ *be three fpfs-matrices over* U. *Then,*

$$([a_{ij}] \wedge [b_{ik}]) \wedge [c_{il}] = [a_{ij}] \wedge ([b_{ik}] \wedge [c_{il}]) \text{ and } ([a_{ij}] \vee [b_{ik}]) \vee [c_{il}] = [a_{ij}] \vee ([b_{ik}] \vee [c_{il}])$$

Remark 2.3. *It must be noted that the products mentioned above of fpfs-matrices are non-commutative and non-distributive.*

3. Prevalence Effect Method (PEM)

In this section, we propose a new soft decision-making method called Prevalence Effect Method (PEM).

Step 1. Construct an *fpfs*-matrix $[a_{ij}]_{m\times n}$ such that $i \in \{0, 1, 2, ..., m-1\}$, $j \in \{1, 2, ..., n\}$, $m \geq 2$, and $n \geq 1$

Step 2. Obtain a matrix $[s_{i1}]$ defined by $s_{i1}{:=}\sum_{j=1}^{n}\left[\left(\frac{1}{m-1}\sum_{k=1}^{m-1} a_{kj}\right)\left(\frac{1}{n}\sum_{t=1}^{n} a_{it}\right) a_{0j}a_{ij}\right]$ such that $i \in \{1, 2, ..., m-1\}$

Step 3. Obtain a decision set $\left\{\frac{s_{k1}}{\max s_{i1}} u_k \mid u_k \in U\right\}$

Here, a_{ij} shows to what extent i^{th} alternative provide the j^{th} parameter such that $i \neq 0$, a_{0j} shows how essential j^{th} parameter is for the user, $\frac{1}{n}\sum_{t=1}^{n} a_{it}$ refers to the prevalence effect value of i^{th} alternative, $\frac{1}{m-1}\sum_{k=1}^{m-1} a_{kj}$ to the prevalence effect value of j^{th} parameter, and s_{i1} to the score value.

4. An Application of PEM

In this section, we apply PEM to a real problem in image denoising. Image denoising (noise removal), which is a preprocess in image processing, positively affects the success rate of other procedures. Therefore, a great many studies have been conducted in this area [44–48].

We, in this study, consider five noise removal methods - Progressive Switching Median Filter (PSMF) [44], Decision-Based Algorithm (DBA) [45], Modified Decision-Based Unsymmetrical Trimmed Median Filter (MDBUTMF) [46], Noise Adaptive Fuzzy Switching Median Filter (NAFSMF) [47], and Different Applied Median Filter (DAMF) [48] - used in [48] for salt-and-pepper noise removal, which is a kind of impulse noise. We compare these methods with regard to performance by using 15 traditional images (Cameraman, Lena, Peppers, Baboon, Plane, Bridge, Pirate, Elaine, Boat, Lake, Flintstones, Living Room, House, Parrot, and Hill) with 512 × 512 pixels and 40 test images with 600 × 600 pixels in the TEST IMAGES Database [49], ranging in noise densities from 10% to 90%, and two image quality metrics: Peak Signal to Noise Ratio (PSNR) and Structural Similarity (SSIM) [50]. The results in Table 1 and 2 show that DAMF outperforms the others in any noise density.

TABLE 1. The mean SSIM results for the 15 traditional images

Algorithm	10%	20%	30%	40%	50%	60%	70%	80%	90%
PSMF	0.9028	0.8715	0.8018	0.6988	0.4903	0.1882	0.0633	0.0318	0.0139
DBA	0.9079	0.8664	0.8097	0.7376	0.6521	0.5552	0.4567	0.3623	0.2937
MDBUTMF	0.8841	0.7994	0.7443	0.7657	0.7963	0.7880	0.7501	0.6443	0.3052
NAFSMF	0.9147	0.8916	0.8669	0.8409	0.8124	0.7796	0.7403	0.6872	0.5736
DAMF	**0.9253**	**0.9113**	**0.8946**	**0.8752**	**0.8523**	**0.8244**	**0.7892**	**0.7398**	**0.6572**

TABLE 2. The mean SSIM results for the 40 test images

Algorithm	10%	20%	30%	40%	50%	60%	70%	80%	90%
PSMF	0.9444	0.9014	0.843	0.7435	0.5663	0.2822	0.0817	0.0383	0.0171
DBA	0.9798	0.9467	0.8964	0.8247	0.7316	0.6218	0.5002	0.3794	0.2998
MDBUTMF	0.9431	0.8349	0.7724	0.8154	0.8748	0.8813	0.8489	0.7407	0.3730
NAFSMF	0.9790	0.9602	0.9411	0.9209	0.8988	0.8724	0.8385	0.7889	0.6648
DAMF	**0.9911**	**0.9819**	**0.9705**	**0.9563**	**0.9392**	**0.9174**	**0.8885**	**0.8451**	**0.7595**

Suppose that the success in high noise densities is more important than in the others. In that case, the values in Table 1 can be represented with an *fpfs*-matrix as follows:

$$[a_{ij}] := \begin{bmatrix} 0.1 & 0.2 & 0.3 & 0.4 & 0.5 & 0.6 & 0.7 & 0.8 & 0.9 \\ 0.9028 & 0.8715 & 0.8018 & 0.6988 & 0.4903 & 0.1882 & 0.0633 & 0.0318 & 0.0139 \\ 0.9079 & 0.8664 & 0.8097 & 0.7376 & 0.6521 & 0.5552 & 0.4567 & 0.3623 & 0.2937 \\ 0.8841 & 0.7994 & 0.7443 & 0.7657 & 0.7963 & 0.7880 & 0.7501 & 0.6443 & 0.3052 \\ 0.9147 & 0.8916 & 0.8669 & 0.8409 & 0.8124 & 0.7796 & 0.7403 & 0.6872 & 0.5736 \\ 0.9253 & 0.9113 & 0.8946 & 0.8752 & 0.8523 & 0.8244 & 0.7892 & 0.7398 & 0.6572 \end{bmatrix}$$

If we apply PEM to the $[a_{ij}]$, then the score matrix and the decision set are as follows:

$$[s_{i1}] = [0.2160\ 0.5171\ 0.7395\ \ 0.8957\ \ 1]^T$$

and

$$\{^{0.2160}\text{PSMF},\ ^{0.5171}\text{DBA},\ ^{0.7395}\text{MDBUTMF},\ ^{0.8957}\text{NAFSMF},\ ^{1}\text{DAMF}\}$$

The scores show that DAMF outperforms the other methods and the order DAMF, NAFSMF, MDBUTMF, DBA, and PSMF is valid.

Similarly, the values in Table 2 can be represented with an *fpfs*-matrix as follows:

$$[b_{ij}] := \begin{bmatrix} 0.1 & 0.2 & 0.3 & 0.4 & 0.5 & 0.6 & 0.7 & 0.8 & 0.9 \\ 0.9444 & 0.9014 & 0.843 & 0.7435 & 0.5663 & 0.2822 & 0.0817 & 0.0383 & 0.0171 \\ 0.9798 & 0.9467 & 0.8964 & 0.8247 & 0.7316 & 0.6218 & 0.5002 & 0.3794 & 0.2998 \\ 0.9431 & 0.8349 & 0.7724 & 0.8154 & 0.8748 & 0.8813 & 0.8489 & 0.7407 & 0.3730 \\ 0.9790 & 0.9602 & 0.9411 & 0.9209 & 0.8988 & 0.8724 & 0.8385 & 0.7889 & 0.6648 \\ 0.9911 & 0.9819 & 0.9705 & 0.9563 & 0.9392 & 0.9174 & 0.8885 & 0.8451 & 0.7595 \end{bmatrix}$$

If we apply PEM to the $[b_{ij}]$, then the score matrix and the decision set are as follows:

$$[s_{i1}] = [0.2114\ 0.5026\ 0.7291\ \ 0.8999\ \ 1]^T$$

and

$$\{^{0.2114}\text{PSMF},\ ^{0.5026}\text{DBA},\ ^{0.7291}\text{MDBUTMF},\ ^{0.8999}\text{NAFSMF},\ ^{1}\text{DAMF}\}$$

The scores show that DAMF outperforms the other methods and the order DAMF, NAFSMF, MDBUTMF, DBA, and PSMF is valid.

5. Conclusion

In this paper, we define the concept of *fpfs*-matrices. We then suggest a new method referred to as PEM. Afterwards, we successfully apply PEM to the determination of the performance of the methods used in [48]. It is clear that PEM, which is a fast and simple method, can be successfully applied to decision-making problems in various areas, such as machine learning and image processing. We also believe that the configuration of the other methods in the literature via *fpfs*-matrices is worth studying.

References

[1] Molodtsov, D., (1999), Soft set theory−first results, Comput. Math. with Appl., 37, pp. 19-31.

[2] Atagün, A. O., Kamacı, H. and Oktay, O., (2018), Reduced soft matrices and generalized products with applications in decision making, Neural Comput & Applic, 29, pp. 445-456.

[3] Atmaca, S., (2017), Relationship between fuzzy soft topological spaces and (X, τ_e) parameter spaces, Cumhuriyet Sci. J., 38, pp. 77-85.

[4] Atmaca, S. and Zorlutuna, İ., (2014), On topological structures of fuzzy parameterized soft sets, Sci. World J., 2014, Article ID 164176, 8 pages.

[5] Bera, S., Roy, S. K., Karaaslan, F. and Çağman, N., (2017), Soft congruence relation over lattice, Hacettepe J. Math. Stat., 46, pp. 1035-1042.

[6] Çağman, N., Çıtak, F. and Enginoğlu, S., (2010), Fuzzy parameterized fuzzy soft set theory and its applications, Turkish J. Fuzzy Syst., 1, pp. 21-35.

[7] Çağman, N., Çıtak, F. and Enginoğlu, S., (2011), FP-soft set theory and its applications, Ann. Fuzzy Math. Informatics, 2, pp. 219-226.

[8] Çağman, N. and Deli, İ., (2012), Means of FP-soft sets and their applications, Hacettepe J. Math. Stat., 41, pp. 615-625.

[9] Çağman, N. and Deli, İ., (2012)., Products of FP-soft sets and their applications, Hacettepe J. Math. Stat., 41, pp. 365-374.

[10] Çağman, N. and Enginoğlu, S., (2010), Soft matrix theory and its decision making, Comput. Math. with Appl., 59, pp. 3308-3314.

[11] Çağman, N. and Enginoğlu, S., (2010), Soft set theory and uni-int decision making, Eur. J. Oper. Res., 207, pp. 848-855.

[12] Çağman, N. and Enginoğlu, S., (2012), Fuzzy soft matrix theory and its application in decision making, Iran. J. Fuzzy Syst., 9, pp. 109-119.

[13] Çağman, N., Enginoğlu, S. and Çıtak, F., (2011), Fuzzy soft set theory and its applications, Iran. J. Fuzzy Syst., 8, pp. 137-147.

[14] Çıtak, F. and Çağman, N., (2015), Soft int-rings and its algebraic applications, J. Intell. Fuzzy Syst., 28, pp. 1225-1233.

[15] Çıtak, F. and Çağman, N., (2017), Soft k-int-ideals of semirings and its algebraic structures, Ann. Fuzzy Math. Informatics, 13, pp. 531-538.

[16] Deli, İ. and Çağman, N., (2015), Relations on FP-soft sets applied to decision making problems, J. New Theory, 3, pp. 98-107.

[17] Enginoğlu, S., (2012), Soft matrices, PhD dissertation, Gaziosmanpaşa University, Tokat, Turkey.

[18] Enginoğlu, S., Çağman, N., Karataş, S. and Aydın, T., (2015), On soft topology, El-Cezerî J. Sci. Eng., 2, pp. 23-38.

[19] Enginoğlu, S. and Memiş, S., (2018), Comment on fuzzy soft sets [The Journal of Fuzzy Mathematics 9(3), 2001, 589-602], Int. J. Latest Eng. Res. Appl., 3, pp. 1-9.

[20] Enginoğlu, S. and Memiş, S., (2018), A configuration of some soft decision-making algorithms via *fpfs*-matrices, Cumhuriyet Sci. J., 39, pp. 871-881.

[21] Enginoğlu, S. and Memiş, S., (2018), A review on an application of fuzzy soft set in multicriteria decision making problem [P. K. Das, R. Borgohain, International Journal of Computer Applications 38 (2012) 33-37], In M. Akgül, İ. Yılmaz and A. İpek (Eds.), Int. Conf. Math. Stud. Appl., Karaman, Turkey, pp. 173-178.

[22] Enginoğlu, S. and Memiş, S., (2018), A review on some soft decision-making methods. In M. Akgül, İ. Yılmaz and A. İpek (Eds.), Int. Conf. Math. Stud. Appl., Karaman, Turkey, pp. 437-442.

[23] Enginoğlu, S., Memiş, S. and Arslan, B., (2018), Comment (2) on soft set theory and uni-int decision-making [European Journal of Operational Research, (2010) 207, 848-855], J. New Theory, 25, pp. 84-102.

[24] Enginoğlu, S., Memiş, S. and Arslan, B., (2018), A fast and simple soft decision-making algorithm: EMA18an. In M. Akgül, İ. Yılmaz and A. İpek (Eds.), Int. Conf. Math. Stud. Appl., Karaman, Turkey, pp. 428-436.

[25] Enginoğlu, S., Memiş, S. and Öngel, T., (2018), Comment on soft set theory and uni-int decision-making [European Journal of Operational Research, (2010) 207, 848-855], J. New Results Sci., 7, pp. 28-43.

[26] Enginoğlu, S., Memiş, S. and Öngel, T., (2018), A fast and simple soft decision-making algorithm: EMO18o. In M. Akgül, İ. Yılmaz and A. İpek (Eds.), Int. Conf. Math. Stud. Appl., Karaman, Turkey, pp. 179-186.

[27] Karaaslan, F., (2016), Soft classes and soft rough classes with applications in decision making, Math. Probl. Eng., 2016, Article ID 1584528, 11 pages.

[28] Maji, P. K., Biswas, R. and Roy, A. R., (2001), Fuzzy soft sets, J. Fuzzy Math., 9, pp. 589-602.

[29] Maji, P. K., Biswas, R. and Roy, A. R., (2003), Soft set theory, Comput. Math. with Appl., 45, pp. 555-562.

[30] Maji, P. K., Roy, A. R. and Biswas, R., (2002), An application of soft sets in a decision making problem, Comput. Math. with Appl., 44, pp. 1077-1083.

[31] Muştuoğlu, E., Sezgin, A. and Türk, Z. K., (2016), Some characterizations on soft uni-groups and normal soft uni-groups, Int. J. Comput. Appl., 155, pp. 1-8.

[32] Riaz, M. and Hashmi, R., (2016), Certain applications of fuzzy parameterized fuzzy soft sets in decision-making problems, Int. J. Algebra Stat., 5, pp. 135-146.

[33] Riaz, M. and Hashmi, R., (2017), Fuzzy parameterized fuzzy soft topology with applications, Ann. Fuzzy Math. Informatics, 13, pp. 593-613.

[34] Riaz, M. and Hashmi, R., (2018), Fuzzy parameterized fuzzy soft compact spaces with decision-making, Punjab Univ. J. Math., 50, pp. 131-145.

[35] Riaz, M., Hashmi, R. and Farooq, A., (2018), Fuzzy parameterized fuzzy soft metric spaces, J. Math. Anal., 9, pp. 25-36.

[36] Şenel, G., (2016), A new approach to Hausdorff space theory via the soft sets, Math. Probl. Eng., 2016, Article ID 2196743, 6 pages.

[37] Şenel, G., (2018), Analyzing the locus of soft spheres: Illustrative cases and drawings, Eur. J. Pure Appl Math., 11, pp. 946-957.

[38] Şenel, G., (2018), The relation between soft topological space and soft ditopological space, Commun. Fac. Sci. Univ. Ank. Ser. A1 Math. Stat., 67, pp. 209-219.

[39] Sezgin, A., (2016), A new approach to semigroup theory I: Soft union semigroups, ideals and bi-ideals, Algebr. Lett., 2016, Article ID 3, 46 pages.

[40] Sezgin, A., Çağman, N. and Çıtak, F., (2019), α-inclusions applied to group theory via soft set and logic, Commun. Fac. Sci. Univ. Ank. Ser. A1 Math. Stat., 68, pp.

334-352.
[41] Tunçay, M. and Sezgin, A., (2016), Soft union ring and its applications to ring theory, Int. J. Comput. Appl., 151, pp. 7-13.
[42] Ullah, A., Karaaslan, F. and Ahmad, I., (2018). Soft uni-Abel-Grassmann's groups, Eur. J. Pure Appl. Math., 11, pp. 517-536.
[43] Zorlutuna, İ. and Atmaca, S., (2016), Fuzzy parameterized fuzzy soft topology, New Trends Math. Sci., 4, pp. 142-152.
[44] Wang, Z. and Zhang, D., (1999), Progressive switching median filter for the removal of impulse noise from highly corrupted images, IEEE Trans. Circuits Syst. II Analog Digit. Signal Process., 46, pp. 78-80.
[45] Pattnaik, A., Agarwal, S. and Chand, S., (2012), A new and efficient method for removal of high density salt and pepper noise through cascade decision based filtering algorithm, Procedia Technol., 6, pp. 108-117.
[46] Esakkirajan, S., Veerakumar, T., Subramanyam, A. N. and PremChand, C. H., (2011), Removal of high density salt and pepper noise through modified decision based unsymmetric trimmed median filter, IEEE Signal Process. Lett., 18, pp. 287-290.
[47] Toh, K. K. V. and Isa, N. A. M., (2010), Noise adaptive fuzzy switching median filter for salt-and-pepper noise reduction, IEEE Signal Process. Lett., 17, pp. 281-284.
[48] Erkan, U., Gökrem, L. and Enginoğlu, S., (2018), Different applied median filter in salt and pepper noise, Comput. Electr. Eng., 70, pp. 789-798.
[49] Asuni, N. and Giachetti, A., (2014), TESTIMAGES: A large-scale archive for testing visual devices and basic image processing algorithms, In A. Giachetti (Ed.), Smart Tools & Apps for Graphics-Eurographics Italian Chapter Conference, Eurographics Assoc.
[50] Wang, Z., Bovik, A. C., Sheikh, H. R. and Simoncelli, E. P., (2004), Image quality assessment: From error visibility to structural similarity, IEEE Trans. Image Process., 13, pp. 600-612.

Serdar Enginoğlu holds a bachelor's degree in Mathematics from Atatürk University, Erzurum, Turkey, in 1998. He received his master's and doctoral degree in Mathematics from Gaziosmanpaşa University, Tokat, Turkey, in 2009 and 2012, respectively. He is currently the Head of Department of Fundamentals of Mathematics and Mathematical Logic at Çanakkale Onsekiz Mart University, Çanakkale, Turkey. His current research interests are in Fuzzy Sets, Soft Sets, Soft Matrices, Soft Decision-Making, Image Processing, and Machine Learning.

Naim Çağman holds a bachelor's degree in Mathematics from Istanbul University, Istanbul, Turkey, in 1991. He received his master's degree in Mathematics from Wales Swansea University, Swansea, UK, in 1996, and his doctoral degree in Mathematics from Leeds University, Leeds, UK, in 2000. He is currently the Head of Department of Fundamentals of Mathematics and Mathematical Logic at Tokat Gaziosmanpaşa University, Tokat, Turkey. His current research interests are in Fuzzy Sets, Soft Sets, Soft Decision-Making, Soft Algebra, and Soft Game Theory.