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HOMOMORPHISM IN BIPOLAR $Q-$FUZZY SOFT $\Gamma-$SEMIRING

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ABSTRACT. In this paper, we discuss bipolar $Q-$fuzzy soft $\Gamma-$Semiring concept and bipolar $Q-$fuzzy soft $\Gamma-$Semiring homomorphism. Indeed, properties and theorems related to these notions are stated and proved.

Keywords: bipolar $Q-$fuzzy soft set, bipolar $Q-$fuzzy soft $\Gamma-$Semiring, bipolar $Q-$fuzzy soft ideal, bipolar $Q-$fuzzy soft $\Gamma-$Semiring homomorphism.

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1. Introduction

Vandiver [1] in 1934 introduced semiring concept which is a common generalization of rings and distributive lattices. Semiring is an universal algebra with two binary operations called addition and multiplication, where one of them is distributive over the other. On the other hand, rings of semigroup theory have considerable impact on the development of the semiring theory. This means that semirings lie between rings and semigroups. Semiring is very helpful for solving problems in information sciences and applied mathematics because semiring provides an algebraic frame work for modeling. Semiring was used in the areas of theoretical computer science as well as in the solutions of graph theory, coding theory and optimization theory. $\Gamma-$ring concept was introduced by Nobusawa [2] in 1964. $\Gamma-$semigrupe was first introduced by Sen [3] in 1981. Muirali Krishna [4] introduced the concept of $\Gamma-$semiring.

Fuzzy set theory studied and discussed by Zadeh [5]. Many papers on fuzzy sets appeared showing the important of the notion and it's applications to ring theory, topology theory, graph theory and group theory. Soft set theory was introduced by Molodtsov [6] as a new mathematical tool for dealing with uncertainties. Maji, Bisuas and Roy [7] generalized soft set theory to fuzzy soft set theory. The ideas of fuzzy soft rings and fuzzy soft ideals were first discussed and studied by Ghoshet, Dinda and Samanta [8]. The concept of soft semirings was investigated by Feng, Jan and Zhao in [9]. However, soft $\Gamma-$Semirings

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was introduced by Bektas, Bayrak and Ersoy [10]. Attanassov in [11] discussed intuitionistic fuzzy sets as a first time. Moreover, Krishna Rao [12] studied fuzzy soft $K-$ideals and fuzzy soft ideals over $\Gamma-$Semiring. However, intuitionistic normal fuzzy soft $K-$ideals over $\Gamma-$Semiring was introduced by Krishna Rao and Venkateswarulu [13]. A bipolar fuzzy sets as a generalization of fuzzy sets was first investigated by Zhang [14, 15] in 1994. Massa'deh [16, 17, 18] discussed and studied bipolar fuzzy coset, bipolar fuzzy ideals in $\Gamma-$nearring and antibipolar $Q-$fuzzy normal semigroup and bipolar $Q-$fuzzy $H-$ideals over $\Gamma-$hemiring. Aslam, Abdullah and ullah [19] studied bipolar fuzzy soft sets concepts. In this paper, we introduce and discuss the concept of bipolar $Q-$fuzzy soft $\Gamma-$semiring homomorphism. Also, we study some properties and theorems of homomorphic image of bipolar $Q-$fuzzy soft $\Gamma-$semiring.

2. Preliminaries

Definition 2.1. [2] *A set S together with associative binary operations called addition and multiplication (denoted by $+$ and $\cdot$ respectively) will be called a semiring provided that:*
1. Addition is a commutative operation.
2. There exist $0 \in S$ such that $a + 0 = a$ and $a \cdot 0 = 0$ for all $a \in S$.
3. Multiplication distributes over addition both from the left and from the right.

Definition 2.2. [3] *Let $(M, +)$ and $(\Gamma, +)$ be commutative semigroups. Then we call M as a $\Gamma-$semiring, if there exists a mapping $M \times \Gamma \times M \to M$ written as (a, α, b) as $a\alpha b$ such that it satisfies the following axioms $\forall a, b, c \in M$ and $\alpha, \vartheta \in \Gamma$:*
1. $a\alpha(b + c) = a\alpha b + a\alpha c$ and $(a + b)\alpha c = a\alpha c + b\alpha c$.
2. $a(\alpha + \vartheta)b = a\alpha b + a\vartheta b$.
3. $a\alpha(b\vartheta c) = (a\alpha b)\vartheta c$.

Definition 2.3. [4] *Let S be a $\Gamma-$semiring and K be a non empty subset of S. K is called a $\Gamma-$subsemiring of S if K is a sub-semigroup of $(S, +)$ and $K\Gamma K \subseteq K$.*

Definition 2.4. [4] *A subset K of S is said to be a right(left) ideals of S if K is closed under addition and $(K\Gamma S \subseteq K)S\Gamma K \subseteq K$. K is called an ideal of S if it is both a left and right ideal.*

Definition 2.5. [17] *If S is a non empty set. A mapping $\lambda : S \times Q \longrightarrow [0, 1]$ is called a $Q-$fuzzy subset of S.*

Definition 2.6. [17] *If S and Q are non empty arbitrary sets. A bipolar $Q-$fuzzy set μ in $S \times Q$ is an object having the form*

$$\mu = \{(s, q); \mu^+(s, q), \mu^-(s, q); s \in S, q \in Q\}$$

such that $\mu^+ : S \times Q \longrightarrow [0, 1]$ and $\mu^- : S \times Q \longrightarrow [-1, 0]$ are mappings.

The positive membership degree $\mu^+(s)$ denotes the satisfaction degree of an element s to the property corresponding to a bipolar $Q-$fuzzy set $\mu = \{(s, q); \mu^+(s, q), \mu^-(s, q); s \in S, q \in Q\}$, and the negative membership degree $\mu^-(s, q)$ denotes the satisfaction degree of an element s to a bipolar valued fuzzy set $\mu = \{(s, q); \mu^+(s, q), \mu^-(s, q); s \in S, q \in Q\}$. If $\mu^+(s, q) \neq (0, q)$ and $\mu^-(s, q) = (0, q)$, then it is the situation that s is regarded as having only positive satisfaction for $\mu = \{(s, q); \mu^+(s, q), \mu^-(s, q); s \in S, q \in Q\}$. If $\mu^+(s, q) = (0, q)$ and $\mu^-(s, q) \neq (0, q)$ then it is the situation that s does not satisfy the property of $\mu = \{(s, q); \mu^+(s, q), \mu^-(s, q); s \in S, q \in Q\}$. But some what satisfies the counter property of $\mu = \{(s, q); \mu^+(s, q), \mu^-(s, q); s \in S, q \in Q\}$. It is possible for an element s to be such that $\mu^+(s, q) \neq (0, q)$ and $\mu^-(s, q) \neq (0, q)$, when the membership function of property over lap S. For the take of simplicity, we shall use the symbol $\mu = (\mu^+, \mu^-)$ for the bipolar $Q-$fuzzy set $\mu = \{(s, q); \mu^+(s, q), \mu^-(s, q); s \in S, q \in Q\}$.

Definition 2.7. *If μ is a $Q-$fuzzy subset of S. For any $(\alpha, \beta) \in [-1, 0] \times [0, 1]$, we define the set $\mu_{<\alpha,\beta>} = \{s \in S; \mu^+(s, q) \leq \alpha$ and $\mu^-(s, q) \geq \beta\}$ is called the level subset of S with respect to μ.*

Definition 2.8. [12] *A $K-$ideal I is an ideal of a $\Gamma-$semiring S such that for any $a, b \in S, a + b \in I, b \in I$ then $a \in I$.*

Definition 2.9. *Let μ and λ be two bipolar $Q-$fuzzy subset of $\Gamma-$semiring S. Then $\mu \circ \lambda$ is given by:*

$$(\mu^+ \circ \lambda^+)(c, q) = \begin{cases} \sup_{(c,q)=(a\alpha b,q)}\{\min\{\mu^+(a, q), \lambda^+(b, q)\}\}, \\ 0, \ otherwise \end{cases}$$

$$(\mu^- \circ \lambda^-)(c, q) = \begin{cases} \inf_{(c,q)=(a\alpha b,q)}\{\max\{\mu^-(a, q), \lambda^-(b, q)\}\}, \\ 0, \ otherwise \end{cases}$$

$\forall a, b, c \in S, \alpha \in \Gamma$.

Definition 2.10. *Let S_1 and S_2 be two sets and $\Psi : S_1 \times Q \to S_2 \times Q$ be any function. A bipolar $Q-$fuzzy subset μ of S is said to be Ψ invariant if $\Psi(a, q) = \Psi(b, q)$ then $\mu^+(a, q) = \mu^+(b, q)$ and $\mu^-(a, q) = \mu^-(b, q)$.*

Definition 2.11. [4] *If S_1 and S_2 are two $\Gamma-$semirings, a map $\Psi : S_1 \to S_2$ is said to be $\Gamma-$semiring homomorphism if $\Psi(s_1 + s_2) = \Psi(s_1) + \Psi(s_1)$ and $\Psi(s_1 \alpha s_2) = \Psi(s_1)\alpha\Psi(s_1)$ for all $s_1, s_2 \in S_1, \alpha \in \Gamma$.*

Definition 2.12. *If S_1 and S_2 are two $\Gamma-$semirings and Ψ is a function from S_1 into S_2. If μ is a biolar $Q-$fuzzy subset of S_2 then the pre image of μ under Ψ is a bipolar $Q-$fuzzy subset of S_1 defined by $\Psi^{-1}(\mu)(s, q) = \mu(\Psi(s), q)$ for all $s \in S_1, q \in Q$, that is, $\Psi^{-1}(\mu^+)(s, q) = \mu^+(\Psi(s), q)$ and $\Psi^{-1}(\mu^-)(s, q) = \mu^-(\Psi(s), q)$.*

Definition 2.13. *Let $\Psi : S_1 \to S_2$ be a homomorphism of semirings and $\mu = (\mu^+, \mu^-)$ be a bipolar $Q-$fuzzy subset of S_1. We define a bipolar $Q-$fuzzy subset $\Psi(\mu)$ of S_2 by:*

$$\Psi(\mu^+)(s_1, q)) = \begin{cases} \sup_{(s_2,q)\in\Psi^{-1}(s_1,q)} \mu^+(s_2, q); \ \ if \ \Psi^{-1}(s_1, q) \neq \phi \\ 0, otherwise \end{cases}$$

$$\Psi(\mu^-)(s_1, q)) = \begin{cases} \inf_{(s_2,q)\in\Psi^{-1}(s_1,q)} \mu^-(s_2, q); \ \ if \ \Psi^{-1}(s_1, q) \neq \phi \\ 0, otherwise \end{cases}$$

for all $s_1 \in S_1$.

Definition 2.14. *If S is a $\Gamma-$semiring. A bipolar $Q-$fuzzy subset μ of S is called bipolar $Q-$fuzzy $\Gamma-$subsemiring of S if it satisfies the following conditions:*

(1) $\mu^+(a + b, q) \geq \min\{\mu^+(a, q), \mu^+(b, q)\}$.
(2) $\mu^-(a + b, q) \leq \max\{\mu^-(a, q), \mu^-(b, q)\}$.
(3) $\mu^+(a\alpha b, q) \geq \min\{\mu^+(a, q), \mu^+(b, q)\}$.
(4) $\mu^-(a\alpha b, q) \leq \max\{\mu^-(a, q), \mu^-(b, q)\}$.

For all $a, b \in S, \alpha \in \Gamma$ and $q \in Q$.

Definition 2.15. *A bipolar $Q-$fuzzy subset δ of $\Gamma-$semiring S is said to be a bipolar $Q-$fuzzy right (left) ideal of S if it satisfies the following axioms:*

(1) $\delta^+(a + b, q) \geq \min\{\delta^+(a, q), \delta^+(b, q)\}$.
(2) $\delta^-(a + b, q) \leq \max\{\delta^-(a, q), \delta^-(b, q)\}$.

(3) $\delta^+(a\alpha b, q) \geq \delta^+(a, q)(\delta^+(b, q))$.
(4) $\delta^-(a\alpha b, q) \leq \delta^-(a, q)(\delta^-(b, q))$.
For all $a, b \in S, \alpha \in \Gamma$ *and* $q \in Q$.

Definition 2.16. [6] *If* U *is an initial universe,* E *is the set of parameters,* $S \subset E$ *and* $P(U)$ *is the power set of* U*. Then* (μ, S) *is called a soft set where* $\mu : S \to P(u)$.

Definition 2.17. *Let* U *be an initial universe set,* E *be the set of parameters and* $S, Q \subseteq E$*. A pair* $(\mu, S \times Q)$ *is called a* $Q-$*fuzzy soft over* U*, where* μ *is mapping given by* $\mu : S \times Q \to I^U$ *where* I^U *denotes the collection of all* $Q-$*fuzzy subset of* U.

3. Bipolar $Q-$fuzzy soft $\Gamma-$semiring

Definition 3.1. [10] *If* S *is* $\Gamma-$*semiring,* E *is a parameter set and* $B \subseteq E$*. If* μ *is a mapping given by* $\mu : B \to P(S)$ *where* $P(S)$ *is the power set of* S*. Then* (μ, B) *is called a soft* $\Gamma-$*semiring over* S *iff for each* $b \in B, \mu(b)$ *is* $\Gamma-$*subsemiring of* S*. That is,*

(1) *If* $s_1, s_2 \in S$ *then* $s_1 + s_2 \in \mu(b)$.
(2) *If* $s_1, s_2 \in S, \alpha \in \Gamma$ *then* $s_1 \alpha s_2 \in \mu(b)$.

Definition 3.2. *Let* U *be a universe set,* E *be the set of parameters and* $S, Q \subset E$*. define* $\mu : S \times Q \to BQF^U$ *where* BQF^U *is the collection of all bipolar* $Q-$*fuzzy subsets of* U*. Then* $(\mu, S \times Q)$ *is said to be a bipolar* $Q-$*fuzzy soft set over a universe* U*. That is,*

$$(\mu, S \times Q) = \{(a, q), \mu_s^+(a, q), \mu_s^-(a, q); s \in U, q \in Q \text{ and } s \in S\}.$$

Definition 3.3. *If* S *is a* $\Gamma-$*semiring,* E *be a parameter set and* $B, Q \subseteq E$*. If* μ *is a mapping given by* $\mu : B \times Q \to ([-1, 0] \times [0, 1])^S$ *where the collection of all bipolar* $Q-$*fuzzy subset's of* S*. Then* $(\mu, B \times Q)$ *is called a bipolar* $Q-$*fuzzy subsets of* S.

Definition 3.4. *Let* S *be a* $\Gamma-$*semiring, then* $(\mu, B \times Q)$ *is called a bipolar* $Q-$*fuzzy soft* $\Gamma-$*semiring over* S *iff for each* $b \in B, \mu(b) = \mu_b$ *is a bipolar* $Q-$*fuzzy* $\Gamma-$*semiring of* S*. This means that:*

(1) $\mu_b^+(s_1 + s_2, q) \geq \min\{\mu_b^+(s_1, q), \mu_b^+(s_2, q)\}$.
(2) $\mu_b^-(s_1 + s_2, q) \leq \max\{\mu_b^-(s_1, q), \mu_b^-(s_2, q)\}$.
(3) $\mu_b^-(s_1 \alpha s_2, q) \geq \min\{\mu_b^+(s_1, q), \mu_b^+(s_2, q)\}$.
(4) $\mu_b^-(s_1 \alpha s_2, q) \leq \max\{\mu_b^+(s_1, q), \mu_b^+(s_2, q)\}$.
For all $s_1, s_2 \in S, \alpha \in \Gamma$ *and* $q \in Q$.

Definition 3.5. [10] *If* S *is a* $\Gamma-$*semiring,* E *is a parameter set and* $B \subseteq E$*. If* μ *is a mapping given by* $\mu : B \to P(S)$*. then* (μ, B) *is called a soft right (left) ideal over* S *iff for each* $b \in B, \mu(b)$ *is a right (left) ideal of* S*. This means that :*

(1) *if* $s_1, s_2 \in \mu(b)$ *then* $s_1 + s_2 \in \mu(b)$.
(2) *if* $s_1 \in \mu(b), \alpha \in \Gamma, s \in S$ *then* $s_1 \alpha s (s \alpha s_1) \in \mu(b)$.

Definition 3.6. [10] *If* S *is a* $\Gamma-$*semiring,* E *be a parameter set and* $B \subseteq E$*. If* μ *is a mapping given by* $\mu : B \to P(S)$*. then* (μ, B) *is called a soft ideal over* S *iff for each* $b \in B, \mu(b)$ *is an ideal of* S*. This means that :*

(1) *if* $s_1, s_2 \in \mu(b)$ *then* $s_1 + s_2 \in \mu(b)$.
(2) *if* $s_1 \in \mu(b), \alpha \in \Gamma, s \in S$ *then* $s_1 \alpha s$ *and* $s \alpha s_1 \in \mu(b)$.

Definition 3.7. *Let* U *be a universe set,* E *be the set of parameters and* $S, Q \subset E$*. define* $\mu : S \times Q \to BQF^U$ *where* BQF^U *is the collection of all bipolar* $Q-$*fuzzy subsets of* U*. Then* $(\mu, S \times Q)$ *is said to be a bipolar* $Q-$*fuzzy soft set over a universe* U*. That is,*

$$(\mu, S \times Q) = \{(a, q), \mu_s^+(a, q), \mu_s^-(a, q); s \in U, q \in Q \text{ and } s \in S\}.$$

Definition 3.8. *If S is a $\Gamma-$semiring, E be a parameter set and $B, Q \subseteq E$. If μ is a mapping given by $\mu : B \times Q \to ([-1,0] \times [0,1])^S$ where the collection of all bipolar $Q-$fuzzy subset's of S. Then $(\mu, B \times Q)$ is called a bipolar $Q-$fuzzy subsets of S.*

4. Bipolar $Q-$fuzzy soft $\Gamma-$semiring homomorphism

Definition 4.1. *If (μ, B_1) and (λ, B_2) are bipolar $Q-$fuzzy soft sets over $\Gamma-$semirings R_1 and R_2 respectively. If $\Psi : R_1 \to R_2$ and $\Phi : B_1 \to B_2$ are two functions where B_1 and B_2 are parameters sets for the crisp set R_1 and R_2 respectively. Then pair (Ψ, Φ) is called a bipolar $Q-$fuzzy soft function from R_1 to R_2.*

Definition 4.2. *If (μ, B_1) and (λ, B_2) are bipolar $Q-$fuzzy soft sets over $\Gamma-$semirings R_1 and R_2 respectively and (Ψ, Φ) is bipolar $Q-$ fuzzy soft function from R_1 to R_2. Then (Ψ, Φ) is called bipolar $Q-$ fuzzy soft $\Gamma-$semiring homomorphism if the following axioms hold:*

(1) *Ψ is a $\Gamma-$semiring homomorphism from R_1 to R_2.*
(2) *Φ is a mapping from B_1 onto B_2.*
(3) *$\Psi(\mu_b) = \lambda\Phi(b)$. for all $b \in B_1$.*

Definition 4.3. *If there exists a bipolar $Q-$fuzzy soft $\Gamma-$semiring homomorphism between (μ, B_1) and (λ, B_2) bipolar $Q-$fuzzy soft semirings, we say that (μ, B_1) is soft homomorphic to (λ, B_2).*

Definition 4.4. *If (Ψ, Φ) is a bipolar $Q-$fuzzy soft function from R_1 to R_2. Then the pre-image (λ, A) under a bipolar $Q-$fuzzy soft function (Ψ, Φ) denoted by $(\Psi, \Phi)^{-1}(\lambda, A)$ defined as $(\Psi, \Phi)^{-1}(\lambda, A) = (\Psi^{-1}(\lambda), \Phi^{-1}(A)$ is a bipolar $Q-$fuzzy soft set.*

Theorem 4.1. *If (μ, B) is a bipolar $Q-$fuzzy soft $\Gamma-$semiring over a $\Gamma-$semiring R_2. And if $\Psi : R_1 \to R_2$ is eipemorphism and for each $b \in B, q \in Q$ define $(\Psi\mu)_b(x, q) = \mu_b(\Psi(x), q)$. For all $x \in R_1, q \in Q$, then $(\Psi\mu, B)$ is a bipolar $Q-$fuzzy soft $\Gamma-$semiring over R_2.*

Proof. If $x_1, x_2 \in R_1, b \in B, \gamma \in \Gamma$ and $q \in Q$. Then

$$\begin{aligned}
1.(\Psi\mu^+)_b(x_1 + x_2, q) &= \mu_b^+(\Psi(x_1 + x_2), q) \\
&= \mu_b^+((\Psi(x_1, q)) + (\Psi(x_2, q))) \\
&\geq \min\{\mu_b^+((\Psi(x_1, q)), \mu_b^+((\Psi(x_2, q))\} \\
&= \min\{((\Psi\mu^+)_b(x_1, q)), ((\Psi\mu^+)_b(x_2, q))\}. \\
2.(\Psi\mu^-)_b(x_1 + x_2, q) &= \mu_b^-(\Psi(x_1 + x_2), q) \\
&= \mu_b^-((\Psi(x_1, q)) + (\Psi(x_2, q))) \\
&\leq \max\{\mu_b^-((\Psi(x_1, q)), \mu_b^-((\Psi(x_2, q))\} \\
&= \max\{((\Psi\mu^-)_b(x_1, q)), ((\Psi\mu^-)_b(x_2, q))\}. \\
3.(\Psi\mu^+)_b(x_1\gamma x_2, q) &= \mu_b^+(\Psi(x_1\gamma x_2), q) \\
&= \mu_b^+((\Psi(x_1)\gamma\Psi(x_2)), q) \\
&\geq \min\{\mu_b^+((\Psi(x_1, q)), \mu_b^+((\Psi(x_2, q))\} \\
&= \min\{((\Psi\mu^+)_b(x_1, q)), ((\Psi\mu^+)_b(x_2, q))\}. \\
4.(\Psi\mu^-)_b(x_1\gamma x_2, q) &= \mu_b^-(\Psi(x_1\gamma x_2), q) \\
&= \mu_b^-((\Psi(x_1)\gamma\Psi(x_2)), q) \\
&\leq \max\{\mu_b^-((\Psi(x_1, q)), \mu_b^-((\Psi(x_2, q))\} \\
&= \max\{((\Psi\mu^-)_b(x_1, q)), ((\Psi\mu^-)_b(x_2, q))\}.
\end{aligned}$$

Therefore $(\Psi\mu)_b$ is a bipolar $Q-$fuzzy $\Gamma-$subsemiring of R_2. Hence $(\Psi\mu, B)$ is a bipolar $Q-$fuzzy soft $\Gamma-$semiring. □

Theorem 4.2. *If (λ, B) is a bipolar $Q-$fuzzy soft $\Gamma-$semiring over a $\Gamma-$semiring R and if Ψ is an endomorphism of R. Define $(\lambda\Psi)_b(x,q) = \lambda_b\Psi$ For all $b \in B$. Then $(\lambda\Psi, B)$ is a bipolar $Q-$fuzzy soft $\Gamma-$semiring R.*

Proof. If $x_1, x_2 \in R_1, b \in B, \gamma \in \Gamma$ and $q \in Q$. Then

$$\begin{aligned}
1.(\lambda^+\Psi)_b(x_1+x_2,q) &= \lambda_b^+(\Psi(x_1+x_2),q)\\
&= \lambda_b^+((\Psi(x_1,q))+(\Psi(x_2,q)))\\
&\geq \min\{\lambda_b^+((\Psi(x_1,q)),\lambda_b^+((\Psi(x_2,q))\}\\
&= \min\{((\lambda^+\Psi)_b(x_1,q)),((\lambda^+\Psi)_b(x_2,q))\}.\\
2.(\lambda^-\Psi)_b(x_1+x_2,q) &= \lambda_b^-(\Psi(x_1+x_2),q)\\
&= \lambda_b^-((\Psi(x_1)+\Psi(x_2)),q)\\
&\leq \max\{\lambda_b^-((\Psi(x_1,q)),\lambda_b^-((\Psi(x_2,q))\}\\
&= \max\{((\lambda^-\Psi)_b(x_1,q)),((\lambda^-\Psi)_b(x_2,q))\}.\\
3.(\lambda^+\Psi)_b(x_1\gamma x_2,q) &= \lambda_b^+(\Psi(x_1\gamma x_2),q)\\
&= \lambda_b^+((\Psi(x_1)\gamma\Psi(x_2)),q)\\
&\geq \min\{\lambda_b^+((\Psi(x_1,q)),\lambda_b^+((\Psi(x_2,q))\}\\
&= \min\{((\lambda^+\Psi)_b(x_1,q)),((\lambda^+\Psi)_b(x_2,q))\}.\\
4.(\lambda^-\Psi)_b(x_1\gamma x_2,q) &= \lambda_b^-(\Psi(x_1\gamma x_2),q)\\
&= \lambda_b^-((\Psi(x_1)\gamma\Psi(x_2)),q)\\
&\leq \max\{\lambda_b^-((\Psi(x_1,q)),\lambda_b^-((\Psi(x_2,q))\}\\
&= \max\{((\lambda^-\Psi)_b(x_1,q)),((\lambda^-\Psi)_b(x_2,q))\}.
\end{aligned}$$

Thus $(\lambda\Psi)_b$ is a bipolar $Q-$fuzzy $\Gamma-$subsemiring of R and hence $(\lambda\Psi, B)$ is a bipolar $Q-$fuzzy soft $\Gamma-$semiring over R. □

Theorem 4.3. *Let $\Psi : R_1 \to R_2$ be an epimorphism of $\Gamma-$semirings and (λ, B) be a bipolar $Q-$fuzzy soft right ideal over $\Gamma-$semiring R_2 if for each $b \in B, \mu_b = \Psi^{-1}(\lambda_b)$ then (μ, B) is a bipolar $Q-$fuzzy soft right ideal over $\Gamma-$semiring R_1.*

Proof. If $b \in B, q \in Q$ and $\gamma \in \Gamma$. Then λ_b is a bipolar $Q-$fuzzy soft right ideal over $\Gamma-$semiring R_2. Take $x_1, x_2 \in R, q \in Q, \gamma \in \gamma$. Then

$$\begin{aligned}
1.\Psi^{-1}(\lambda_b^+)(x_1+x_2,q) &= \lambda_b^+(\Psi(x_1+x_2),q)\\
&= \lambda_b^+((\Psi(x_1)+\Psi(x_2),q))\\
&\geq \min\{\lambda_b^+(\Psi(x_1,q)),\lambda_b^+(\Psi(x_2,q))\}\\
&= \min\{\Psi^{-1}(\lambda_b^+)(x_1,q),\Psi^{-1}(\lambda_b^+)(x_2,q)\}.\\
2.\Psi^{-1}(\lambda_b^-)(x_1+x_2,q) &= \lambda_b^-(\Psi(x_1+x_2),q)\\
&= \lambda_b^-((\Psi(x_1)+\Psi(x_2),q))\\
&\leq \max\{\lambda_b^-(\Psi(x_1,q)),\lambda_b^-(\Psi(x_2,q))\}\\
&= \max\{\Psi^{-1}(\lambda_b^-)(x_1,q),\Psi^{-1}(\lambda_b^-)(x_2,q)\}.\\
3.\Psi^{-1}(\lambda_b^+)(x_1\gamma x_2,q) &= \lambda_b^+(\Psi(x_1\gamma x_2),q)\\
&= \lambda_b^+((\Psi(x_1)\gamma\Psi(x_2),q))\\
&\geq \lambda_b^+(\Psi(x_1,q))\\
&= \Psi^{-1}(\lambda_b^+)(x_1,q).\\
4.\Psi^{-1}(\lambda_b^-)(x_1\gamma x_2,q) &= \lambda_b^-(\Psi(x_1\gamma x_2),q)\\
&= \lambda_b^-((\Psi(x_1)\gamma\Psi(x_2),q))\\
&\leq \lambda_b^-(\Psi(x_1,q))\\
&= \Psi^{-1}(\lambda_b^-)(x_1,q).
\end{aligned}$$

Hence $\mu_b = \Psi^{-1}(\lambda_b)$ is a bipolar $Q-$fuzzy soft right ideal of $\Gamma-$semiring R_1. Thus (μ, B) is a bipolar $Q-$fuzzy soft right ideal over $\Gamma-$semiring R_1. □

Corollary 4.1. *If $\Psi : R_1 \to R_2$ is an epimorphism of $\Gamma-$semirings (λ, B) is a bipolar $Q-$fuzzy left ideal over $\Gamma-$semiring R_2. If for each $b \in B, \mu_b = \Psi^{-1}(\lambda_b)$ then (μ, B) is a bipolar $Q-$fuzzy soft right ideal over $\Gamma-$semiring R_1.*

Proof. Straightforward. □

Proposition 4.1. *If R_1 and R_2 are $\Gamma-$semirings, $\Psi : R_1 \to R_2$ is a $\Gamma-$semiring-homomorphism and f is a ϕ invariant bipolar $Q-$fuzzy subset of R_1. If $(x, q) = (\Psi(y), q)$ then $\Psi(f)(x, q) = (f(y), q); y \in R_1$.*

Proof. Assume that $y \in R_1$, $(x, q) = (\Psi(y), q)$ and f is a ϕ invariant bipolar $Q-$fuzzy ideal of R_1. Then $(\Psi^{-1}(x), q) = (y, q)$. Let $(z, q) \in (\Psi^{-1}(x), q)$ then $(\Psi(z), q) = (x, q) = (\Psi(y), q)$, hence f is a ϕ invariant bipolar $Q-$fuzzy subset of $R_1, (f(z), q) = (f(y), q)$. Therefore $\Psi(f)(x, q) = \sup_{(z,q)\in(\Psi^{-1}(x),q)}(f(z), q) = (f(y), q)$ and hence $\Psi(f)(x, q) = (f(y), q)$. □

Theorem 4.4. *Let (λ, β) be a bipolar $Q-$fuzzy soft right ideal over $\Gamma-$semiring R_1 and Ψ be a homomorphism from R_1 onto R_2. For each $z \in \beta, \lambda_z$ is a ϕ invariant bipolar $Q-$fuzzy right ideal of R_1, if $\mu_z = \Psi(\lambda_z), z \in \beta$ then (μ, β) is a bipolar $Q-$fuzzy soft right ideal over $\Gamma-$semiring R_2.*

Proof. Let $y_1, y_2 \in R_2, z \in \beta, \gamma \in \Gamma$ and $q \in Q$. Then there exist $x_1, x_2 \in R_1$ such that $\Psi(x_1) = y_1, \Psi(x_2) = y_2, y_1 + y_2 = \Psi(x_1 + x_2), y_1\gamma y_2 = \Psi(x_1\gamma x_2)$ and since λ_z is $\phi-$invariant and by Proposition 4.1, we have:

$$\begin{aligned}
1.\mu_z^+(y_1 + y_2, q) &= \Psi(\lambda_z^+)(y_1 + y_2, q)\\
&= \lambda_z^+(x_1 + x_2, q)\\
&\geq \min\{\lambda_z^+(x_1, q), \lambda_z^+(x_2, q)\}\\
&= \min\{\Psi(\lambda_z^+)(y_1, q), \Psi(\lambda_z^+)(y_2, q)\}\\
&= \min\{\mu_z^+(y_1, q), \mu_z^+(y_2, q)\}.\\
2.\mu_z^-(y_1 + y_2, q) &= \Psi(\lambda_z^-)(y_1 + y_2, q)\\
&= \lambda_z^-(x_1 + x_2, q)\\
&\leq \max\{\lambda_z^-(x_1, q), \lambda_z^-(x_2, q)\}\\
&= \max\{\Psi(\lambda_z^-)(y_1, q), \Psi(\lambda_z^-)(y_2, q)\}\\
&= \max\{\mu_z^-(y_1, q), \mu_z^-(y_2, q)\}.\\
3.\mu_z^+(y_1\gamma y_2, q) &= \Psi(\lambda_z^+)(y_1\gamma y_2, q)\\
&= \lambda_z^+(\Psi(x_1\gamma x_2), q)\\
&= \lambda_z^+(\Psi(x_1)\gamma\Psi(x_2), q)\\
&\geq \lambda_z^+(x_1, q)\\
&= \Psi(\lambda_z^+)(y_1, q)\\
&= \mu_z^+(y_1, q).\\
4.\mu_z^-(y_1\gamma y_2, q) &= \Psi(\lambda_z^-)(y_1\gamma y_2, q)\\
&= \lambda_z^-(\Psi(x_1\gamma x_2), q)\\
&= \lambda_z^-(\Psi(x_1)\gamma\Psi(x_2), q)\\
&\leq \lambda_z^-(x_1, q)\\
&= \Psi(\lambda_z^-)(y_1, q)\\
&= \mu_z^-(y_1, q).
\end{aligned}$$

We get μ_z is a bipolar $Q-$fuzzy right ideal of R_2. Thus (μ, β) is a bipolar $Q-$fuzzy soft right ideal over R_2. □

Corollary 4.2. *If (λ, β) is a bipolar $Q-$fuzzy soft left ideal over $\Gamma-$semiring R_1 and Ψ is a homomorphism from R_1 onto R_2. For each $z \in \beta, \lambda_z$ is a $\phi-$invariant bipolar $Q-$fuzzy left ideal of R_1, if $\mu_z = \Psi(\lambda_z), z \in \beta$, then (μ, β) is a bipolar $Q-$fuzzy soft left ideal over $\Gamma-$semiring R_2.*

Proof. Straightfoward. □

Theorem 4.5. *If $(\lambda,\beta_1),(\mu,\beta_2)$ are bipolar $Q-$fuzzy soft $\Gamma-$semirings over R_1 and R_2 respectively, and (Ψ,ϕ) is a bipolar $Q-$fuzzy soft $\Gamma-$semiring homomorphism from (λ,β_1) onto (μ,β_2). Then $(\Psi(\mu),\beta_2)$ is a bipolar $Q-$fuzzy soft $\Gamma-$semiring over $\Gamma-$semiring R_2.*

Proof. By Definition 4.2, Ψ is a $\Gamma-$semiring homomorphism from R_1 onto R_2 and ϕ is a mapping from β_1 onto β_2. For each $b_2 \in \beta_2$ then there exist $b_1 \in \beta_1$ such that $\phi(b_1) = b_2$. Define $(\Psi(\lambda))_{b_2} = \Psi(\lambda_{b_1})$. Let $y_1, y_2 \in R_2, q \in Q$ and $\gamma \in \Gamma$. Then there exist $x_1, x_2 \in R$ such that $\Psi(x_1) = y_1, \Psi(x_2) = y_2, \Psi(x_1 + x_2) = y_1 + y_2$ and $\Psi(x_1\gamma x_2) = y_1\gamma y_2$. Thus

$$\begin{aligned}
1.(\Psi(\lambda))^+_{\phi(b_1)}(y_1+y_2,q) &= \Psi^+(\lambda_{b_1})(y_1+y_2,q)\\
&= \lambda^+_{b_1}(x_1+x_2,q)\\
&\geq \min\{\lambda^+_{b_1}(x_1,q),\lambda^+_{b_1}(x_2,q)\}\\
&= \min\{\Psi^+(\lambda_{b_1})(y_1,q),\Psi^+(\lambda_{b_1})(y_2,q)\}\\
&= \min\{(\Psi(\lambda))^+_{\phi(b_1)}(y_1,q),(\Psi(\lambda))^+_{\phi(b_1)}(y_2,q)\}.\\
2.(\Psi(\lambda))^-_{\phi(b_1)}(y_1+y_2,q) &= \Psi^-(\lambda_{b_1})(y_1+y_2,q)\\
&= \lambda^-_{b_1}(x_1+x_2,q)\\
&\leq \max\{\lambda^-_{b_1}(x_1,q),\lambda^-_{b_1}(x_2,q)\}\\
&= \max\{\Psi^-(\lambda_{b_1})(y_1,q),\Psi^-(\lambda_{b_1})(y_2,q)\}\\
&= \max\{\Psi^-(\lambda)_{\phi(b_1)}(y_1,q),\Psi^-(\lambda)_{\phi(b_1)}(y_2,q)\}.\\
3.(\Psi(\lambda))^+_{\phi(b_1)}(y_1\gamma y_2,q) &= \Psi^+(\lambda_{b_1})(y_1\gamma y_2,q)\\
&= \lambda^+_{b_1}(x_1\gamma x_2,q)\\
&\geq \min\{\lambda^+_{b_1}(x_1,q),\lambda^+_{b_1}(x_2,q)\}\\
&= \min\{\Psi^+(\lambda_{b_1})(y_1,q),\Psi^+(\lambda_{b_1})(y_2,q)\}\\
&= \min\{\Psi^+(\lambda)_{\phi(b_1)}(y_1,q),\Psi^+(\lambda)_{\phi(b_1)}(y_2,q)\}.\\
4.(\Psi(\lambda))^-_{\phi(b_1)}(y_1\gamma y_2,q) &= \Psi^-(\lambda_{b_1})(y_1\gamma y_2,q)\\
&= \lambda^-_{b_1}(x_1\gamma x_2,q)\\
&\leq \max\{\lambda^-_{b_1}(x_1,q),\lambda^-_{b_1}(x_2,q)\}\\
&= \max\{\Psi^-(\lambda_{b_1})(y_1,q),\Psi^-(\lambda_{b_1})(y_2,q)\}\\
&= \max\{\Psi^-(\lambda)_{\phi(b_1)}(y_1,q),\Psi^-(\lambda)_{\phi(b_1)}(y_2,q)\}.
\end{aligned}$$

Then we get $\Psi(\lambda)_{b_2}$ is a bipolar $Q-$fuzzy soft $\Gamma-$subsemiring of R_2. Thus $(\Psi(\lambda),\beta_2)$ is a bipolar $Q-$fuzzy soft $\Gamma-$semiring over $\Gamma-$semiring R_2. □

Definition 4.5. *If $(\lambda,\beta_1),(\mu,\beta_2)$ are bipolar $Q-$fuzzy soft $\Gamma-$semirings over a semiring R. Then (λ,β_1) is a bipolar $Q-$fuzzy soft $\Gamma-$subsemiring of (μ,β_2) if it satisfies the following axioms:*

(1) $\beta_1 \subseteq \beta_2$
(2) λ_b *is a bipolar $Q-$fuzzy soft $\Gamma-$subsemirings of μ_b for all $b \in \beta_1$.*

Proposition 4.2. *If $(\lambda,\beta_1),(\mu,\beta_2)$ are two bipolar $Q-$fuzzy soft $\Gamma-$semirings over R. Then the following statements are hold:*

(1) *If $\mu_{b_2} \subset \lambda_{b_2}$ for all $b_2 \in \beta_2 \subset \beta_1$, then (μ,β_2) is a bipolar $Q-$fuzzy soft $\Gamma-$subsemiring of (λ,β_1).*
(2) *$(\lambda,\beta_1) \cap (\mu,\beta_2)$ is a bipolar $Q-$fuzzy soft $\Gamma-$subsemiring of (λ,β_1) and (μ,β_2) if it is non null.*

Proof. (1) By Definition 4.5, (μ,β_2) is a bipolar $Q-$fuzzy soft $\Gamma-$subsemiring of (λ,β_1).
(2) Suppose that $(\lambda,\beta_1) \cap (\mu,\beta_2) = (\delta,\beta_3)$ such that $\beta_3 = \beta_1 \cap \beta_2$. Thus (δ,β_3) is a bipolar $Q-$fuzzy soft $\Gamma-$semiring over R, on the other hand $\beta_3 = \beta_1 \cap \beta_2 \subset \beta_1$

and $\beta_3 \subset \beta_2$, by Definition 4.5, (δ, β_3) is a bipolar $Q-$fuzzy soft $\Gamma-$subsemiring of (λ, β_1) as well as (μ, β_2).

□

Proposition 4.3. *If (λ, β_1) and (μ, β_2) are two bipolar $Q-$fuzzy soft $\Gamma-$semirings over R and (λ, β_1) is a bipolar $Q-$fuzzy soft $\Gamma-$subsemiring of (μ, β_2). If $\Psi : R_1 \to R_2$ is homomorphism of $\Gamma-$semiring then:*

(1) *$(\Psi(\lambda), \beta_1)$ and $(\Psi(\mu), \beta_2)$ are bipolar $Q-$fuzzy soft $\Gamma-$subsemiring over R_2.*
(2) *$(\Psi(\lambda), \beta_1)$ is a bipolar $Q-$fuzzy soft $\Gamma-$subsemiring of $(\Psi(\mu), \beta_2)$.*

Proof. (1) $(\Psi(\lambda))_{b_1} = \Psi(\lambda_{b_1})$ is a bipolar $Q-$fuzzy soft $\Gamma-$subsemiring of R_2 for all $b_1 \in \beta_1$, also $(\Psi(\mu))_{b_2} = \Psi(\mu_{b_2})$ is a bipolar $Q-$fuzzy soft $\Gamma-$subsemiring of R_2 for all $b_2 \in \beta_2$. Hence $(\Psi(\lambda), \beta_1)$ and $(\Psi(\mu), \beta_2)$ are bipolar $Q-$fuzzy soft $\Gamma-$subsemiring over R_2.

(2) We know that (λ, β_1) is a bipolar $Q-$fuzzy soft $\Gamma-$subsemiring of $(\mu, \beta_2), \lambda_{b_1}$ is a bipolar $Q-$fuzzy $\Gamma-$subsemiring of μ_{b_1}. Therefore $\Psi(\lambda_{b_1})$ is a bipolar $Q-$fuzzy $\Gamma-$subsemiring of $\Psi(\mu_{b_1})$ for all $b_1 \in \beta_1$. Thus $(\Psi(\lambda), \beta_1)$ is a bipolar $Q-$fuzzy soft $\Gamma-$subsemiring of $(\Psi(\mu), \beta_2)$.

□

Theorem 4.6. *If (λ, β_1) and (μ, β_2) are two bipolar $Q-$fuzzy soft $\Gamma-$semirings over R_1 and R_2 respectively and (Ψ, ϕ) is a bipolar $Q-$fuzzy soft homomorphism from (λ, β_1) onto (μ, β_2), then the pre-image of (μ, β_2) under bipolar $Q-$fuzzy soft $\Gamma-$semiring homomorphism (Ψ, ϕ) is a bipolar $Q-$fuzzy soft $\Gamma-$semiring of (λ, β_1) over R_1.*

Proof. By Definition 4.4, $(\Psi, \Phi)^{-1}(\mu, \beta_2) = (\Psi^{-1}(\mu), \Phi^{-1}(\beta_2))$. Define $(\Psi^{-1}(\mu))_{b_1}(x, q) = \mu\Phi(b_1)(\Psi(x), q)$ for all $x \in R_1, q \in Q$ and $b_1 \in \Phi^{-1}(\beta_2)$. Take $x, y \in R_1, q \in Q$ and $\gamma \in \Gamma$. Then

$$
\begin{aligned}
1.(\Psi^{-1}(\mu^+))_{b_1}(x+y, q) &= \mu^+\Phi(b_1)(\Psi(x+y), q) \\
&= \mu^+\Phi(b_1)(\Psi(x) + \Psi(y), q) \\
&\geq \min\{\mu^+\Phi(b_1)(\Psi(x), q), \mu^+\Phi(b_1)(\Psi(y), q)\} \\
&= \min\{(\Psi^{-1}(\mu^+))_{b_1}(x, q), (\Psi^{-1}(\mu^+))_{b_1}(y, q)\}. \\
2.(\Psi^{-1}(\mu^-))_{b_1}(x+y, q) &= \mu^-\Phi(b_1)(\Psi(x+y), q) \\
&= \mu^-\Phi(b_1)(\Psi(x) + \Psi(y), q) \\
&\leq \max\{\mu^-\Phi(b_1)(\Psi(x), q), \mu^-\Phi(b_1)(\Psi(y), q)\} \\
&= \max\{(\Psi^{-1}(\mu^-))_{b_1}(x, q), (\Psi^{-1}(\mu^-))_{b_1}(y, q)\}. \\
3.(\Psi^{-1}(\mu^+))_{b_1}(x\gamma y, q) &= \mu^+\Phi(b_1)(\Psi(x\gamma y), q) \\
&= \mu^+\Phi(b_1)(\Psi(x)\gamma\Psi(y), q) \\
&\geq \min\{\mu^+\Phi(b_1)(\Psi(x), q), \mu^+\Phi(b_1)(\Psi(y), q)\} \\
&= \min\{(\Psi^{-1}(\mu^+))_{b_1}(x, q), (\Psi^{-1}(\mu^+))_{b_1}(y, q)\}. \\
4.(\Psi^{-1}(\mu^-))_{b_1}(x\gamma y, q) &= \mu^-\Phi(b_1)(\Psi(x\gamma y), q) \\
&= \mu^-\Phi(b_1)(\Psi(x)\gamma\Psi(y), q) \\
&\leq \max\{\mu^-\Phi(b_1)(\Psi(x), q), \mu^-\Phi(b_1)(\Psi(y), q)\} \\
&= \max\{(\Psi^{-1}(\mu^-))_{b_1}(x, q), (\Psi^{-1}(\mu^-))_{b_1}(y, q)\}.
\end{aligned}
$$

Thus $(\Psi^{-1}(\mu))_{b_1}$ is a bipolar $Q-$fuzzy $\Gamma-$subsemiring of R_1 for all $b_1 \in \Phi^{-1}(\beta_2)$. Hence $(\Psi^{-1}(\mu), \Phi^{-1}(\beta))$ is a bipolar $Q-$fuzzy soft $\Gamma-$subsemiring of (λ, β_1) over R_1. □

5. Conclusion

In this paper, the concept of bipolar $Q-$fuzzy soft $\Gamma-$semiring homomorphism and some properties of homomorphic image of bipolar $Q-$fuzzy soft $\Gamma-$semiring are discussed. In our future work, we will study the concept and properties of bipolar $Q-$fuzzy soft prime ideals, bipolar $Q-$fuzzy soft integral domaine over $\Gamma-$semirings and kernel of bipolar $Q-$fuzzy soft $\Gamma-$semiring.

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Oqlah Al-Refai for the photography and short autobiography, see TWMS J. App. Eng. Math., current issue, pp. 250-261