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I-CORDIAL LABELING OF SPIDER GRAPHS

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Abstract. Let G= (V, E) be a graph with p vertices and q edges. A graph G=(V,E)with p vertices and q edges is said to be an I-cordial labeling of a graph if there exists an injective map f from V to$\lfloor \frac{-p}{2} .. \frac{p}{2} \rfloor^* or [-\lfloor \frac{p}{2} \rfloor .. \lfloor \frac{p}{2} \rfloor]$ as p is even or odd respectively such that the injective mapping is defined for $f(u) + f(v) \neq 0$ that induces an edge labeling $f^* : E \rightarrow \{0, 1\}$ where $f^*(uv) = 1$ if $f(u) + f(v) > 0$ and $f^*(uv) = 0$ otherwise, such that the number of edges labeled with 1 and the number of edges labeled with 0 differ atmost by 1. If a graph satisfies the condition then graph is called I-Cordial labeling graph or I - Cordial graph. In this paper we intend to prove the spider graph $SP(1^m, 2^t)$ is integer I-cordial labeling graph and obtain some characteristics of I cordial labeling on the graph and we define M-Joins of Spider graph $SP(1^m, 2^t)$ and study their characteristics.
Here we use the notation $\lfloor -p..p \rfloor^* = \lfloor -p..p \rfloor - [0]$ and
$\lfloor -p..p \rfloor = [x/x$ is an integer such that $\mid x \mid \leq p]$

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1. Introduction

A graph G is a finite nonempty set of objects called vertices and edges. All graphs considered here are finite, simple and undirected. Gallian J A[1] has given a dynamic survey of graph labelling. The origin of graph labelings can be attributed to Rosa. The vertex set is denoted by $V(G)$ and the edge set is denoted by$E(G)$. The concept of integer I - Cordial labeling was introduced by Nicholas T and Maya P[2] and they have proved some standard graphs are I-cordial labeling. Jeyanthi.P and Saratha Devi. T[3] studied on edge pair sum labeling. Sriram.S and Govindarajan .R[4][5][6] discussed on homocordial labeling of spider graphs and Pell labeling of Joins of square of path graph.Motivated towards the study of integer I-cordial labeling of graphs we study on the spider graphs $SP(1^m, 2^t)$ and prove that they are I-Cordial graph and also study joins of $SP(1^m, 2^t)$. We also identify some characteristics based on construction of $SP(1^m, 2^t)$ and joins of $SP(1^m, 2^t)$. We also study on M-joins of $SP(1^m, 2^t)$ graph. The basic preliminary concept of graphs is from Handbook of graph theory[7]

2. Preliminaries

Definition 2.1. *A tree is called a spider if it has a centre vertex C of degree$R > 1$ and all the other vertex is either a leaf or with degree 2. Thus a spider is an amalgamation of*

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k paths with various lengths. If it has $X_1's$ of length a_1, $X_2's$ paths of length a_2 etc. We shall denote the spider by $SP({a_1}^{x_1}, {a_2}^{x_2}, {a_3}^{x_3}...{x_m}^{a_m})$ where $a_1 < a_2 < a_3 < ... < a_m$ and $x_1 + x_2 + x_3... + x_m = R$

Definition 2.2. *A graph G=(V, E) with p vertices and q edges is said to be an integer I-cordial labeling of a graph if there exists and injective map f from V to $\lfloor \frac{-p}{2}..\frac{p}{2} \rfloor^*$ or $[-\lfloor \frac{p}{2} \rfloor..\lfloor \frac{p}{2} \rfloor]$ as p is even or odd respectively and f be an injective mapping such that $f(u) + f(v) \neq 0$ that induces an edge labeling $f^* : E \rightarrow \{0, 1\}$ where $f^*(uv) = 1$ if $f(u) + f(v) > 0$ and $f^*(uv) = 0$ otherwise, such that the number of edges labeled with 1 and the number of edges labeled with 0 differ atmost by 1. If a graph satisfies the condition then graph is called integer I-Cordial labeling graph or I - Cordial graph*

In this paper we study on Spider graph $SP(1^m, 2^t)$ and introduce joins to the spider graph $SP(1^m, 2^t)$. We study specifically on construction of the Spider graph $SP(1^m, 2^t)$ with a special understanding by including for each of the value of t the corresponding value of m which composes the spider graph $SP(1^m, 2^t)$ as $SP(1^2, 2^1)$, $SP(1^4, 2^2)$ ... . We prove that $SP(1^{2t}, 2^t)$ is integer I-Cordial labeling graph. Subsequently we define 1-join of Spider graph$SP(1^{2t}, 2^t)$ which we call them as $J(SP(1^{2t}, 2^t))$. we also extend the study by defining M-join of Spider graph $SP(1^{2t}, 2^t)$ which we call them as $MJ(SP(1^{2t}, 2^t))$ Interestingly we look up on in this paper the effective way of constructing a Spider graph $SP(1^m, 2^t)$ by reasonably studying the number of vertices and edges required to label them so as to prove that the spider graph $SP(1^m, 2^t)$ is integer I-Cordial labeling graph. For this we study by choosing the value of t=1 which in turn leads to the number of vertices as 2 and hence on labeling the vertices as desired we understand that it requires two more vertices so we consider the value of m=2 so as to get the basic spider graph $SP(1^2, 2^1)$ which in resultant by labeling we can obtain the spider graph $SP(1^2, 2^1)$ to be an integer I - Cordial labeling. This study is recognised and due importance is given in this paper to create corresponding vertices in the spider graph $SP(1^{2t}, 2^t)$.

3. Main Results

Theorem 3.1. *The Spider graph $SP(1^m, 2^t)$ is an integer I-Cordial labeling graph for all values of t and for $m = 2t$*

Proof. Consider the graph $G = SP(1^m, 2^t)$. The graph G consists of the vertex set $V(SP(1^m, 2^t)) = \{u, v_i, u_j : 1 \leq i \leq m, 1 \leq j \leq 2t\}$ and the edge set
$E(SP(1^m, 2^t)) = \{e_i = uv_i : 1 \leq i \leq m, e_i^{'} = uu_i : 1 \leq i \leq t, e_i^{''} = u_iu_{i+1} : 1 \leq i \leq t\}$. Now let us prove that the graph G is integer I-Cordial labeling graph for all values of t and for m=2t. Here we construct a spider graph by choosing m=2t .Let us label the vertices of the graph $G = SP(1^m, 2^t)$ as follows
$f(u) = 0$
For every introduction of vertices in the graph G namely u_i we introduce vertices v_i for $1 \leq i \leq 2n$. That is we consider for $t = i, m = 2i$ so that we can label to prove that it is an integer I cordial labeling graph.
$f(v_{2i-1}) = -i$ when i is odd
$f(u_i) = i$ when i is odd
$f(v_{2i}) = 2i$ when i is even
$f(u_{2i}) = -2i$ when i is even
Then the induced edge labeling is given as
$f^*(uu_i) = 1$ for $i = 1(mod2)$
$f^*(uv_i) = 0$ for $i = 1(mod2)$
$f^*(uv_i) = 1$ for $i = 0(mod2)$

$f^*(u_{2i-1}, u_{2i}) = 0$ for $1 \leq i \leq 2n$
Hence we can find that the induced edge labeling satisfies the condition that the number of edges labeled with 0 differ at most by 1 to that of the number of edges labeled with 1. Hence the graph G is integer I -Cordial labeling graph. Hence the proof. □

The following is an example of the above labeling schema to verify the spider graph $SP(1^4, 2^2)$ is integer I cordial labeling of graph

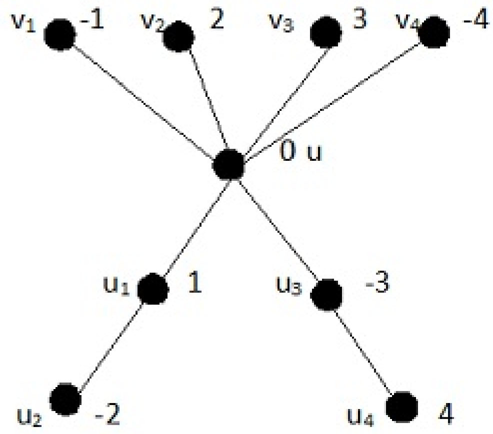

Fig.1: Spider graph SP(1^4,2^2) – I Cordial Labeling graph

Now for the Spider graph $SP(1^m, 2^t)$ we define one part by taking m=2 and t=1 which inturn is the spider graph $SP(1^2, 2^1)$. Which has 5 vertices and 4 edges. which we call it as basic graph Now introducing one part to the basic graph $SP(1^2, 2^1)$ will give rise to the spider graph $SP(1^4, 2^2)$ which has 9 vertices and 8 edges. In the following process if we systematically add one part to each of the spider graph that we arrive at each step we can construct a general spider graph $SP(1^m, 2^t)$ for any m=2t . Now we illustrate this in the form of the table given below

Type of Spider graph $SP(1^m, 2^t)$	Number of Vertices	Number of Edges
$SP(1^2, 2^1)$	5	4
$SP(1^4, 2^2)$	9	8
$SP(1^6, 2^3)$	13	12
and so on	...	...

Table 1. Spider Graph$SP(1^m, 2^t)$ with number of Vertices and Edges

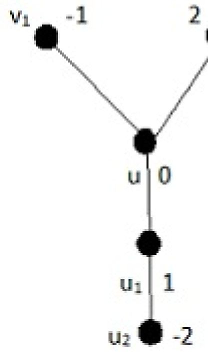

Fig.2: One Part of Spider graph $SP(1^{2t},2^{t})$

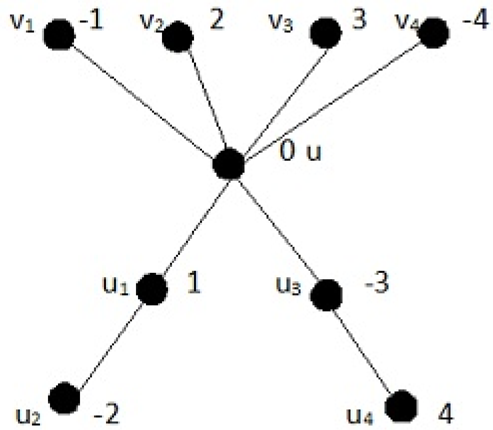

Fig.3: Adding One Part of Spider graph $SP(1^{2t},2^{t})$ to the Basic graph

Now we state the following remark which is obvious

Remark 3.1. *For a Spider graph $SP(1^{2t}, 2^t)$ for 1≤ t ≤ n the number of vertices are 5,9,13,... which forms an arithmetic progression with common difference 4 . The general term to find the number of vertices is given by* $5 + 4(t - 1)$

Remark 3.2. *For a Spider graph $SP(1^{2t}, 2^t)$ for 1≤ t ≤ n the number of edges are 4,8,12,... which forms an arithmetic progression with common difference 4.The general term to find the number of edges is given by* $4 + 4(t - 1)$

Remark 3.3. *For a Spider graph $SP(1^{2t}, 2^t)$ for 1≤ t ≤ n the number of edges and number of vertices differ by 1 for each value of t for 1≤ t ≤ n*

Application 3.1. *It is easy to compute the number of vertices and number of edges for any spider graph $SP(1^{2t}, 2^t)$ if we know the value of t.*
For suppose t=100 then the spider graph $SP(1^{200}, 2^{100})$ will have 401 vertices and 400 edges using the general term given in Remark.3.1 and Remark.3.2

Definition 3.1. *K-part of a Spider graph $SP(1^{2t}, 2^t)$ is the graph which is obtained from the basic spider graph $SP(1^2, 2^1)$ by adding one part consequently according to the value of K where* $1 \le K \le n$

Remark 3.4. *For a K part of Spider graph $SP(1^{2t}, 2^t)$ for 1≤ $t \leq n$ the number of vertices is calculated using 4K+1 where $1 \leq K \leq n$ and the number of edges is calculated as 4K where $1 \leq K \leq n$*

Definition 3.2. *A Spider graph $SP(1^{2t}, 2^t)$ joined with another Spider graph $SP(1^{2t}, 2^t)$ by an edge between them is called a 1-Join of Spider graph $SP(1^{2t}, 2^t)$ and is denoted by$J(SP(1^{2t}, 2^t))$. Here we consider only attaching Spider graph $SP(1^{2t}, 2^t)$ of same order.*

Now let us think about constructing 1-join of Spider graph with the basic spider graph being attached with another basic spider graph by an edge.We know that the total number of vertices in a basic spider graph is 5 and simliarly the attached basic spider graph also consists of 5 vertices which in total for 1-Join of Spider graph is 10 vertices. We also notice that the number of edges in a basic spider graph is 4 and attaching another basic spider graph to it which also will have 4 edges and therefore in total the number of edges 1-join of Spider graph is 9 as there is one common edge between the two basic spider graphs. Subsequently attaching 1 part to the basic spider graph increases the number of vertices to 9 from the original vertices 5 and the number of edges increases to 8 from the original edges 4 and the total number of vertices in 1-Join of Spider graph is 18 and the total number of edges is 17 as there is one edge between the two spider graphs.

In this way the process can be continued. The following is illustrated in the table given below.

Type of 1- Join of Spider graph $SP(1^m, 2^t)$	Number of Vertices	Number of Edges
$SP(1^2, 2^1)$	10	9
$SP(1^4, 2^2)$	18	17
$SP(1^6, 2^3)$	26	25
and so on	...	...

TABLE 2. 1-Join of Spider Graph$SP(1^{2t}, 2^t)$ with number of Vertices and Edges

and we can find the following remarks obvious

Remark 3.5. *For a 1- Join of Spider graph $SP(1^{2t}, 2^t)$ for 1≤ $t \leq n$ by subsequently adding 1 part to the basic spider graph we find the number of vertices are 10,18,26... respectively which forms an arithmetic progression with common difference 8 . The general term to compute the number of vertices is $10 + 8(n - 1)$*

Remark 3.6. *For a 1- Join of Spider graph $SP(1^{2t}, 2^t)$ for 1≤ $t \leq n$ by subsequently adding 1 part to the basic spider graph we find the number of edges are 9,17,25... respectively which forms an arithmetic progression with common difference 8. The general term to compute the number of edges is $9 + 8(n - 1)$*

Remark 3.7. *For a 1- Join of Spider graph $SP(1^{2t}, 2^t)$ for 1≤ $t \leq n$ by subsequently adding 1 part to the basic spider graph we find the number of edges and number of vertices differ by 1*

Remark 3.8. *For a K-part 1- Join of Spider graph $SP(1^{2t}, 2^t)$ for 1≤ $t \leq n$ by subsequently adding k-1 part to the basic spider graph we have the general term to find the number of vertices is $KV(G_t) = 10 + 8K(t - 1)$ where $2 \leq t \leq n$ and the number of edges is number of edges is $KE(G_t)$=$KV(G_t) - 1$ where 2≤ $t \leq n$*

Application 3.2. *It is easy to compute the number of vertices and number of edges for any 1- join of spider graph $SP(1^{2t}, 2^t)$ if we know the value of t.*

For suppose t=100 then the 1- join of spider graph $SP(1^{200}, 2^{100})$ *will have 802 vertices and 801 edges using the general term given in Remark.3.5 and Remark.3.6, Remark.3.7 and Remark.3.8*
From the above we understand that 1-Join of Spider graph $SP(1^{2t}, 2^t)$ *for* $1 \leq t \leq n$ *will result in the number of vertices to be even. Hence the labeling technique adopted in theorem.3.1 is to be revised as for theorem.3.1 refers to labeling the vertices only when the number of vertices is odd. If the number of vertices is even then we go by the definition 2.2 which states that that the vertices to be labeled as* $\lfloor \frac{-p}{2} .. \frac{p}{2} \rfloor^* or [-\lfloor \frac{p}{2} \rfloor .. \lfloor \frac{p}{2} \rfloor]$ *as p is even or odd.*
We know for the basic graph 1-join of $SP(1^2, 2^1)$ *attached by an edge to another basic graph the number of vertices is 10 which is even hence the possible labels that can be used for the vertices are [-1,1,-2,2,-3,3,-4,4,-5,5]*

The typical labeling procedure adopted to prove that 1-Join $SP(1^2, 2^1)$ basic graph with another $SP(1^2, 2^1)$ basic graph is integer I Cordial labeling graph is given below

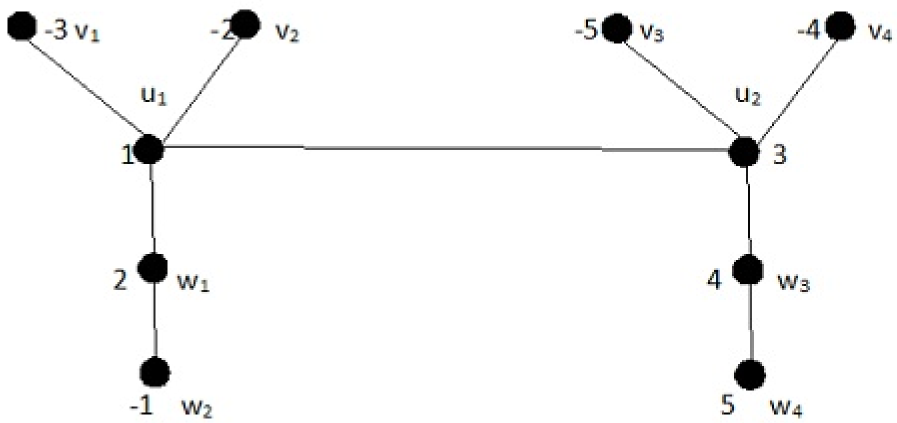

Fig.4 : 1- Join of Basic Spider graph with another Basic Spider graph

Theorem 3.2. *The 1-join of K part of Spider graph* $J(SP(1^{2t}, 2^t))$ *is an integer I-Cordial labeling graph*

Proof:
We prove the theorem by method of Mathematical induction applied on the number of parts that we add to the basic Spider graph. We know that 1-join of basic spider graph attached to another basic spider graph is integer I Cordial Labeling graph. Now let us assume that the theorem is true for k-1 part i.e 1-Join of k-1 part of spider graph attached to another k-1 part of spider graph is integer I Cordial labeling graph. Now let us prove that the theorem is true for k part i.e 1- join of k part of spider graph attached to another k part of spider graph then it is to be proved that integer I cordial labeling graph. For we attach to k-1 part of spider graph a 1 part which makes the spider graph as k part and also join with another k-1 part of spider which is also attached with 1 part so as to make k part spider graph. Clearly 1- Join of K-part of spider graph with another K-part of spider graph is integer I cordial labeling as k-1 part and 1 part are individually an integer I cordial labeling. Hence the proof.

Now in a similar way we can construct 2-join k part spider graph which will result in odd number of vertices which can be labelled with the method adopted in theorem.2.1.

and can be proved integer I cordial labeling graph. Similarly we can proceed to analyse on M-Joins of K-Part of Spider graph which can also be proved to be integer I cordial labeling of graph. Here M represent the number of joins connected with the K-part of Spider graph. If M is even then the number of vertices required to be labeled is odd and hence we adopt procedure given in theorem.2.1 and in case of M is odd then the number of vertices required to be labeled is even and hence we adopt the procedure that we have used for labeling 1-join of K-part of spider graph.

Remark 3.9. *For M-joins of K part of Spider graph we notice that if M is even then the number of edges labeled with 1 and number of edges labeled with 0 differ by 1 and if M is odd then the number of edges labeled with 1 is equal to the number of edges labeled with 0.*

4. Conclusion

In this paper we have identified Spider graph $J(SP(1^m, 2^t))$ and have proved that the Spider graph $SP(1^{2t}, 2^t))$ is integer I Cordial Labeling of graph. We have also considered the Joins of Spider graph $SP(1^{2t}, 2^t)$ and proved to integer I cordial labeling and also discussed some characteristics of the spider graph $SP(1^{2t}, 2^t)$. We have also discussed on M-Joins of spider graph $SP(1^{2t}, 2^t)$. We are further investigating some graph in order to prove that the graph is integer I cordial labeling graph.

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