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FCSTP WITH POSSIBILITY AND EXPECTED VALUE APPROACHES IN HYBRID UNCERTAIN ENVIRONMENTS

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Abstract. The objective of this investigation is to formulate a fixed charge (FC) solid transportation problem (STP) under a hybrid uncertain environment where both fuzziness and roughness coexist. A fuzzy rough STP model is developed by integrating the classical STP, fuzzy set theory, and rough set theory, which apparently provides a way to accommodate the uncertainty. For solving the problem, we apply the fuzzy rough expected value operator and propose the possibility based STP model with fuzzy rough parameters on a rough space. At the end, a mathematical illustration is provided to describe the fuzzy rough approach using LINGO 14.0 optimization software. As particular cases, the proposed model is also solved for single impreciseness. Finally, a graphical presentation is also shown to describe the comparison between two proposed approaches. In a particular case, the expressions of an earlier investigator have been derived from the present expressions. Important managerial decisions are made after observation of optimal results.

Keywords: Expected value, Solid transportation problem (STP), Fuzzy rough set, Hybrid uncertain environment, Possibility value approach.

AMS Subject Classification: 90B06, 90B50

1. Introduction

Haley [1] in the year 1962, first developed the concept of STP. The STP is the process of distributing certain products from its manufacturing point (source) to the different demand points (destination) using different conveyences keeping in mind the factor of different transportation capacities and transportation costs, fixed charge costs, etc so that the total transportation cost is minimum. While dealing with practical life, vagueness appears in

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the transportation system due to insufficient information about the system or for some unforeseen trouble like strikes, natural disaster, festivals etc. Therefore, the study of these kinds of problems is important for practical purposes. STP is nothing but an extension of traditional transportation problem (TP) where conveyance constraint is added with source and demand constraints. Also in the literature, there are so many works in which TP/STPs are considered under the different uncertain environments. The basic TP was originally developed by F. L. Hitchcock [2] and later discussed by Koopmans [3] in details. Later Dantzig [4] formulated the TP as a spacial class of linear programming problems and developed a special form of simplex technique. Based on Das et al. [5], the interval number TPs were converted into deterministic multi-objective problems. Omar and Samir [6] and Chanas and Kuchta [7] discussed the solution algorithm for solving the TPs in fuzzy environment. Yang and Liu [8] investigated a fixed charge STP under fuzzy environment with fuzzy direct costs, fuzzy supplies, fuzzy demands and fuzzy conveyance capacities as a expected value model / chance- constrained programming model / dependent -chance programming model and solved using fuzzy simulation and a heuristic method- tabu search algorithm.

Recently Yang and Liu [8] investigated the Fixed Charge STP (FCSTP) under fuzzy environment. Das et al. [21],[22] described about type-II fuzzy variables with different types of membership functions like Gaussian, Trapezoidal etc. Sinha et al. [23] presented a profit maximizing STP with trapezoidal interval type-II fuzzy number. Das et al. [16] described broadly about rough interval approach in their paper. There are different types of uncertainties like, theory of fuzzy set, stochastic process, theory of rough, randomness, theory of uncertainty etc. But in real life, sometimes there is a need of hybrid tools which can handle the uncertainty in a better way for fuzzy rough, fuzzy random, etc.

Rough sets compared to precise set cannot be characterized in terms of information about their elements. When we consider a rough set, the lower and the upper approximation of the rough set are always associated. Objects which are sure to belong to the set constitutes the lower approximation and objects which possibly belongs to the set constitutes the upper approximation. The boundary region is the difference between the upper approximation and the lower approximation.

Recently Shiraz et al. [9] extended data envelopment analysis (DEA) models to a fuzzy rough frame work. Fuzzy rough DEA was created by integrating classical DEA, fuzzy set theory, and rough set theory. At the beginning, fuzzy rough expected value operator was applied to calculate the efficiency measures of the decision making units. Then possibility DEA model with fuzzy rough parameters on a rough space and a fuzzy expected value approach were proposed and illustrated. Tao and Xu [25] developed rough multi-objective programming for rough multi-objective STP considering an appropriately large feasible region as a universe and equivalent relationship is induced to generate an approximate space. Kundu et al. [26] presented some practical solid transportation models with transportation cost as rough variables.

Approximations are fundamental concepts of rough set theory. Rough set theory share many common ideas with many other theories which deals with inaccurate knowledge like evidence, Bayesian inference, fuzzy sets and others. Nonetheless, the theory can be considered as an independent discipline in its own merits.

With the above directions, in this article we consider a the hybrid uncertain environment of rough set and fuzzy set theory to crack the uncertainty that exist in the parameters of STP. As per our knowledge, this is the first attempt to solve such type of problem in hybrid uncertain environment. In this investigation, there are sources at which some amounts of an item are stored i.e available for transportation. There are some destinations with some demands for the said item. The item is transported from sources to destinations. For transportation, there are some conveyances with different capacities. The unit transportation and fixed charge costs are uncertain and hybrid imprecise in nature, say fuzzy-rough. Now, the problem is to find the optimum transported amounts so that total transportation is minimum. Following Shiraz et al [9] the formulated problem has been made crisp in two different ways and solved using GRG method through LINGO 14.0. The model has been illustrated numerically. Some sensitivity analysis are performed. Models under some particular cases are derived and solved. Results of an earlier model are obtained from the present model. Thus, the main findings of this investigation are as follows:

- An unbalanced solid transportation problem is considered with unit transportation cost and fixed charge on the conveyance.
- The model is constructed with fuzzy-rough cost parameters as a hybrid uncertainty which are not considered till now.
- We have proposed two different approaches to convert these hybrid uncertain variables into crisp form, one is possibility value approach and the other is expected value approach. This concept is not used by others.
- General character of our analysis is demonstrated. Results of an earlier investigator (Giri et al) are obtained as a particular case of the present model.
- Effects of different type of impreciseness on the total cost are illustrated.

With this introductory part, the rest of the article is organized as follows. Section 2 gives some necessary and preliminary ideas; section 3 is about the mathematical model formulation with statements. Next section 4 discusses the solutions approaches. A numerical experiment is presented in section 5, discussion on result and managerial insights are shown in section 6. Last section 7 draws the conclusion of this article with some managerial discussion.

2. Preliminary

Some preliminaries which are associated with the research concept are presented.

2.1. Fuzzy set.

The credit of first introducing of the fuzzy set goes to professor Lofti A. Zadeh [24] in 1965.

Definition 1 (Zadeh [24]): If X is the universal set, then a fuzzy set $\widetilde{A}$ in X is a set of ordered pairs:

$$\widetilde{A} = \{x, \mu_{\widetilde{A}}(x) | x \epsilon X\}$$

$\mu_{\widetilde{A}}(x)$ is called the membership function (generalization of characteristic function) which maps X to the membership space M. Its range is the subset of non negative real numbers whose supremum is finite.

Definition 2 (Liu and Liu [27]): Let ξ be a fuzzy variable on the possibility space$(U, P(U), Pos)$. The possibility, necessity, and credibility of a fuzzy event $\{\xi \geq r\}$ are represented by: $Pos\{\xi \geq r\} = Sup\mu_\xi(t)$, $Nes\{\xi \geq r\} = 1 - Sup\mu_\xi(t)$, $Cr(\xi \geq r) = \frac{1}{2}[Pos\{\xi \geq r\} + Nec\{\xi \geq r\}]$

Lemma. Let $\bar{\lambda}_1 = (\alpha_1, m_1, \bar{m}_1, \beta_1)_{LR}$ and $\bar{\lambda}_2 = (\alpha_2, m_2, \bar{m}_2, \beta_2)_{LR}$ be two L-R type fuzzy numbers with continuous membership function. For a given confidence level $\alpha\epsilon[0,1]$. If $Pos\{\bar{\lambda}_1 \geq \bar{\lambda}_2\} \geq \alpha$, then we have: $\bar{m}_1 + \beta_1 R^{-1}(\alpha) \geq m_2 - \alpha_2 R^{-1}(\alpha)$

Proof. Suppose that $\bar{\lambda}_1 = (\alpha_1, m_1, \bar{m}_1, \beta_1)_{LR}$ and $\bar{\lambda}_2 = (\alpha_2, m_2, \bar{m}_2, \beta_2)_{LR}$ are the two L-R type fuzzy variables. The fuzzy arithmetic yields $\bar{\lambda} = \bar{\lambda}_1$ - $\bar{\lambda}_2 = (\alpha_1 + \beta_2, m_1 - \bar{m}_2, \bar{m}_1 - m_2, \alpha_2 + \beta_1)_{LR}$. Thus $\bar{\lambda}$ is a L-R type fuzzy number and accordingly with respect to Definition-2, the possibility of the fuzzy event $Pos\{\bar{\lambda} \geq 0\}$ is expressed as follows:

$$Pos(\bar{\lambda} \geq 0) = \begin{cases} 1 & \text{if } r \leq n_1 - \bar{\alpha} \\ R\left(\frac{r - n_2}{\bar{\beta}}\right) & \text{if } n_2 < 0 \leq n_2 + \bar{\beta} \\ 0 & \text{if } r > n_2 + \bar{\beta} \end{cases}$$

where $\bar{\alpha} = \alpha_1 + \beta_2$, $\bar{\beta} = \alpha_2 + \beta_1$, $n_1 = m_1 - \bar{m}_2$, and $n_2 = \bar{m}_1 - m_2$. Let us consider $Pos\{\bar{\lambda} \geq 0\} \geq \alpha$. Then, we have:
$R\left(\frac{-n_2}{\bar{\beta}}\right) \geq \alpha \Leftrightarrow \frac{-n_2}{\bar{\beta}} \leq R^{-1}(\alpha) \Leftrightarrow (\alpha_2 + \beta_1)R^{-1}(\alpha) \geq -(\bar{m}_1 - m_2) \Leftrightarrow \bar{m}_1 + \beta_1 R^{-1}(\alpha) \geq m_2 - \alpha_2 R^{-1}(\alpha)$

2.2. **Rough set.** As a new mathematical tool Zdzislaw Pawlak in the early 1980s ([13], [15]) introduced Rough set theory to handle with vagueness and uncertainty. Rough set theory has a great advantage compared to the other theories so far introduced in Mathematics. For details, reader may refer to [20]. Rough set comprises of approximation and boundary regions. The definitions of approximations as well as boundary region are as follows: R-lower approximation is defined by

$$R_*(x) = \bigcup_{x \in U} R(x) : R(x) \subseteq X$$

R-upper approximation is defined by

$$R^*(x) = \bigcup_{x \in U} R(x) : R(x) \cap X \neq \phi$$

R-boundary region of X defined as

$$RN_B(X) = R^*(X) - R_*(X)$$

For the rough variable ξ, we have

$$Tr\{\check{\xi} \geq r\} = \begin{cases} 0 & \text{for } d \leq r \\ \frac{d-r}{2(d-c)} & \text{for } b \leq r \leq d, \\ \frac{1}{2}\left(\frac{d-r}{d-c} + \frac{b-r}{b-a}\right) & \text{for } a \leq r \leq b, \\ \frac{1}{2}\left(\frac{d-r}{d-c} + 1\right) & \text{for } c \leq r \leq a, \\ 1 & \text{for } r \leq c \end{cases}$$

Definition (Liu [28]**):** Let ξ be a rough variable, and $\alpha \epsilon (0, 1]$. Then, $\xi_{sup}(\alpha) = sup\{r : Tr\{\xi \geq r\} \geq \alpha\}$ is called the α-optimistic value to ξ, and $\xi_{inf}(\alpha) = inf\{r : Tr\{\xi \leq r\} \geq \alpha\}$ is called α-pessimistic value to ξ.

2.3. **Fuzzy Rough Variable (FRV).** Fuzzy rough variable (FRV) is a measurable function from a rough space $(\Lambda; \Delta; A; \pi)$ to the fuzzy set which is again defuzzified $Pos\xi(\lambda)\epsilon B$ measurable function λ in any Boral set B ϵ R^n. For details reader are refer to [9].

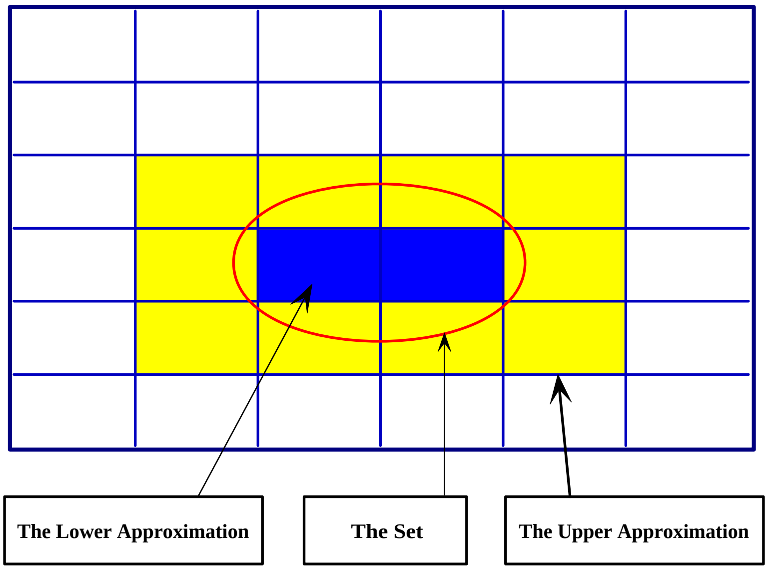

Figure 1. A graphical representation of a rough set environment

2.4. **General and Solid Transportation Problems:** Normally, a General Transportation Problem (GTP) is defined as follows: There are some sources or availabilities where some amount of different items are stocked and some destinations (or demands) where some amounts of the above items are required. Here, the items are transported from sources to destinations as per the requirements and availabilities. In this process, the problem of the decision maker is to find the optimum transported amount from the sources to the destinations satisfying the conditions of availabilities and destinations so that total transportation cost including other necessary cost such as loading and unloading costs, fixed charge cost, etc is minimum. This problem is called simply transportation problem (TP) or GTP. It may also be called as 2DTP. If the total amount of the availabilities is equal to the total amount of requirements at destination then it is called balanced transportation problem, otherwise it is unbalanced transportation problem. In this formulation, the requirement of vehicle to transport the goods from sources to destinations is ignored. In real life, it is very much required. Again, there may be several conveyenances available at sources for transportation. If the vehicles and their capacities for transportation from sources to destinations are considered, then it is called solid transportation problem (STP) or 3DTP. In this formulation, in addition to the availabilities and requirement constraints of 2DTP / an other vehicle capacity constrain is imposed. This differences is illustrated in Fig-2.

2.5. **Fixed Charge:** When some amounts of an item or items are transported from one place to another, then some costs such as toll taxes, festival collections etc are incurred. These costs do not depend on the transported amount of the items, rather depends on the path (or route) of travel and vehicle used for transportation.

3. Model Description and Formulation

Here a STP is formulate with the following notations and assumptions.

3.1. **Notations.**

- $\overline{\widetilde{C}}_{ijk}$ is the unit transportation cost.

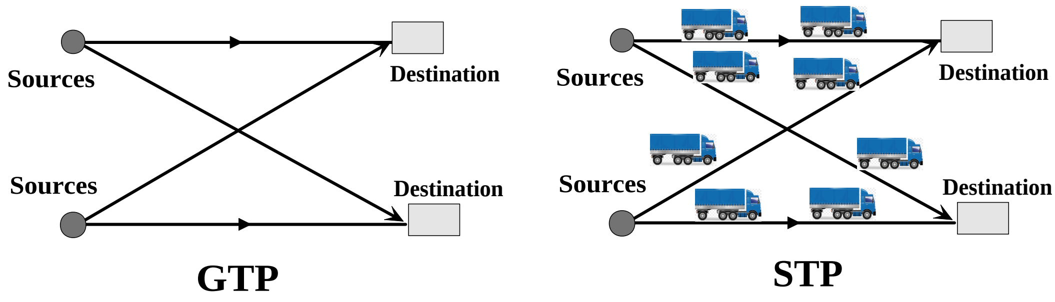

FIGURE 2. Illustration of GTP and STP

- x_{ijk} is the quantity transported from $i-th$ source to $j-th$ goal by $k-th$ vehicle.
- $\overset{\simeq e}{f}_{ijk}$ is the unit restricted fixed charge.
- a_i represents the total available source.
- b_j represents the total available demands
- e_k represents the total conveyance capacities in deterministic forms.

3.2. **Assumptions.**

(i) The model is an unbalanced transportation where $a_i \neq b_j \neq e_k$.
(ii) All the conveyances are fully loaded here. We have not considered partially loaded case.

3.3. **Formulation of the model.** In this article, a STP for a single item is considered with M origins, N destinations and K number of conveyances. There are fixed charges f_{ijk} on the conveyance along with the unit of transportation cost C_{ijk} Let X_{ijk} be the quantity transported for the $i-th$ sources to $j-th$ destination by $k-th$ vehicle. Let a_i, b_j, and e_k are respectively the total amount available at the $i-th$ source, total amount required at $j-th$ destination and the total capacity of the $k-th$ vehicle. Here, objective is to minimise the total transportation cost subject to the availability requirement and vehicle capacity constraints. Hence the mathematical formulation is given as below.
In this article, a STP is considered with M origins, N destinations and K number of conveyances. There are fixed charges $\overset{\simeq e}{f}_{ijk}$ on the conveyances along with unit transportation costs $\tilde{\tilde{C}}_{ijk}$. So the mathematical model, where objective function is the minimization of

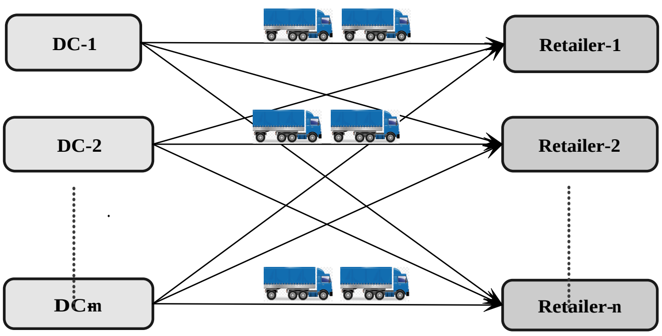

FIGURE 3. One stage solid transportation problem

total transportation cost consists unit transportation cost and restricted fixed charge is of the form (cf. Figure-3)

$$\text{Minimize } Z = \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}\{\tilde{\tilde{C}}_{ijk} + \tilde{\tilde{f}}^{e}_{ijk}\}x_{ijk} \tag{1}$$

$$\text{subject to} \quad \sum_{j=1}^{N}\sum_{k=1}^{K} x_{ijk} \leq a_i, \forall i = 1,2,...,M \tag{2}$$

$$\sum_{i=1}^{M}\sum_{k=1}^{K} x_{ijk} \geq b_j, \forall j = 1,2,...,N \tag{3}$$

$$\sum_{i=1}^{M}\sum_{j=1}^{N} x_{ijk} \leq e_k, \forall k = 1,2,...,K \tag{4}$$

Where $x_{ijk} \geq 0, \forall i, j$ and k and it denotes the unknown quantities to be transported. Here $\tilde{\tilde{C}}_{ijk}$ is unit transportation cost and $\tilde{\tilde{f}}^{e}_{ijk}$ is the fixed charge on conveyance considered in a hybrid uncertain environment of fuzzy and rough set theory. Also a_i, b_j and e_k are the available source, demand and conveyance capacities in deterministic form.

4. **Solution approaches**

Here in the objective function the parameters are fuzzy rough variable. So direct solution of this model is not possible and hence we proposed two new approaches to solve this type of problem in hybrid uncertain environment.

4.1. **Solution using possibility value approach.** The possibility approach is based on the possibility measures and chance constrained programming (CCP). It deals with the uncertainty that exist in the objective function and in the set of constraints by possibility measures. Applying this possibility approach to our proposed model then it became.
Minimize ψ
Subject to

$$Pos(\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}\{\tilde{\tilde{C}}_{ijk} + \tilde{\tilde{f}}^{e}_{ijk}\}x_{ijk} \leq \psi) \geq \delta \tag{5}$$

and constraints with $x_{ijk} \geq 0, \forall i, j$ and k.
Here $\delta \in [0;1]$ is the predetermined threshold specified by the decision maker (DM). The model (5) is a possibility rough solid transportation problem (PRSTP). Now following steps discussed by Shiraz et al. [8], we obtain the deterministic form of the possibility constraint of model (5)and hence the model became as follows.

$$(\psi^*)^{inf(\alpha)} = min\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}\{{C_{ijk}}^{m_2-sup(\alpha)} + R^{-1}(\delta)C^{\beta}_{ijk}x_{ijk} + {f^{e}_{ijk}}^{m_2-inf(\alpha)}R^{-1}(\delta){f^{e}_{ijk}}^{m_2-inf(\alpha)}\}x_{ijk} \tag{6}$$

$$(\psi^*)^{sup(\alpha)} = min\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}\{{C_{ijk}}^{m_2-sup(\alpha)} + R^{-1}(\delta)C^{\beta}_{ijk}x_{ijk} + {f^{e}_{ijk}}^{m_2-inf(\alpha)}R^{-1}(\delta){f^{e}_{ijk}}^{m_2-inf(\alpha)}\}x_{ijk} \tag{7}$$

subject to constraints (2) - (4) with $x_{ijk} \geq 0, \forall i, j$ and k.

4.2. **Solution using expected value approach.** When we apply the expected value approach according to Shiraz et al.[8], then our model became,

$$(min1/8(\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}\{{C_{ijk}}^{a-m_1} - {C_{ijk}}^{b-m_1} - {C_{ijk}}^{c-m_1} - {C_{ijk}}^{d-m_1} - {C_{ijk}}^{a-m_2}$$
$$-{C_{ijk}}^{b-m_2} - {C_{ijk}}^{c-m_2} - {C_{ijk}}^{d-m_2} + {f^e_{ijk}}^{a-m_1} - {f^e_{ijk}}^{b-m_1}$$
$$-{f^e_{ijk}}^{c-m_1} - {f^e_{ijk}}^{d-m_1} - {f^e_{ijk}}^{a-m_2} - {f^e_{ijk}}^{b-m_2} - {f^e_{ijk}}^{c-m_2} - {f^e_{ijk}}^{d-m_2}))$$
$$+4\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}(-C^{\alpha}_{ijk}\int_0^1 L(t)dt + \int_0^1 R(t)dt)x_{ijk}$$

and constraints $(2)-(4)$ with $x_{ijk} \geq 0, \forall i, j$ and k. where the last terms indicating the right and left expansion of the fuzzy number respectively

5. **Numerical Experiments:**

To demonstrate the modelling idea and the efficiency of the rough conception, we consider a STP composed of two sources, two destinations, and two conveyances. The corresponding fuzzy rough input data of the transportation components are given below Table-1. The fixed charge of the conveyances are presented in Table-2. The others STP parameters are, like availabilities a_1=16 and a_2=13, the demands b_1=11 and b_2=17 and capacities of the conveyances e_1=18 and e_2=16. Here it is needed to maintain that all the data are taken from hypothetical assumptions.

Table-1: Unit transportation costs

$\tilde{\overline{C}}_{111}$	$\tilde{\overline{C}}_{121}$	$\tilde{\overline{C}}_{211}$	$\tilde{\overline{C}}_{112}$
(4,[15,26],[13,38],4)	(6,[35,40],[23,46],6)	(2,[16,20],[8,22],2)	(5,[8,12],[6,14],5)
$\tilde{\overline{C}}_{221}$	$\tilde{\overline{C}}_{212}$	$\tilde{\overline{C}}_{221}$	$\tilde{\overline{C}}_{222}$
(2,[10,12],[33,35],3)	(4,[15,20],[23,46],4)	(5,[11,12],[6,12],5)	(2,[9,14],[3,12],2)

Table-2: Fixed cost

$\tilde{\overline{f}}_{111}$	$\tilde{\overline{f}}_{121}$	$\tilde{\overline{f}}_{211}$	$\tilde{\overline{f}}_{112}$
(1,[5,6],[4,5],1)	(3,[6,7],[5,6],3)	(2,[6,8],[7,9],2)	(4,[8,9],[6,7],4)
$\tilde{\overline{f}}_{221}$	$\tilde{\overline{f}}_{212}$	$\tilde{\overline{f}}_{221}$	$\tilde{\overline{f}}_{222}$
(2,[10,12],[5,7],2)	(1,[5,6],[4,6],1)	(3,[7,9],[6,8],5)	(2,[5,6],[6,8],2)

5.1. **Results.** With these numerical data presented in Table 1 and -2, we solve the problem by the two proposed solution approaches and for computing, we have used the Lingo 14.0 software. We got the optimal results in both case and the results are presented in Table 3. For this problem we choose $\alpha = \delta = 0.5$. From the Table 3, we can see that the expected value cost (408) belongs to the interval $[348, 482]$, which we obtained using the possibility value approach. This fact strongly supports the convergence of the proposed methods for solving such type of real life problems.

Table-3: Optimum results for the Fuzzy Rough Model

Using possibility value approach	Using expected value approach
Total cost=(12,[348,462],[320,480],12)	Total cost=421.7
Transported amount (Solution)	
x_{111}=4.1, x_{221}=7.8,x_{211}=13, x_{222}=10.1, and other variables are zero.	x_{121}=3.6, x_{122}=8.2, x_{221}=12, x_{222}=9.1 and other variables are zero.

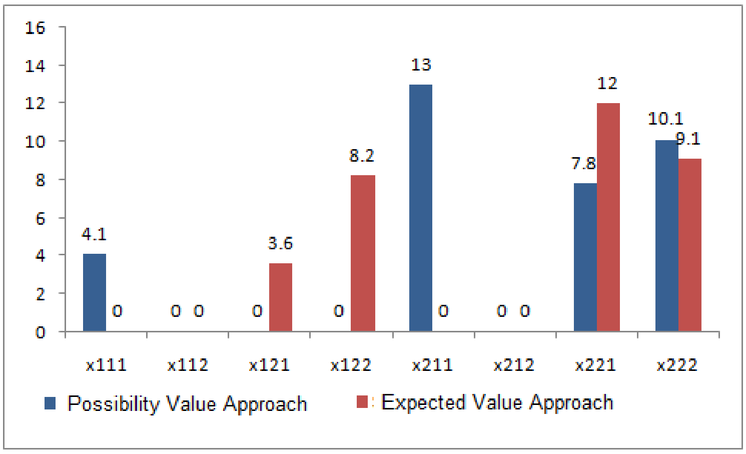

FIGURE 4. Comparison between transported amounts in two different approaches for fuzzy rough model

Table-4: Optimum results for the Fuzzy Model

Using possibility value approach	Using expected value approach
Total cost=[12, 418, 12]	Total cost= 417
Transported amount (Solution)	
x_{111}=5.7, x_{221}=9.18,x_{211}=10.8, x_{222}=9.1, and other variables are zero.	x_{121}=5.2, x_{122}=8.1, x_{221}=11, x_{222}=7.3 and other variables are zero.

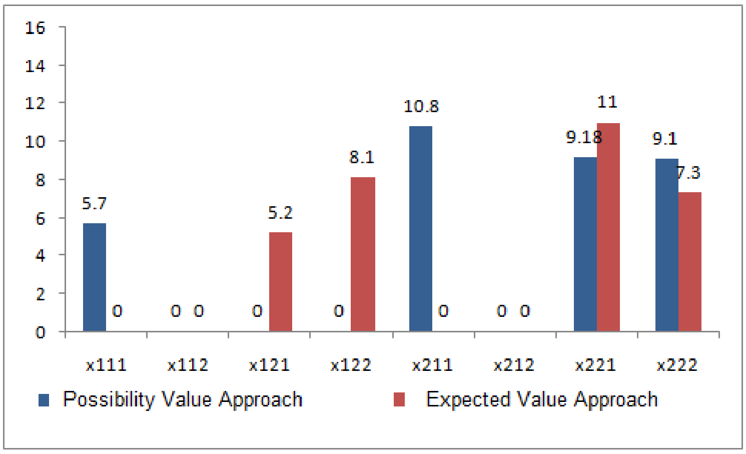

FIGURE 5. Comparison between transported amounts in two different approaches for fuzzy model

Table-5: Optimum results for the Rough Model

Using possibility value approch	Using expected value approach
Total cost=([355, 421], [321, 464])	Total cost= 408
Transported amount (Solution)	
x_{111}=6.1, x_{221}=8.9,x_{211}=12, x_{222}=5.1, and other variables are zero.	x_{121}=6, x_{122}=8, x_{221}=13, x_{222}=3 and other variables are zero.

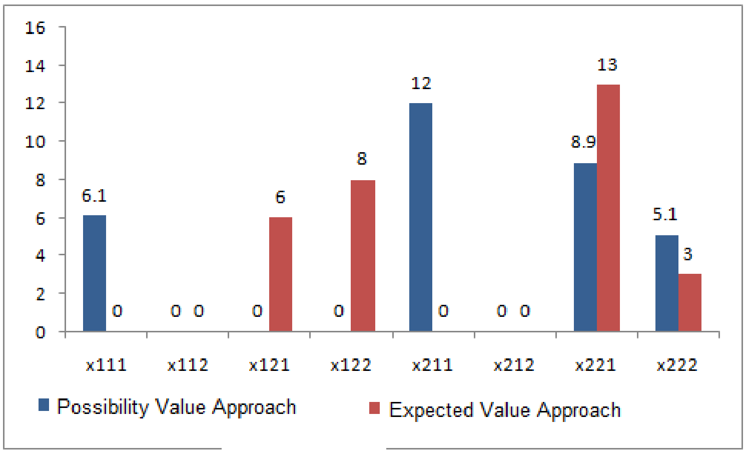

FIGURE 6. Comparison between transported amounts in two different approaches for rough model

From the optimum results it is seen that, more impreciseness create worse results to the decision maker. Here all the results are non-degenerate in nature, since the number of nonzero values is $(M + N + K - 2)$.

5.2. **Comparison between two solution approaches.** Figure 4, Figure 5 and Figure 6 shows the graphical representation of the two approaches. From Figure 4, Figure 5 and Figure 6, it can be clearly stated that result obtained in expected value approach is superior to the possibility value approach.

5.3. **Results of Giri et al.'s** [29] **Model.** In the expression of Z (of equation (1) and (6)) of the model, if we ignored the conception of rough, the same expression of Giri et al [29] is obtained. More over the input of Giri et al [29] by ignoring roughness of our model gives exactly the same results with our results.

6. Discussion on result and some managerial insights

The solution of the above model has been obtained in two different approaches. Here the input parameters are in fuzzy rough set. The discussion and managerial insights are as follows:

- Fuzzy rough set is a special case of uncertainty. Rough set is bounded by upper and lower approximation. Here fuzzy set is bounded by upper and lower approximation of the rough set. This type of variable is very critical to define.
- Here variables are taken according to experts' opinions. Experts and decision makes have help for such type of research

- Here we obtained the optimal results using two different approaches that proposed in this paper. It can be observed that the formats of solutions are of two types. One is in interval form and another is in number form.
- So it will not be possible to compare these two results. But here it will be very helpful for the decision maker (DM), because the inputs are same and outputs are in two different forms.
- In such a case it becomes easy for the DM to choose the appropriate solution approach according to the requirement of the problem.
- Also from the results, we see that the expected value approaches solution is lying in the interval that obtained using the possibility approach which justifies the convergence test.
- From the Table-3, 4, and 5, it is observed that hybrid impreciseness furnishes worse result. Hence, more impreciseness is not desirable.

7. Conclusion and further scope:

An unbalanced solid transportation problem is formed here with unit transportation cost and fixed charge. Here all the cost parameters are fuzzy rough in nature. More-over, accommodation of the uncertainty has been achieved in the form of hybrid fuzzy-rough parameters which are converted into deterministic ones by using two different approaches and finally optimized using Generalised Reduced Graphics (GRG) method through the LINGO 14.0 software. A comparison is shown between the two approaches. Observing the results, it can be concluded that both the solutions are nearest to each other which guaranteed the convergence of the methods. Also it is noticed here the possibility approach is giving the result in interval form and the expected value approach is giving it in a number. The discussion of the model for the considerable hypothetical input data helps the decision mangers to take their managerial insights.

The concept of the proposed model is quite simple and easy to understand. It will help the transportation engineering science to solve their complicated problem under uncertainty. Hence there is a great scope for future study in the transportation engineering science.

The illustrated problem is a STP (3D-TP). In this problem, the different routs between the sources and destinations have been ignored. Recently, taking different routes into consideration, Halder et al.[30] formulated and solved 4DTP (4-Dimensional Transportation Problem) in fuzzy environment. The present investigated problem can be formulated and solved as 4DTPs taking different available routes into consideration.

Here, only one hybrid uncertainly i.e. fuzzy rough has been considered. Other hybrid uncertainties such as fuzzy random, rough fuzzy, interval Type-2 fuzzy [cf.Muhun et.al [31], bi-fuzzy rough, etc can be taken for representation of uncertainties.

For simplicity, in this investigation, uncertainties have been introduced in the cost parameters only (unit transportation and fixed charge). The resources, (availabilities), demands and conveyances capacities also can be taken as hybrid uncertain.

For solution, we have used the generalised gradient method (GRG) using LINGO-14.0 software. However, the meta-heuristic method such as Genetic Algorithm(GA), Ant Colony Optimization (ACO), Particle Swarm Optimization (PSO)(cf. Ashraf et al.[32]), etc. can be appropriately used for the solution of the proposed 4DTPs, etc.

In this investigation, the analysis of uncertainty is quite general. This type of Hybrid uncertainty can be incorporated in the other research areas such as inventory control, supply chain, portfolio management etc.

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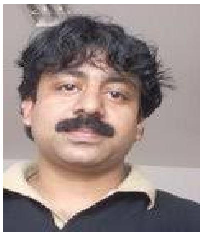

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