TWMS J. App. and Eng. Math. V.11, N.2, 2021, pp. 386-394

# NONLINEAR DARK SOLITARY SH WAVES IN A HETEROGENEOUS LAYER

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Abstract. In this study, we consider the nonlinear propagation of shear horizontal (SH) waves in a layer of finite thickness. The materials of the layer are assumed to be heterogeneous, isotropic, and generalized neo-Hookean. We assume that heterogeneity varies only with the thickness and we choose hyperbolic functions for heterogeneity type. We also assume that the traction is free on the upper surface of the layer. Furthermore, the lower boundary is rigidly fixed. Using a perturbation method and keeping the balance of the nonlinearity and the dispersion in the analysis, we show that the self-modulation of nonlinear SH waves can be given by the nonlinear Schrödinger (NLS) equation. Using well known solutions of NLS equation, we find that the dark solitary SH waves can exist depending on the nonlinear constitution of the layer. Consequently, the effects of the heterogeneity and the nonlinearity on the deformation field are considered for these waves.

Keywords: Nonlinear SH waves, dark solitary waves, rigid substratum, heterogeneity.

AMS Subject Classification: 35-XX, 35B20, 35Q55.

## 1. Introduction

In seismology we see some examples of the seismic waves such as body waves, surface waves, Love waves, and Rayleigh waves. In this study, we consider shear waves which are one type of body waves. If shear waves are polarized horizontally, then such waves are called shear horizontal (SH) waves. In general, the seismic waves are not dispersive in an unbounded homogeneous medium. Using the wave guides it is possible to obtain dispersive waves. Seismology, geophysics, and electronic signal processing devices are some examples of application areas for dispersive elastic waves. More information about applications and for reviews, we refer to [1] and references therein.

This work includes nonlinear elastic waves in a heterogeneous medium, so we need to overview some nonlinear works such as from [1] to [10]. In these works, the effect of constitutional nonlinearity on the propagation characteristics of dispersive elastic waves are studied in a homogeneous medium. Furthermore, for some linear works in a heterogeneous medium we refer to from [11] to [13]. In [14], we investigate nonlinear bright solitary

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§ Manuscript received: June 26, 2019; accepted: September 10, 2019.

SH waves in a hyperbolically heterogeneous layer; whereas, in this paper we investigate nonlinear dark solitary SH waves in a hyperbolically heterogeneous layer. Moreover, In [15, 16, 17], we consider nonlinear SH waves in a plate. Plate case may be considered a little bit an improved version of a layer in terms of geometry, i.e. a layer may be though as a half of a plate. Due to different boundary conditions, we need to consider as different problems.

The present work searches for the propagation of nonlinear SH waves in a heterogeneous, isotropic, and generalized neo-Hookean layer overlying a rigid substratum. We assume that heterogeneity varies with the thickness and we choose hyperbolic function of one variable as a heterogeneity type. Moreover, the traction is free on the upper surface of the layer and the lower boundary is rigidly fixed. Applying the method of multiple scales and striking a balance between the weak nonlinearity and the dispersion in the asymptotic analysis, we derive a nonlinear Schrödinger (NLS) equation describing a self-modulation of nonlinear SH waves. We claim that the existence of dark solitary SH waves depends on the nonlinear constitution of the layer, and consider the effects of the heterogeneity and the nonlinearity on the propagation characteristics of SH waves via NLS equation. Furthermore, the effects of the heterogeneity and the nonlinearity on the deformation field are considered for these waves.

## 2. Formulation of the Problem

The work [14] can be seen for the anti-plane motion of the considered problem. Our aim is to overcome with the propagation of nonlinear SH waves, so it is necessary to define SH wave such that

$$x = X,\ y = Y,\ z = Z + u(X, Y, t). \tag{1}$$

Here, $(x, y, z)$ and $(X, Y, Z)$ are the spatial and material coordinates respectively, $u = u(X, Y, t)$ is the displacement in the $Z$-direction, and $t$ is the time. Furthermore, the thickness of this layer is $h$, $X$-axis is for the propagation of the waves, and $Z$-axis is for the motion of the particle. The layer is in the region between the planes $Y = 0$ and $Y = h$, and also the semi-infinite substratum occupies the region $Y < 0$. The displacement in the $Z$-direction is zero at the rigid boundary $Y = 0$ and the boundary $Y = h$ is assumed to be free of traction. Moreover, we consider heterogeneous, isotropic, and generalized neo-Hookean materials in such a layer. Under some restriction for more detail see [14], the governing equation and the boundary conditions can be written as

$$\begin{aligned}&\frac{\partial^2 u}{\partial t^2} - c_T^2\left(\frac{\partial^2 u}{\partial X^2} + \frac{\partial^2 u}{\partial Y^2}\right) - \frac{1}{\rho}\frac{\partial(\rho c_T^2)}{\partial Y}\frac{\partial u}{\partial Y} = \\ &n_T\left[\frac{\partial}{\partial X}\left(\frac{\partial u}{\partial X}\mathcal{K}(u)\right) + \frac{\partial}{\partial Y}\left(\frac{\partial u}{\partial Y}\mathcal{K}(u)\right)\right] + \frac{\mathcal{K}(u)}{\rho}\frac{\partial(\rho n_T)}{\partial Y}\frac{\partial u}{\partial Y},\end{aligned} \tag{2}$$

$$\frac{\partial u}{\partial Y} + \frac{n_T}{c_T^2}\mathcal{K}(u)\frac{\partial u}{\partial Y} = 0 \quad \text{on} \quad Y = h \quad \text{and} \quad u = 0 \quad \text{on} \quad Y = 0. \tag{3}$$

Here, the linear shear wave velocity and the nonlinear material function can be denoted by $c_T = \sqrt{\mu/\rho}$ and $n_T$ respectively, and $\mathcal{K}$ can be defined by

$$\mathcal{K}(u) = (\partial u/\partial X)^2 + (\partial u/\partial Y)^2. \tag{4}$$

Because of the homogeneity, we note that the functions $\mu$, $\rho$ and $n_T$ are constants in [10], but here they are not. The constituent material of the layer softens in shear if $n_T < 0$, but if $n_T > 0$ it hardens. In this study, $n_T$ indicates a continuously differentiable function of $Y$, and also because of heterogeneity we need to put the following conditions on the functions $\mu$ and $\rho$

$$\mu = \mu_0 \cosh^2(\alpha Y), \quad \rho = \rho_0 \cosh^2(\alpha Y). \tag{5}$$

Here, $\mu_0$ and $\rho_0$ are constants, and $\alpha$ is a parameter. We choose condition (5) for simplicity. In this paper, we consider not only heterogeneity but also nonlinearity. Therefore, it is not easy to overcome with two difficulties.

## 3. Nonlinear Self-Modulation of SH Waves

The method of multiple scales [18] is a good method for solving such a nonlinear problem. For applying the method, it is necessary to introduce new variables as

$$x_i = \varepsilon^i X, \quad t_i = \varepsilon^i t, \quad y = Y, \quad i = 0, 1, 2. \tag{6}$$

Here, $x_1, x_2, t_1, t_2$ indicate the slow variables, $x_0, y, t_0$ indicate the fast variables, and $\varepsilon$ indicates a small positive parameter. Then, $u$ can be considered as a function of (6), and $u$ can be expanded in the following asymptotic series in $\varepsilon$;

$$u = \sum_{n=1}^{\infty} \varepsilon^n u_n(x_0, x_1, x_2, y, t_0, t_1, t_2). \tag{7}$$

If we plug this asymptotic series with (6) into equation (2) and the boundary conditions (3), then we can get a system of equations and boundary conditions for determining $u_n$ successively. Up to the third-order in $\varepsilon$ they can be written as follows:

$$\mathcal{O}(\varepsilon): \qquad \mathcal{L}_0 u_1 = 0, \tag{8}$$

$$\frac{\partial u_1}{\partial y} = 0 \quad \text{on} \quad y = h \quad \text{and} \quad u_1 = 0 \quad \text{on} \quad y = 0; \tag{9}$$

$$\mathcal{O}(\varepsilon^2): \qquad \mathcal{L}_0 u_2 = \mathcal{L}_1 u_1, \tag{10}$$

$$\frac{\partial u_2}{\partial y} = 0 \quad \text{on} \quad y = h \quad \text{and} \quad u_2 = 0 \quad \text{on} \quad y = 0; \tag{11}$$

$$\mathcal{O}(\varepsilon^3): \qquad \mathcal{L}_0 u_3 = \mathcal{L}_1 u_2 + \mathcal{L}_2 u_1 + \mathcal{N} u_1, \tag{12}$$

$$\frac{\partial u_3}{\partial y} = 0 \quad \text{on} \quad y = h \quad \text{and} \quad u_3 = 0 \quad \text{on} \quad y = 0; \tag{13}$$

The differential operators $\mathcal{L}_0, \mathcal{L}_1, \mathcal{L}_2, \mathcal{K}$, and $\mathcal{N}$ can be defined by

$$\begin{aligned}
\mathcal{L}_0 &= \frac{\partial^2}{\partial t_0^2} - c_T^2 \left( \frac{\partial^2}{\partial x_0^2} + \frac{\partial^2}{\partial y^2} \right) - \frac{\mu'}{\rho} \frac{\partial}{\partial y}, \\
\mathcal{L}_1 &= 2 \left( c_T^2 \frac{\partial^2}{\partial x_0 \partial x_1} - \frac{\partial^2}{\partial t_0 \partial t_1} \right), \\
\mathcal{L}_2 &= c_T^2 \left( \frac{\partial^2}{\partial x_1^2} + 2 \frac{\partial^2}{\partial x_0 \partial x_2} \right) - \frac{\partial^2}{\partial t_1^2} - 2 \frac{\partial^2}{\partial t_0 \partial t_2}, \\
\mathcal{K} &= \left( \frac{\partial}{\partial x_0} \right)^2 + \left( \frac{\partial}{\partial y} \right)^2, \\
\mathcal{N} &= \frac{1}{\rho} \left[ \frac{\partial}{\partial x_0} \left( \rho n_T \mathcal{K} \frac{\partial}{\partial x_0} \right) + \frac{\partial}{\partial y} \left( \rho n_T \mathcal{K} \frac{\partial}{\partial y} \right) \right].
\end{aligned} \tag{14}$$

We emphasize that it is possible to reduce the obtained order problems to the order problems [10]. We also emphasize that the obtained order problems are linear at each step and the solution of the linear case of this problem is identical with the solution of the first-order problem. Furthermore, we observe that the main analysis is same as the analysis in [14], but the results are different. So, we consider only the first-order perturbation problem here, let

$$u_1 = \mu_0^{-\frac{1}{2}} \mathrm{sech}(\alpha y) \sum_{l=1}^{\infty} \left[ A_1^{(l)} e^{ilks_l y} + B_1^{(l)} e^{-ilks_l y} \right] e^{il\phi} + \text{c.c.} \tag{15}$$

be for the solution of equation (8) where

$$s_l = [(c^2/c_{0_T}^2) - (\alpha^2/(k^2l^2)) - 1]^{1/2}, \quad \phi = kx_0 - \omega t_0, \quad c_{0_T} = \sqrt{\mu_0/\rho_0}, \tag{16}$$

$k$, $\omega$, and $c = \omega/k$ indicate the wave number, the angular frequency, and the phase velocity respectively. Moreover, $l$ indicates a positive integer, c.c. indicates the complex conjugate to the preceding terms, and also $A_1^{(l)}$ and $B_1^{(l)}$ indicate the first-order amplitude functions of wave propagation depending on the slow variables $x_1, x_2, t_1$, and $t_2$. The case $c > c_{0_T}[1 + (\alpha^2/(k^2l^2))]^{1/2}$ is for the validity of the analysis; otherwise, the propagation of SH waves is not possible in this layer. Plugging the form of the solution (15) into the boundary conditions (9) yields

$$\mathbf{W}_l \mathbf{U}_1^{(l)} = \mathbf{0} \tag{17}$$

where $\mathbf{W}_l$ denotes the dispersion matrix

$$\mathbf{W}_l = \begin{bmatrix} (-\alpha \tanh(\alpha h) + ilks_l)\, e^{ilks_l h} & (-\alpha \tanh(\alpha h) - ilks_l)\, e^{-ilks_l h} \\ 1 & 1 \end{bmatrix} \tag{18}$$

and

$$\mathbf{U}_1^{(l)} = [\ A_1^{(l)},\quad B_1^{(l)}\ ]^T. \tag{19}$$

For this paper the nonlinear self-modulation is enough to talk, and the harmonic-resonance phenomena is not included, so the following condition may be assumed

$$\det \mathbf{W}_l \neq 0 \quad \text{for} \quad l \neq 1. \tag{20}$$

To obtain the nontrivial solution of system (17) for $l = 1$, the condition $\det\mathbf{W}_1 = 0$ is a must step. Then, using the condition $\det\mathbf{W}_1 = 0$ we can find the dispersion relation as follows:

$$kp\cos(kph) - \alpha\tanh(\alpha h)\sin(kph) = 0 \tag{21}$$

where $p = s_1$. Therefore,

$$kph = \arctan\left[kp/(\alpha \tanh(\alpha h))\right] + n\pi. \tag{22}$$

Here, the branches of the dispersion relation can be denoted by $n$ ($n = 0, 1, 2, ...$). We observe that dispersion relation (22) as $\alpha \to 0$ is the same as the dispersion relation of [10]. Under the consideration above, we give the solution of system (17) as

$$\begin{aligned} \mathbf{U}_1^{(l)} &= \mathcal{A}_1 \mathbf{R} \quad \text{for} \quad l = 1 \quad \text{and} \\ \mathbf{U}_1^{(l)} &= \mathbf{0} \quad \text{for} \quad l \neq 1 \end{aligned} \tag{23}$$

where $\mathcal{A}_1$ is a complex function of the slow variables and $\mathbf{R}$ is a vector satisfying

$$\mathbf{W}_1 \mathbf{R} = \mathbf{0}. \tag{24}$$

Using (24), we get the vector $\mathbf{R}$, as

$$\mathbf{R} = [R_1,\ R_2]^T = \left[1, \frac{-\alpha \tanh(\alpha h) + ikp}{\alpha \tanh(\alpha h) + ikp} e^{2ikph}\right]^T. \tag{25}$$

Then, the first-order solution can be given by

$$u_1 = \mu_0^{-\frac{1}{2}} \mathcal{A}_1 \operatorname{sech}(\alpha y)(R_1 e^{ikpy} + R_2 e^{-ikpy}) e^{i\phi} + \text{c.c.}. \tag{26}$$

For solution (26), it is necessary to find the function $\mathcal{A}_1$, explicitly. Using the compatibility condition of the second-order problem we can find the function $\mathcal{A}_1$ such that $\mathcal{A}_1 = \mathcal{A}_1(x_1 - V_g t_1, x_2, t_2)$, but it is not clear exactly. Moreover, using the third-order problem and defining the following nondimensional variables and constants

$$\tau = \omega t_2, \quad \xi = k(x_1 - V_g t_1), \quad \mathcal{A} = k\mathcal{A}_1, \quad \Gamma = k^2\tilde{\Gamma}/\omega, \quad \Delta = \tilde{\Delta}/\omega k^2 \tag{27}$$

then we get nonlinear Schrödinger (NLS) equation as follows:

$$i\frac{\partial \mathcal{A}}{\partial \tau} + \Gamma \frac{\partial^2 \mathcal{A}}{\partial \xi^2} + \Delta |\mathcal{A}|^2 \mathcal{A} = 0 \tag{28}$$

where

$$\tilde{\Gamma} = \frac{1}{2}\frac{d^2\omega}{dk^2} \quad \text{and} \quad \tilde{\Delta} = -\frac{\mathbf{L}\cdot\mathbf{F}}{\mathbf{L}\frac{\partial \mathbf{W}_1}{\partial \omega}\mathbf{R}}. \tag{29}$$

Here, we define the vectors $\mathbf{L}$ and $\mathbf{F}$ in [14] and the analysis is same as the work [14], so for more detail we refer to [14]. Using known properties of solutions of the NLS equation in the following section, we can discuss the existence of dark solitary SH waves in such a layer.

## 4. Conclusion

In wave propagation, the NLS equation is a famous characteristic equation. We know that the sign of $\Gamma\Delta$ behaves as a criteria for determining how a given initial data will evolve for long times for the asymptotic wave field governed by the NLS equation. For $\Gamma\Delta < 0$, a solution of the form

$$\mathcal{A}(\xi,\tau) = \phi(\eta) e^{i[\Gamma^2\Delta\phi_0^2\tau - F(\eta)]}, \quad \eta = \xi - V_0\tau \tag{30}$$

which tends to the uniform solution $\phi_0 e^{i\Gamma^2\Delta\phi_0^2\tau}$ as $|\eta| \to \infty$ exists. The solutions for $\phi$ and $F$ are found to be

$$\phi^2 = \phi_0^2(1 - \sin^2 B_0 \operatorname{sech}^2\psi) \quad \text{and} \quad F = \arctan(\tan B_0 \tanh\psi) \tag{31}$$

where $B_0$ is a constant, $\psi$ and $V_0$ are given as

$$\psi = (-\Gamma\Delta/2)^{1/2}\phi_0\eta\sin B_0 \quad \text{and} \quad V_0 = \pm 2^{-3/2}\Gamma(-\Gamma\Delta)^{1/2}\phi_0. \tag{32}$$

Solution (30) is called as a dark solitary wave solution, for more details see [19, 20, 21]. In this study, the following limits are valid for all branches of dispersion relation (22)

$$C \to \infty,\ V_G \to 0 \ \text{as} \ K \to 0 \ \text{and} \ C \to 1,\ V_G \to 1 \ \text{as} \ K \to \infty \tag{33}$$

where $C = c/c_{0_T}, V_G = V_g/c_{0_T}$, and $K = kh$ denote the nondimensional phase velocity, the nondimensional group velocity, and the nondimensional wave number, respectively. Furthermore, $W = wh/c_{0_T}$ denotes the nondimensional angular frequency and $A = \alpha h$. We depict the changes of $W$ and $C$ versus $K$ for the first three branches of dispersion relation (22) with the constant $A = 0.01$ in Fig. 1 and Fig. 2, respectively. Moreover, we depict the effect of the heterogeneity on the change of $W$ versus $K$ for the first branch of dispersion relation (22) with the constants $A = 0.01, A = 0.03$, and $A = 0.05$ in Fig. 3.

In this study, it is possible to choose the nonlinear material function as

$$n_T = n_{0_T}\cosh^2(\gamma y). \tag{34}$$

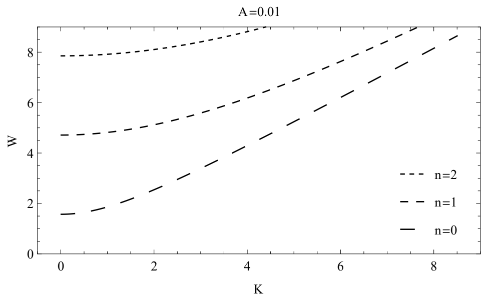


FIGURE 1. The change of $W$ versus $K$ for the first three branches of (22).

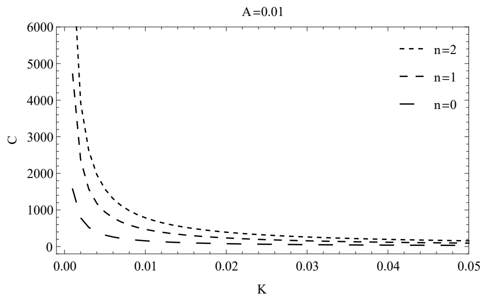


FIGURE 2. The change of $C$ versus $K$ for the first three branches of (22).

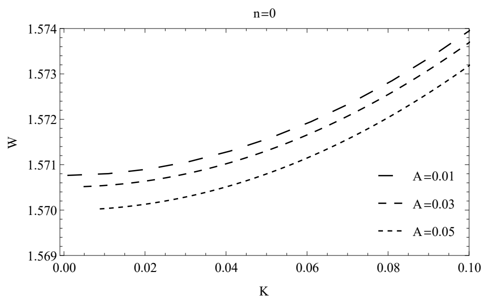


FIGURE 3. Heterogeneous effect on the change of $W$ versus $K$ for $n = 0$.

Here, $n_{0_T}$ indicates a constant and $\gamma$ indicates a parameter. As a result of this study, we claim that if the layer consists of a hardening material, i.e. $n_{0_T} > 0$, then $\Gamma\Delta < 0$ for all phase velocities $c > c_{0_T}[1 + (\alpha^2/(k^2l^2))]^{1/2}$; but if the layer consists of a softening material, i.e. $n_{0_T} < 0$, we refer to [14]. Hence, using the above review, we claim that if the layer consists of a hardening material, then the dark solitary SH waves will exist and propagate in such a layer.

We define a nondimensional constant as $\Lambda = \gamma h$ and depict the changes of $\Gamma$, $|\Delta|$, and $\Gamma|\Delta|$ versus $K$ for the first branch of dispersion relation (22) with the constants $A = 0.01$, $\Lambda = 0.04$, $n_{0_T} = 1$ and $\mu_0 = \rho_0 = 1$ in Fig. 4. Moreover, we depict the effects of the heterogeneities on the changes of $\Delta$ and $\Gamma\Delta$ versus $K$ for the first branch of dispersion relation (22) with the constants $A = 0.01, A = 0.03, A = 0.05$, $\Lambda = 0.04$, $n_{0_T} = 1$ and $\mu_0 = \rho_0 = 1$ in Fig. 5 and Fig. 6, respectively.

The following specific hardening material is considered

$$n_{0_T} = 1,\ \mu_0 = \rho_0 = 1,\ K = 0.01,\ \varepsilon = 0.01,\ \phi_0 = 0.01,\ B_0 = 0.01. \tag{35}$$

We show the effect of the heterogeneity on the deformation field of the plane $Z = 0$ of the layer for dark solitary SH wave solution (30) considering the first branch of dispersion relation (22) with the constants $A = 0.01, A = 0.03$, and $A = 0.05$ in Fig. 7.

We depict the changes of $\Gamma$, $|\Delta|$, and $\Gamma|\Delta|$ versus $K$ for the first branch of dispersion relation (22) with the constants $A = 0.01$, $\Lambda = 0.5$, $n_{0_T} = 1$ and $\mu_0 = \rho_0 = 1$ in Fig. 8. We also depict the effects of the nonlinearities on the changes of $\Delta$ and $\Gamma\Delta$ versus $K$ for the first branch of dispersion relation (22) with the constants $A = 0.01, \Lambda = 0.5, \Lambda = 0.7, \Lambda = 0.9$, $n_{0_T} = 1$ and $\mu_0 = \rho_0 = 1$ in Fig. 9 and Fig. 10, respectively.

Using specific hardening material (35), we show the effect of the nonlinearity on the deformation field of the plane $Z = 0$ of the layer for dark solitary SH wave solution (30) considering the first branch of dispersion relation (22) with $\Lambda = 0.5, \Lambda = 0.7$, and $\Lambda = 0.9$ in Fig. 11.

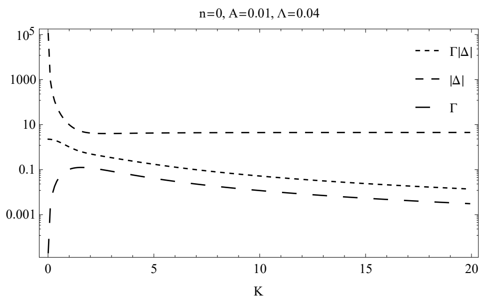


FIGURE 4. The changes of $\Gamma$, $|\Delta|$, and $\Gamma|\Delta|$ versus $K$ for $n = 0$.

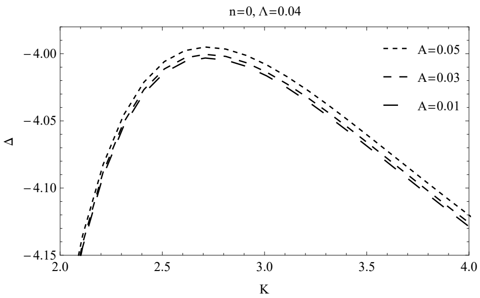


FIGURE 5. Heterogeneous effect on the change of $\Delta$ versus $K$ for $n = 0$.

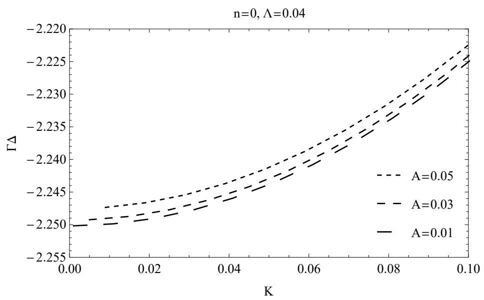


FIGURE 6. Heterogeneous effect on the change of $\Gamma\Delta$ versus $K$ for $n = 0$.

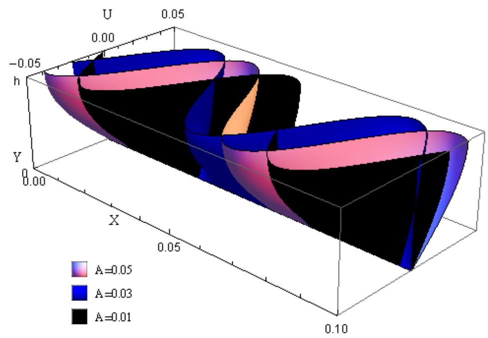


FIGURE 7. Heterogeneous effect on the deformation field of the plane $Z = 0$ of the layer for (30).

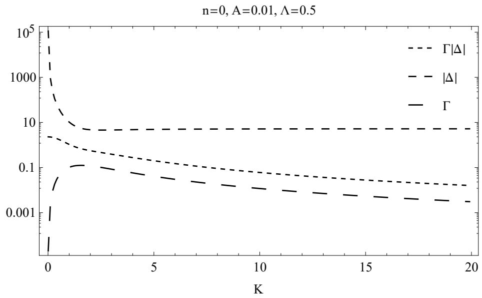


FIGURE 8. The changes of $\Gamma$, $|\Delta|$, and $\Gamma|\Delta|$ versus $K$ for $n = 0$.

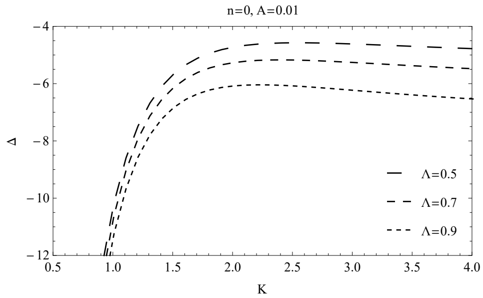


FIGURE 9. Nonlinear effect on the change of $\Delta$ versus $K$ for $n = 0$.

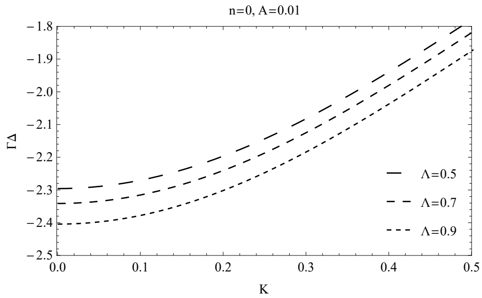


FIGURE 10. Nonlinear effect on the change of $\Gamma\Delta$ versus $K$ for $n = 0$.

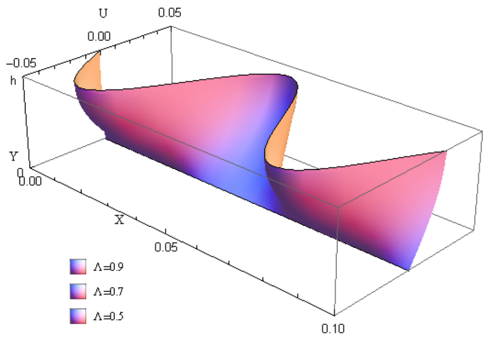


FIGURE 11. Nonlinear effect on the deformation field of the plane $Z = 0$ of the layer for (30).

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