

## INITIAL BOUNDS FOR CERTAIN CLASSES OF BI-UNIVALENT FUNCTIONS DEFINED BY THE $(p, q)$ -LUCAS POLYNOMIALS

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ABSTRACT. Our present investigation is motivated essentially by the fact that, in Geometric Function Theory, one can find many interesting and fruitful usages of a wide variety of special functions and special polynomials. The main purpose of this article is to make use of the  $(p, q)$ -Lucas polynomials  $L_{p,q,n}(x)$  and the generating function  $\mathcal{G}_{L_{p,q,n}(x)}(z)$ , in order to introduce three new subclasses of the bi-univalent function class  $\Sigma$ . For functions belonging to the defined classes, we then derive coefficient inequalities and the Fekete-Szegő inequalities. Some interesting observations of the results presented here are also discussed. We also provide relevant connections of our results with those considered in earlier investigations.

Keywords: Univalent functions, bi-univalent functions, bi-Mocanu-convex functions, bi- $\alpha$ -starlike functions, bi-starlike functions, bi-convex functions, Fekete-Szegő problem, Chebyshev polynomials,  $(p, q)$ -Lucas polynomials.

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### 1. INTRODUCTION

Let  $\mathbb{R} = (-\infty, \infty)$  be the set of real numbers,  $\mathbb{C}$  be the set of complex numbers and

$$\mathbb{N} := \{1, 2, 3, \dots\} = \mathbb{N}_0 \setminus \{0\}$$

be the set of positive integers. Let also  $\mathcal{A}$  denote the class of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \tag{1}$$

which are analytic in the open unit disk  $\Delta = \{z : z \in \mathbb{C} \text{ and } |z| < 1\}$ . Further, by  $\mathcal{S}$  we shall denote the class of all functions in  $\mathcal{A}$  which are univalent in  $\Delta$ .

It is well known that every function  $f \in \mathcal{S}$  has an inverse  $f^{-1}$ , defined by

$$f^{-1}(f(z)) = z \quad (z \in \Delta)$$

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and

$$f(f^{-1}(w)) = w(|w| < r_0(f); r_0(f) \geq \frac{1}{4}),$$

where

$$g(w) = f^{-1}(w) = w - a_2w^2 + (2a_2^2 - a_3)w^3 - (5a_2^3 - 5a_2a_3 + a_4)w^4 + \dots$$

A function  $f \in \mathcal{A}$  is said to be bi-univalent in  $\Delta$  if both a function  $f$  and its inverse  $f^{-1}$  are univalent in  $\Delta$ . Let  $\Sigma$  denote the class of bi-univalent functions in  $\Delta$  given by (1).

In 2010, Srivastava *et al.* [24] revived the study of bi-univalent functions by their pioneering work on the study of coefficient problems. Various subclasses of the bi-univalent function class  $\Sigma$  were introduced and non-sharp estimates on the first two coefficients  $|a_2|$  and  $|a_3|$  in the Taylor-Maclaurin series expansion (1) were found in the recent investigations (see, for example, [1, 2, 4, 5, 6, 7, 10, 11, 12, 13, 14, 15, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27]) and including the references therein. The afore-cited all these papers on the subject were actually motivated by the work of Srivastava *et al.* [24]. However, the problem to find the coefficient bounds on  $|a_n|$  ( $n = 3, 4, \dots$ ) for functions  $f \in \Sigma$  is still an open problem.

For analytic functions  $f$  and  $g$  in  $\Delta$ ,  $f$  is said to be subordinate to  $g$  if there exists an analytic function  $w$  such that

$$w(0) = 0, \quad |w(z)| < 1 \quad \text{and} \quad f(z) = g(w(z)) \quad (z \in \Delta).$$

This subordination will be denoted here by

$$f \prec g \quad (z \in \Delta)$$

or, conventionally, by

$$f(z) \prec g(z) \quad (z \in \Delta).$$

In particular, when  $g$  is univalent in  $\Delta$ ,

$$f \prec g \quad (z \in \Delta) \Leftrightarrow f(0) = g(0) \quad \text{and} \quad f(\Delta) \subset g(\Delta).$$

Let  $p(x)$  and  $q(x)$  be polynomials with real coefficients. The  $(p, q)$ -polynomials  $L_{p,q,n}(x)$ , or briefly  $L_n(x)$  are given by the following recurrence relation (see [8, 9]):

$$L_n(x) = p(x)L_{n-1}(x) + q(x)L_{n-2}(x) \quad (n \in \mathbb{N} \setminus \{1\}),$$

with

$$\begin{aligned} L_0(x) &= 2, \\ L_1(x) &= p(x), \\ L_2(x) &= p^2(x) + 2q(x), \\ L_3(x) &= p^3(x) + 3p(x)q(x), \\ &\vdots \end{aligned}$$

The generating function of the Lucas polynomials  $L_n(x)$  (see [16]) is given by:

$$\mathcal{G}_{L_n(x)}(z) := \sum_{n=0}^{\infty} L_n(x)z^n = \frac{2 - p(x)z}{1 - p(x)z - q(x)z^2}. \tag{2}$$

Note that for particular values of  $p$  and  $q$ , the  $(p, q)$ -polynomial  $L_n(x)$  leads to various polynomials, among those, we list few cases here (see, [16] for more details, also [3]):

- (1) For  $p(x) = x$  and  $q(x) = 1$ , we obtain the Lucas polynomials  $L_n(x)$ .
- (2) For  $p(x) = 2x$  and  $q(x) = 1$ , we attain the Pell-Lucas polynomials  $Q_n(x)$ .
- (3) For  $p(x) = 1$  and  $q(x) = 2x$ , we attain the Jacobsthal-Lucas polynomials  $j_n(x)$ .
- (4) For  $p(x) = 3x$  and  $q(x) = -2$ , we attain the Fermat-Lucas polynomials  $f_n(x)$ .
- (5) For  $p(x) = 2x$  and  $q(x) = -1$ , we have the Chebyshev polynomials  $T_n(x)$  of the first kind.

We want to remark explicitly that, in [3] Altınkaya and S. Yalçın, first introduced a subclass of bi-univalent functions by using the  $(p, q)$ -Lucas polynomials. This methodology builds a bridge between the Theory of Geometric Functions and that of Special Functions, which are known as different areas. Thus, we aim to introduce several new classes of bi-univalent functions defined through the  $(p, q)$ -Lucas polynomials. Furthermore, we derive coefficient estimates and Fekete-Szegő inequalities for functions defined in those classes.

## 2. COEFFICIENT ESTIMATES AND FEKETE-SZEGŐ INEQUALITIES

In this section, we introduce three new subclasses  $\mathcal{S}_\Sigma^*(\alpha, x)$ ,  $\mathcal{M}_\Sigma(\alpha, x)$ ,  $\mathcal{L}_\Sigma(\alpha, x)$  of the bi-univalent function class  $\Sigma$ .

A function  $f \in \Sigma$  of the form (1) belongs to the class  $\mathcal{S}_\Sigma^*(\alpha, x)$ ,  $\alpha \geq 0$  and  $z, w \in \Delta$ , if the following conditions are satisfied:

$$\frac{zf'(z)}{f(z)} + \alpha \frac{z^2 f''(z)}{f(z)} \prec \mathcal{G}_{L_n(x)}(z) - 1$$

and for  $g = f^{-1}$

$$\frac{wg'(w)}{g(w)} + \alpha \frac{w^2 g''(w)}{g(w)} \prec \mathcal{G}_{L_n(x)}(w) - 1.$$

Note that  $S_\Sigma^*(x) \equiv \mathcal{S}_\Sigma^*(0, x)$  was introduced and studied by [3].

A function  $f \in \Sigma$  of the form (1) belongs to the class  $\mathcal{M}_\Sigma(\alpha, x)$ ,  $0 \leq \alpha \leq 1$  and  $z, w \in \Delta$ , if the following conditions are satisfied:

$$(1 - \alpha) \frac{zf'(z)}{f(z)} + \alpha \left( 1 + \frac{zf''(z)}{f'(z)} \right) \prec \mathcal{G}_{L_n(x)}(z) - 1$$

and for  $g = f^{-1}$

$$(1 - \alpha) \frac{wg'(w)}{g(w)} + \alpha \left( 1 + \frac{wg''(w)}{g'(w)} \right) \prec \mathcal{G}_{L_n(x)}(w) - 1.$$

Note that the class  $\mathcal{M}_\Sigma(\alpha, x)$ , unifies the classes  $\mathcal{S}_\Sigma^*(x)$  and  $K_\Sigma(x)$  like  $\mathcal{M}_\Sigma(0, x) \equiv \mathcal{S}_\Sigma^*(x)$  and  $\mathcal{M}_\Sigma(1, x) \equiv K_\Sigma(x)$ . For functions in the class  $\mathcal{M}_\Sigma(\alpha, x)$ , the following coefficient estimates and Fekete-Szegő inequality are obtained.

Next, a function  $f \in \Sigma$  of the form (1) belongs to the class  $\mathcal{L}_\Sigma(\alpha, x)$ ,  $0 \leq \alpha \leq 1$ , and  $z, w \in \Delta$ , if the following conditions are satisfied:

$$\left( \frac{zf'(z)}{f(z)} \right)^\alpha \left( 1 + \frac{zf''(z)}{f'(z)} \right)^{1-\alpha} \prec \mathcal{G}_{L_n(x)}(z) - 1$$

and for  $g = f^{-1}$

$$\left( \frac{wg'(w)}{g(w)} \right)^\alpha \left( 1 + \frac{wg''(w)}{g'(w)} \right)^{1-\alpha} \prec \mathcal{G}_{L_n(x)}(w) - 1.$$

Now, for functions in the classes  $\mathcal{S}_\Sigma^*(\alpha, x)$ ,  $\mathcal{M}_\Sigma(0, x)$ ,  $\mathcal{L}_\Sigma(\alpha, x)$ , the following coefficient estimates and Fekete-Szegő inequality are obtained.

**Theorem 2.1.** *Let  $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$  be in the class  $\mathcal{S}_\Sigma^*(\alpha, x)$ . Then*

$$|a_2| \leq \frac{|p(x)| \sqrt{|p(x)|}}{\sqrt{|4\alpha^2 p^2(x) + 2q(x)(1 + 2\alpha)^2|}}, \quad |a_3| \leq \frac{|p(x)|}{2 + 6\alpha} + \frac{p^2(x)}{(1 + 2\alpha)^2}$$

and for  $\nu \in \mathbb{R}$

$$|a_3 - \nu a_2^2| \leq \begin{cases} \frac{|p(x)|}{2 + 6\alpha}, & |\nu - 1| \leq \frac{|2\alpha^2 p^2(x) + q(x)(1 + 2\alpha)^2|}{2p^2(x)(1 + 3\alpha)} \\ \frac{|p(x)|^3 |\nu - 1|}{|4\alpha^2 p^2(x) + 2q(x)(1 + 2\alpha)^2|}, & |\nu - 1| \geq \frac{|2\alpha^2 p^2(x) + q(x)(1 + 2\alpha)^2|}{2p^2(x)(1 + 3\alpha)} \end{cases}.$$

*Proof.* Let  $f \in \mathcal{S}_\Sigma^*(\alpha, x)$  be given by Taylor-Maclaurin expansion (1). Then, there are two analytic functions  $u$  and  $v$  such that

$$u(0) = 0, \quad v(0) = 0,$$

$$|u(z)| = |u_1 z + u_2 z^2 + \dots| < 1, \quad |v(w)| = |v_1 w + v_2 w^2 + \dots| < 1 \quad (\forall z, w \in \Delta).$$

Hence, we can write

$$\frac{z f'(z)}{f(z)} + \alpha \frac{z^2 f''(z)}{f(z)} = \mathcal{G}_{L_n(x)}(u(z)) - 1$$

and

$$\frac{w g'(w)}{g(w)} + \alpha \frac{w^2 g''(w)}{g(w)} = \mathcal{G}_{L_n(x)}(v(w)) - 1.$$

Or, equivalently,

$$\frac{z f'(z)}{f(z)} + \alpha \frac{z^2 f''(z)}{f(z)} = -1 + L_0(x) + L_1(x)u(z) + L_2(x)[u(z)]^2 + \dots$$

and

$$\frac{w g'(w)}{g(w)} + \alpha \frac{w^2 g''(w)}{g(w)} = -1 + L_0(x) + L_1(x)v(w) + L_2(x)[v(w)]^2 + \dots$$

From the above equalities, we obtain

$$\frac{z f'(z)}{f(z)} + \alpha \frac{z^2 f''(z)}{f(z)} = 1 + L_1(x)u_1 z + [L_1(x)u_2 + L_2(x)u_1^2]z^2 + \dots \tag{3}$$

and

$$\frac{w g'(w)}{g(w)} + \alpha \frac{w^2 g''(w)}{g(w)} = 1 + L_1(x)v_1 w + [L_1(x)v_2 + L_2(x)v_1^2]w^2 + \dots \tag{4}$$

Additionally, it is fairly well known that

$$|u_k| \leq 1, \quad |v_k| \leq 1 \quad (k \in \mathbb{N}). \tag{5}$$

Thus upon comparing the corresponding coefficients in (3) and (4), we have

$$(1 + 2\alpha) a_2 = L_1(x)u_1 \tag{6}$$

$$2(1 + 3\alpha) a_3 - (1 + 2\alpha) a_2^2 = L_1(x)u_2 + L_2(x)u_1^2 \tag{7}$$

$$-(1 + 2\alpha) a_2 = L_1(x)v_1 \tag{8}$$

and

$$(3 + 10\alpha) a_2^2 - 2(1 + 3\alpha) a_3 = L_1(x)v_2 + L_2(x)v_1^2. \tag{9}$$

From (6) and (8), we can easily see that

$$u_1 = -v_1 \tag{10}$$

and

$$\begin{aligned} 2(1 + 2\alpha)^2 a_2^2 &= [L_1(x)]^2 (u_1^2 + v_1^2) \\ a_2^2 &= \frac{[L_1(x)]^2 (u_1^2 + v_1^2)}{2(1 + 2\alpha)^2}. \end{aligned} \tag{11}$$

If we add (7) to (11), we get

$$2(1+4\alpha)a_2^2 = L_1(x)(u_2 + v_2) + L_2(x)(u_1^2 + v_1^2). \quad (12)$$

By substituting (11) in (12), we reduce that

$$a_2^2 = \frac{[L_1(x)]^3(u_2 + v_2)}{2(1+4\alpha)[L_1(x)]^2 - 2L_2(x)(1+2\alpha)^2} \quad (13)$$

which yields

$$|a_2| \leq \frac{|p(x)|\sqrt{|p(x)|}}{\sqrt{|4\alpha^2 p^2(x) + 2q(x)(1+2\alpha)^2|}}.$$

By subtracting (9) from (7) and in view of (10), we obtain

$$\begin{aligned} 4(1+3\alpha)a_3 - 4(1+3\alpha)a_2^2 &= L_1(x)(u_2 - v_2) + L_2(x)(u_1^2 - v_1^2) \\ a_3 &= \frac{L_1(x)(u_2 - v_2)}{4(1+3\alpha)} + a_2^2. \end{aligned} \quad (14)$$

Then in view of (11), (14) becomes

$$a_3 = \frac{L_1(x)(u_2 - v_2)}{4(1+3\alpha)} + \frac{[L_1(x)]^2(u_1^2 + v_1^2)}{2(1+2\alpha)^2}.$$

Applying (5), we deduce that

$$|a_3| \leq \frac{|p(x)|}{2+6\alpha} + \frac{p^2(x)}{(1+2\alpha)^2}.$$

From (14), for  $\nu \in \mathbb{R}$ , we write

$$a_3 - \nu a_2^2 = \frac{L_1(x)(u_2 - v_2)}{4(1+3\alpha)} + (1-\nu)a_2^2. \quad (15)$$

By substituting (13) in (15), we have

$$\begin{aligned} a_3 - \nu a_2^2 &= \frac{L_1(x)(u_2 - v_2)}{4(1+3\alpha)} + \left( \frac{(1-\nu)[L_1(x)]^3(u_2 + v_2)}{2[(1+4\alpha)[L_1(x)]^2 - 2L_2(x)(1+2\alpha)^2]} \right) \\ &= L_1(x) \left\{ \left( \Omega(\nu, x) + \frac{1}{4(1+3\alpha)} \right) u_2 + \left( \Omega(\nu, x) - \frac{1}{4(1+3\alpha)} \right) v_2 \right\}, \end{aligned} \quad (16)$$

where

$$\Omega(\nu, x) = \frac{(1-\nu)[L_1(x)]^2}{2(1+4\alpha)[L_1(x)]^2 - 2L_2(x)(1+2\alpha)^2}.$$

Hence, in view of (5), we conclude that

$$|a_3 - \nu a_2^2| \leq \begin{cases} \frac{|L_1(x)|}{2+6\alpha} & ; 0 \leq |\Omega(\nu, x)| \leq \frac{1}{4(1+3\alpha)} \\ 2|L_1(x)||\Omega(\nu, x)| & ; |\Omega(\nu, x)| \geq \frac{1}{4(1+3\alpha)} \end{cases},$$

which evidently completes the proof of Theorem 2.1.  $\square$

Analysis similar to that in the proof of the previous Theorem shows that

**Theorem 2.2.** Let  $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$  be in the class  $\mathcal{M}_{\Sigma}(\alpha, x)$ . Then

$$|a_2| \leq \frac{|p(x)| \sqrt{|p(x)|}}{\sqrt{|\alpha(1+\alpha)p^2(x) + 2q(x)(1+\alpha)^2|}}, \quad |a_3| \leq \frac{|p(x)|}{2+4\alpha} + \frac{p^2(x)}{(1+\alpha)^2}$$

and for  $\nu \in \mathbb{R}$

$$|a_3 - \nu a_2^2| \leq \begin{cases} \frac{|p(x)|}{2+4\alpha}, & |\nu - 1| \leq \frac{|\alpha(1+\alpha)p^2(x) + 2q(x)(1+\alpha)^2|}{p^2(x)(2+4\alpha)} \\ \frac{|p(x)|^3 |\nu - 1|}{|\alpha(1+\alpha)p^2(x) + 2q(x)(1+\alpha)^2|}, & |\nu - 1| \geq \frac{|\alpha(1+\alpha)p^2(x) + 2q(x)(1+\alpha)^2|}{p^2(x)(2+4\alpha)} \end{cases}$$

**Theorem 2.3.** Let  $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$  be in the class  $\mathcal{L}_{\Sigma}(\alpha, x)$ . Then

$$|a_2| \leq \frac{|p(x)| \sqrt{2|p(x)|}}{\sqrt{|(\alpha^2 - 5\alpha + 4)p^2(x) + 4q(x)(2 - \alpha)^2|}}, \quad |a_3| \leq \frac{|p(x)|}{6 - 4\alpha} + \frac{p^2(x)}{(2 - \alpha)^2}$$

and for  $\nu \in \mathbb{R}$

$$|a_3 - \nu a_2^2| \leq \begin{cases} \frac{|p(x)|}{6 - 4\alpha}, & |\nu - 1| \leq \frac{|(\alpha^2 - 5\alpha + 4)p^2(x) + 4q(x)(2 - \alpha)^2|}{4p^2(x)(3 - 2\alpha)} \\ \frac{2|p(x)|^3 |\nu - 1|}{|(\alpha^2 - 5\alpha + 4)p^2(x) + 4q(x)(2 - \alpha)^2|}, & |\nu - 1| \geq \frac{|(\alpha^2 - 5\alpha + 4)p^2(x) + 4q(x)(2 - \alpha)^2|}{4p^2(x)(3 - 2\alpha)} \end{cases}$$

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**Sahsene Altınkaya** for the photography and short autobiography, see *TWMS J. App. and Eng. Math.* V.8, No.1a, 2018.