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A NEW TYPE TIMELIKE SURFACE GIVEN WITH MONGE PATCH IN $\mathbb{E}_1^4$

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Abstract. In this work, we think about timelike Aminov surface of hyperbolic type in four dimensional Minkowski space $\mathbb{E}_1^4$. We investigate and classify this surface with respect to its Gaussian curvature, mean curvature and normal curvature. We get some results on flat, minimal and semiumbilical hyperbolic type Aminov surfaces. Further, we obtain necessary and sufficient condition for this type of surface to become Wintgen Ideal Surface in Minkowski space-time.

Aminov surface, Minkowski 4−space, Monge patch, mean curvature, Gaussian curvature.

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1. Introduction

In mathematical physics, Minkowski space or Minkowski space-time is the mathematical setting in which special relativity theory of Einstein is most properly systematized. 3 ordinary dimensions of space are compounded with a single dimension of time to shape a 4−dimensional manifold for representing a space-time, in this setting. Minkowski 4−space or Minkowski space-time is 4−dimensional semi-Euclidean space with 1−index which is indicated by $\mathbb{E}_1^4$. The metric associated with four-dimensional Minkowski space is given by

$$g\left(u,v\right) = -u_1v_1 + u_2v_2 + u_3v_3 + u_4v_4 \tag{1}$$

where $u = (u_1, .., u_4)$, $v = (v_1, .., v_4)$. Any surface S, represented by $X = X(\theta,\varphi) : (\theta,\varphi) \in D$ ($D \subset \mathbb{E}^2$) is said to be timelike in $\mathbb{E}_1^4$ if the induced metric g on S is a metric with index 1. Therefore, for the timelike surface S, at each point p, we can refer to the decomposition

$$\mathbb{E}_1^4 = T_pS \oplus T_p^{\perp}S,$$

where T_pS is the tangent space of S, $T_p^{\perp}S$ is the normal space of S.

Let $\widetilde{\nabla}$ and ∇ denote the Levi-Civita connections on $\mathbb{E}_1^4$ and S, respectively. If X_1 and X_2 are tangent vector fields and ξ is a normal vector field, the tangent and normal component

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of the vectors $\widetilde{\nabla}_{X_1}X_2$ and $\widetilde{\nabla}_{X_1}\xi$ can be seen by Gauss and Weingarten formulas:

$$\begin{aligned}\widetilde{\nabla}_{X_1}X_2 &= \nabla_{X_1}X_2 + h(X_1, X_2),\\ \widetilde{\nabla}_{X_1}\xi &= -A_\xi X_1 + D_{X_1}\xi.\end{aligned} \tag{2}$$

A_ξ is the shape operator with respect to ξ, D is the normal connection and h is the second fundamental tensor in these formulas [11].

Assume that the timelike surface S in $\mathbb{E}^4_1$ has the parametric representation $S : X = X(\theta, \varphi) : (\theta, \varphi) \in D$ ($D \subset \mathbb{E}^2$). The tangent space T_pS at any point $p = X(\theta, \varphi)$ is spanned by the vectors X_θ and X_φ. The first fundamental form is written by

$$I(\lambda, \mu) = E\lambda^2 + 2F\lambda\mu + G\mu^2, \ \lambda, \mu \in IR \tag{3}$$

where the first fundamental form coefficients are $E = g(X_\theta, X_\theta)$, $F = g(X_\theta, X_\varphi)$, $G = g(X_\varphi, X_\varphi)$. Due to fact that our chosen surface is timelike, we suppose $g(X_\theta, X_\theta) < 0$, $g(X_\varphi, X_\varphi) > 0$. Therefore, $E < 0$, $F > 0$ and for the later use we set $W = \sqrt{F^2 - EG}$ and prefer to use a normal frame field $\{\xi_1, \xi_2\}$ such that $g(\xi_1, \xi_1) = 1$, $g(\xi_2, \xi_2) = 1$, $g(\xi_1, \xi_2) = 0$. Then, the second derivatives are linear combinations of these vector fields:

$$\begin{aligned}\widetilde{\nabla}_{X_\theta}X_\theta &= X_{\theta\theta} = -\Gamma^1_{11}X_\theta + \Gamma^2_{11}X_\varphi + c^1_{11}\xi_1 + c^2_{11}\xi_2,\\ \widetilde{\nabla}_{X_\theta}X_\varphi &= X_{\theta\varphi} = -\Gamma^1_{12}X_\theta + \Gamma^2_{12}X_\varphi + c^1_{12}\xi_1 + c^2_{12}\xi_2,\\ \widetilde{\nabla}_{X_\varphi}X_\varphi &= X_{\varphi\varphi} = -\Gamma^1_{22}X_\theta + \Gamma^2_{22}X_\varphi + c^1_{22}\xi_1 + c^2_{22}\xi_2.\end{aligned} \tag{4}$$

Cristoffel symbols and second fundamental form coefficients are denoted by Γ^k_{ij} and c^k_{ij}, $i, j, k = 1, 2$. Hence, the second fundamental form coefficients can be calculated by

$$\begin{aligned}c^1_{11} &= g(X_{\theta\theta}, \xi_1); \quad c^2_{11} = g(X_{\theta\theta}, \xi_2);\\ c^1_{12} &= g(X_{\theta\varphi}, \xi_1); \quad c^2_{12} = g(X_{\theta\varphi}, \xi_2);\\ c^1_{22} &= g(X_{\varphi\varphi}, \xi_1); \quad c^2_{22} = g(X_{\varphi\varphi}, \xi_2).\end{aligned} \tag{5}$$

With the help of Gauss formula, it is obvious that the second fundamental tensors are

$$\begin{aligned}h(X_\theta, X_\theta) &= c^1_{11}\xi_1 + c^2_{11}\xi_2,\\ h(X_\theta, X_\varphi) &= c^1_{12}\xi_1 + c^2_{12}\xi_2,\\ h(X_\varphi, X_\varphi) &= c^1_{22}\xi_1 + c^2_{22}\xi_2.\end{aligned} \tag{6}$$

By the use of Gram-Schmidt orthonormalization method, we yield the orthonormal tangent vectors as:

$$\begin{aligned}X_1 &= \frac{X_\theta}{\sqrt{|E|}},\\ X_2 &= \frac{\sqrt{|E|}}{W}\left(X_\varphi - \frac{F}{E}X_\theta\right).\end{aligned} \tag{7}$$

Thus, with the help of the orthonormal tangent vectors, the second fundamental form can be written as

$$\begin{aligned}h(X_1, X_1) &= h^1_{11}\xi_1 + h^2_{11}\xi_2,\\ h(X_1, X_2) &= h^1_{12}\xi_1 + h^2_{12}\xi_2,\\ h(X_2, X_2) &= h^1_{22}\xi_1 + h^2_{22}\xi_2,\end{aligned} \tag{8}$$

where the functions h^k_{ij}, $i,j,k=1,2$ are given by

$$\begin{aligned} h^k_{11} &= -\frac{c^1_{11}}{E}, \\ h^k_{12} &= \frac{Ec^k_{12}-Fc^1_{11}}{EW}, \\ h^k_{22} &= -\frac{E^2c^k_{22}-2EFc^k_{12}+F^2c^k_{11}}{EW^2}. \end{aligned} \tag{9}$$

The entries of the shape operator matrices consist of these coefficients. The mean curvature vector field can be calculated by $H=\frac{1}{2}trh$. Thus, if S is a timelike surface, then the mean curvature vector field is

$$H=\frac{1}{2}\left(-h\left(X_1,X_1\right)+h\left(X_2,X_2\right)\right), \tag{10}$$

where $\{X_1,X_2\}$ is a local orthonormal frame of the tangent bundle such that $g\left(X_1,X_1\right)=-1$, $g\left(X_2,X_2\right)=1$ [12]. The mean curvature of S is the norm of the mean curvature vector.

Further, Gaussian curvature of the timelike surface S in $\mathbb{E}^4_1$ is obtained by

$$K=\sum_{k=1}^{2}h^k_{11}h^k_{22}-\left(h^k_{12}\right)^2, \tag{11}$$

(see, [8]). A surface S is said to be flat (minimal) if its Gaussian curvature (mean curvature vector) vanishes identically [10].

Moreover, for the orthonormal bases $\{X_1,X_2\}$ and $\{\xi_1,\xi_2\}$ of the tangent and normal spaces, the normal curvature is defined by

$$K_\xi=g\left(R^\perp\left(X_1,X_2\right)\xi_2,\ \xi_1\right), \tag{12}$$

(see, [12]).

According to this relation, a surface S is said to be semiumbilical if its normal curvature vanishes for all points on S [13].

A special surface which is called as Wintgen Ideal surface is defined with the help of Gaussian curvature, mean curvature and normal curvature. Hence, the surface satisfies the condition

$$K+|K_\xi|=\|H\| \tag{13}$$

(see, [6, 7, 14, 15]).

In [1], Yu. A. Aminov focused on the notion of Monge patch in $\mathbb{E}^4$ with the representation

$$z=f(\theta,\varphi), \qquad w=g(\theta,\varphi) \tag{14}$$

where θ,φ,z,w are Cartezian coordinates in $\mathbb{E}^4$. Also in [5], the authors studied some surfaces given with the Monge patch by using the parametrization

$$X(\theta,\varphi)=(\theta,\varphi,f(\theta,\varphi),g(\theta,\varphi)).$$

If the functions f and g in (14) is given by the form

$$f(\theta,\varphi)=r(\theta)\cos\varphi, \quad g(\theta,\varphi)=r(\theta)\sin\varphi,$$

then this representation is congruent to Aminov surface in $\mathbb{E}^4$. Aminov surfaces have been studied in different categories. In [4], the authors studied the pedal surface of these type of surfaces. Also in [2], [3], the authors characterized Aminov surfaces in Euclidean 4−space $\mathbb{E}^4$ and isotropic 4−space $\mathbb{I}_4$, respectively.

In this work, we consider timelike Aminov surfaces in Minkowski 4-space $\mathbb{E}_1^4$ with hyperbolic type. We characterize these types of surfaces in terms of their Gaussian curvature, mean curvature and normal curvature. Finally, we give the necessary and sufficient condition for this surface to become Wintgen Ideal surface.

2. Classification of Timelike Aminov Surfaces of Hyperbolic Type in $\mathbb{E}_1^4$

Definition 2.1. *Let S be a timelike surface in Minkowski four space $\mathbb{E}_1^4$. If the surface is given by the parametrization*

$$X(\theta,\varphi) = (r(\theta)\cosh\varphi, r(\theta)\sinh\varphi, \theta, \varphi), \tag{15}$$

where $r(\theta)$ is a smooth function, then S is called timelike Aminov surface of hyperbolic type in $\mathbb{E}_1^4$.

Let S be a timelike Aminov surface of hyperbolic type with the parametrization (15). By choosing normal vectors as $g(\xi_1,\xi_1) = 1$, $g(\xi_2,\xi_2) = 1$, we have the followings:

The tangent space of S is spanned by the vector fields

$$\begin{aligned} X_\theta &= (r'(\theta)\cosh\varphi, r'(\theta)\sinh\varphi, 1, 0), \\ X_\varphi &= (r(\theta)\sinh\varphi, r(\theta)\cosh\varphi, 0, 1). \end{aligned} \tag{16}$$

Hence the coefficients of the first fundamental form of the surface are

$$\begin{aligned} E &= g(X_\theta, X_\theta) = 1 - (r')^2, \\ F &= g(X_\theta, X_\varphi) = 0, \\ G &= g(X_\varphi, X_\varphi) = 1 + r^2, \end{aligned} \tag{17}$$

where g is the Lorentzian metric in $\mathbb{E}_1^4$. Since the surface S is timelike, then $W = \sqrt{F^2 - EG}$.

The second partial derivatives of $X(\theta,\varphi)$ are expressed as following

$$\begin{aligned} X_{\theta\theta} &= (r''(\theta)\cosh\varphi, r''(\theta)\sinh\varphi, 0, 0), \\ X_{\theta\varphi} &= (r'(\theta)\sinh\varphi, r'(\theta)\cosh\varphi, 0, 0), \\ X_{\varphi\varphi} &= (r(\theta)\cosh\varphi, r(\theta)\sinh\varphi, 0, 0). \end{aligned} \tag{18}$$

Further, the normal space of S is spanned by the orthonormal vector fields

$$\begin{gathered} \xi_1 = \frac{1}{\sqrt{A}}(1, 0, r'(\theta)\cosh\varphi, r(\theta)\sinh\varphi), \\ \xi_2 = \frac{1}{\sqrt{AD}}(-B, A, -Br'(\theta)\cosh\varphi - Ar'(\theta)\sinh\varphi, -Br(\theta)\sinh\varphi - Ar(\theta)\cosh\varphi), \end{gathered} \tag{19}$$

where

$$\begin{aligned} A &= -1 + (r'\cosh\varphi)^2 + (r\sinh\varphi)^2, \\ B &= -\cosh\varphi\sinh\varphi((r')^2 + r^2), \\ C &= 1 + (r'\sinh\varphi)^2 + (r\cosh\varphi)^2, \\ D &= AC - B^2. \end{aligned} \tag{20}$$

Since S is timelike surface in $\mathbb{E}_1^4$ with respect to choosen orthonormal frame, A and D are positive definite. Using (18) and (19), we can calculate the coefficients of the second fundamental form as follows;

$$
\begin{aligned}
c_{11}^{1} &= \frac{-r'' \cosh\varphi}{\sqrt{A}}, \quad c_{22}^{1} = \frac{-r\cosh\varphi}{\sqrt{A}}, \\
c_{12}^{1} &= \frac{-r' \sinh\varphi}{\sqrt{A}}, \quad c_{12}^{2} = \frac{r'(A\cosh\varphi + B\sinh\varphi)}{\sqrt{AD}}, \\
c_{11}^{2} &= \frac{r''(A\sinh\varphi + B\cosh\varphi)}{\sqrt{AD}}, \\
c_{22}^{2} &= \frac{r(B\cosh\varphi + A\sinh\varphi)}{\sqrt{AD}}.
\end{aligned} \tag{21}
$$

By the use of the equations (9) and (21), one can calculate the functions h_{ij}^{k}, $i, j, k = 1, 2$ as:

$$
\begin{aligned}
h_{11}^{1} &= \frac{r'' \cosh\varphi}{E\sqrt{A}}, \quad h_{22}^{1} = \frac{Er\cosh\varphi}{W^2\sqrt{A}}, \\
h_{12}^{1} &= \frac{-r' \sinh\varphi}{W\sqrt{A}}, \quad h_{12}^{2} = \frac{r'(A\cosh\varphi + B\sinh\varphi)}{W\sqrt{AD}}, \\
h_{11}^{2} &= \frac{-r''(B\cosh\varphi + A\sinh\varphi)}{E\sqrt{AD}}, \\
h_{22}^{2} &= \frac{-Er(B\cosh\varphi + A\sinh\varphi)}{W^2\sqrt{AD}}.
\end{aligned} \tag{22}
$$

Now, we classify timelike Aminov surface of hyperbolic type with respect to Gaussian, mean and normal curvature.

Theorem 2.1. [8] *Let S be a timelike Aminov surface of hyperbolic type in $\mathbb{E}_1^4$. Then the Gaussian curvature of the surface is given by*

$$
K = \frac{r''r\left(1 + r^2\right) + (r')^2(1 - (r')^2)}{W^2 D}. \tag{23}
$$

Proof. Let S be a timelike Aminov surface of hyperbolic type in $\mathbb{E}_1^4$. By the use of (11) and (22) we have

$$
K = \frac{\left(r''r(\cosh\varphi)^2 - (r')^2(\sinh\varphi)^2\right)C + 2\cosh\varphi\sinh\varphi(r''r - (r')^2)B + \left(r''r(\sinh\varphi)^2 - (r')^2(\cosh\varphi)^2\right)A}{DW^2}.
$$

Then, substituting (20) into this equation, we get the result. □

Theorem 2.2. [9] *Let S be a timelike Aminov surface of hyperbolic type with the parametrization (15) in $\mathbb{E}_1^4$. Then S is flat if and only if*

$$
\theta = \int \sqrt{\frac{ar^2(\theta) + 1}{r^2(\theta) + 1}}\, dr(\theta) \tag{24}
$$

holds.

Proof. Let S be a timelike Aminov surface of hyperbolic type in $\mathbb{E}_1^4$. If it is flat, then we get

$$
r''r\left(r^2 + 1\right) + (r')^2(1 - (r')^2) = 0 \tag{25}
$$

which has a solution (24). The converse statement is trivial. □

Theorem 2.3. [9] *Let S be a timelike Aminov surface of hyperbolic type in $\mathbb{E}_1^4$. Then the mean curvature vector of the surface is given by*

$$\vec{H} = \frac{\cosh\varphi\left(r''G + rE\right)}{2\sqrt{A}W^2}\xi_1 - \frac{\left(A\sinh\varphi + B\cosh\varphi\right)\left(r''G + rE\right)}{2\sqrt{AD}W^2}\xi_2. \tag{26}$$

Proof. By the use of the equations (10), (8) and (22), we get the result. □

Theorem 2.4. [9]*Let S be a timelike Aminov surface of hyperbolic type in $\mathbb{E}_1^4$. Then S is minimal if and only if*

$$r(\theta) = \frac{-1 - a^2 + e^{\pm 2a(a\theta+b)}}{2ae^{\pm 2a(a\theta+b)}} \tag{27}$$

holds.

Proof. Let S be a timelike Aminov surface of hyperbolic type with the parametrization (15) in $\mathbb{E}_1^4$. If S is minimal, from the mean curvature vector, we get

$$r''G + rE = 0. \tag{28}$$

Substituting (17) into the equation (28), we have

$$r''\left(1 + r^2\right) + r\left(1 - \left(r'\right)^2\right) = 0 \tag{29}$$

which has the solution (27). □

Example 2.1. *Substituting $a = -1$ and $b = 2$ in equality (27), we can plot timelike minimal Aminov surface of hyperbolic type with maple command plot3d($[x + y, z, t]$, $x = a..b$, $y = c..d$) :*

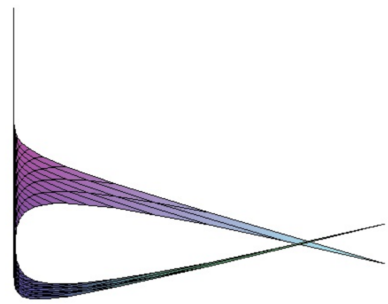

FIGURE 1. Timelike Minimal Aminov Surface of Hyperbolic Type

Theorem 2.5. *Let S be a timelike Aminov surface of hyperbolic type in $\mathbb{E}_1^4$. Then, the normal curvature of the surface is given by*

$$K_\xi = \frac{r'\left(r''\left(1 + r^2\right) - r\left(1 - r'^2\right)\right)}{W^3\sqrt{D}}. \tag{30}$$

Proof. Let S be a timelike Aminov surface of hyperbolic type with the parametrization (15) in $\mathbb{E}_1^4$. By the use of the equations (12), we get

$$K_\xi = g\left(R^\perp(X_1, X_2)\xi_2,\ \xi_1\right) = g\left(h\left(X_1, A_{\xi_2}X_2\right), \xi_1\right) - g\left(h\left(X_2, A_{\xi_2}X_1\right), \xi_1\right).$$

Then, we obtain

$$g\left(R^\perp(X_1, X_2)\xi_2,\ \xi_1\right) = h_{12}^1\left(h_{22}^2 - h_{11}^2\right) + h_{12}^2\left(h_{11}^1 - h_{22}^1\right). \tag{31}$$

Substituting h_{ij}^k in (22) into the equation (31), we have

$$K_\xi = \frac{r'\left(Gr'' - Er\right)}{W^3\sqrt{D}} \tag{32}$$

where $E = 1 - (r')^2$ and $G = 1 + r^2$. Hence, we get the result. □

Theorem 2.6. *Let S be a timelike Aminov surface of hyperbolic type in $\mathbb{E}_1^4$. Then, S is semiumbilical if and only if*

$$r(\theta) = c_1 \qquad or \qquad \theta = \pm\int\sqrt{\frac{1 + r^2(\theta)}{r^2(\theta) + c_1}}dr(\theta) \tag{33}$$

where c_1, c_2 are real constants.

Proof. Let S be a timelike Aminov surface of hyperbolic type in $\mathbb{E}_1^4$. Suppose that, S is semiumbilical. By the use of (30), we get the differential equation

$$r'\left(r''\left(1 + r^2\right) - r\left(1 - r'^2\right)\right) = 0$$

which has the solution (33). The converse statement is trivial. □

Theorem 2.7. *Let S be a timelike Aminov surface of hyperbolic type in $\mathbb{E}_1^4$. Then, S is congruent to Wintgen Ideal surface if and only if*

$$\pm 2W\sqrt{D}r'\left(Gr'' - Er\right) = \left(r''G + rE\right)^2\left(C\cosh^2\theta + 2B\cosh\theta\sinh\theta + A\sinh^2\theta\right) - 2EG\left(r''rG + r'^2E\right).$$

Proof. Substituting (23), (26) and (30) into (13), we get the result. □

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