

REMARKS ON THE MULTIPLICATIVE OPERATIONS OF INTUITIONISTIC FUZZY MATRICES

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ABSTRACT. In this paper, using the definition of the multiplicative operations X_1 and X_2 of intuitionistic fuzzy matrices, we construct nA and A^n of an intuitionistic fuzzy matrix A and establish their algebraic properties. Also, we discuss some results on nA and A^n combined with max-min and min-max compositions of intuitionistic fuzzy matrices.

Keywords: Intuitionistic Fuzzy Set, Intuitionistic Fuzzy Matrix, Multiplicative Operations.

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1. INTRODUCTION

Atanassov [1] introduced the concept of intuitionistic fuzzy set (IFS) which is the generalization of fuzzy set introduced by Zadeh [14]. Since its appearance, intuitionistic fuzzy set has been investigated by many researchers and applied to many fields, such as decision making, clustering analysis etc., Using the fuzzy sets Kim and Roush [7] studied fuzzy matrices as a generalization of matrices over the two element Boolean algebra. Im et al. [5] defined the notion of intuitionistic fuzzy matrix (IFM) as a generalization of fuzzy matrix. Simultaneously Pal et al. [6] defined the IFM. Shyamal and Pal [10] studied the two new operators on fuzzy matrices and their desirable properties.

Boobalan and Sriram [3, 9] studied the algebraic sum and algebraic product of two intuitionistic fuzzy matrices and their algebraic properties. Also they proved the set of all intuitionistic fuzzy matrices forms a commutative monoid with respect to these operations. Muthuraji et al. [8] introduced a new composition operator and studied the desirable properties also obtained a decomposition of an IFM.

Emam and Fndh [4] defined some kinds of IFMs, the max-min and min-max composition of IFMs. Also they derived several important results by these compositions and construct an idempotent intuitionistic fuzzy matrix from any given one through the min-max composition.

Atanassov et al. [2] defined five novel operations from multiplicative type operations of

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intuitionistic fuzzy pairs and studied their algebraic properties. Venkatesan and Sriram [11, 12] defined the multiplicative operations X_1, X_2, X_3 and X_4 of an IFMs and investigated their desirable properties. In [13], they studied some new equalities connected with IFMs and established their algebraic properties.

In this paper, in Section 2, we give to some basic definitions of IFM that are necessary for this paper. In Section 3, using the definition of the multiplicative operations X_1 and X_2 of IFMs, In Section 4, we construct nA and A^n of an IFM A and establish their algebraic properties. In Section 4, we discuss some results on nA and A^n combined with max-min and min-max compositions of IFMs.

2. PRELIMINARIES

In this section, we give to some basic definitions of intuitionistic fuzzy matrix that are necessary for this paper.

Definition 2.1. [5] *A intuitionistic fuzzy matrix(IFM) is a matrix of pairs $A = (\langle a_{ij}, a'_{ij} \rangle)$ of a non negative real numbers $a_{ij}, a'_{ij} \in [0, 1]$ satisfying the condition $0 \leq a_{ij} + a'_{ij} \leq 1$ for all i, j .*

Definition 2.2. [11] *For any two IFMs A and B of the same size, we have*

(i) *The max-min composition of A and B is defined by*

$$A \vee B = (\langle \max(a_{ij}, b_{ij}), \min(a'_{ij}, b'_{ij}) \rangle).$$

(ii) *The min-max composition of A and B is defined by*

$$A \wedge B = (\langle \min(a_{ij}, b_{ij}), \max(a'_{ij}, b'_{ij}) \rangle).$$

Definition 2.3. [11] *For any two IFMs A and B of the same size, $A \geq B$ iff $a_{ij} \geq b_{ij}$ and $a'_{ij} \leq b'_{ij}$ for all i, j .*

Definition 2.4. [11] *The $m \times n$ zero IFM O is an IFM all of whose entries are $\langle 0, 1 \rangle$. The $m \times n$ universal IFM J is an IFM all of whose entries are $\langle 1, 0 \rangle$.*

Definition 2.5. [11] *The complement of an IFM A which is denoted by A^C and is defined by $A^C = (\langle a'_{ij}, a_{ij} \rangle)$.*

Definition 2.6. [9] *For any two IFMs A and B of the same size, then we have*

(i) $A \oplus B = (\langle a_{ij} + b_{ij} - a_{ij}b_{ij}, a'_{ij}b'_{ij} \rangle),$

(ii) $A \odot B = (\langle a_{ij}b_{ij}, a'_{ij} + b'_{ij} - a'_{ij}b'_{ij} \rangle).$

3. MAIN RESULTS

In this section, using the definition of the multiplicative operations X_1 and X_2 of IFMs, we construct nA and A^n of an IFM A and establish their algebraic properties.

Definition 3.1. [11] *For any two IFMs A and B of the same size, then we have*

(i) $AX_1B = (\langle \max(a_{ij}, b_{ij}), a'_{ij}b'_{ij} \rangle),$

(ii) $AX_2B = (\langle a_{ij}b_{ij}, \max(a'_{ij}, b'_{ij}) \rangle).$

Where $'.'$ is the ordinary multiplication.

Proposition 3.1. *For any two IFMs A and B of the same size, then we have*

(i) $A \odot B \leq AX_1B,$

(ii) $A \oplus B \geq AX_2B$.

Proof : (i) Let $A = (\langle a_{ij}, a'_{ij} \rangle)$ and $B = (\langle b_{ij}, b'_{ij} \rangle)$, be IFMs of the same size, $A \odot B = (\langle a_{ij}b_{ij}, a'_{ij} + b'_{ij} - a'_{ij}b'_{ij} \rangle)$ and $AX_1B = (\langle \max(a_{ij}, b_{ij}), a'_{ij}b'_{ij} \rangle)$. Since, $a_{ij}b_{ij} \leq \max(a_{ij}, b_{ij})$ and $a'_{ij} + b'_{ij} - a'_{ij}b'_{ij} \geq a'_{ij}b'_{ij}$, for all i, j . Hence, $A \odot B \leq AX_1B$.

(ii) The proof of (ii) is similar to that of (i).

Proposition 3.2. For any two IFMs A and B of the same size, then we have

$$(i) AX_2B = (A^C X_1 B^C)^C,$$

$$(ii) AX_1B = (A^C X_2 B^C)^C.$$

Proof : (i) From Definition 3.1, we have $A^C X_1 B^C = (\langle \max(a'_{ij}, b'_{ij}), a_{ij}b_{ij} \rangle)$.

Then it follows that $(A^C X_1 B^C)^C = (\langle a_{ij}b_{ij}, \max(a'_{ij}, b'_{ij}) \rangle) = AX_2B$.

Hence, $AX_2B = (A^C X_1 B^C)^C$.

(ii) The proof of (ii) is similar to that of (i).

Remark 3.1. Based on the multiplicative operations defined in Definition 3.1, we can get

the following operations: For any IFM $A = (\langle a_{ij}, a'_{ij} \rangle)$, $n > 0$ is a positive integer, the scalar multiplication operation is $nA = \underbrace{AX_1 \dots X_1 A}_n = (\langle a_{ij}, a'_{ij}{}^n \rangle)$ and the exponentia-

tion operation is $A^n = \overbrace{AX_2 \dots X_2 A}^n = (\langle a_{ij}^n, a'_{ij} \rangle)$.

Proof : Let $A = (\langle a_{ij}, a'_{ij} \rangle)$ be an IFM and $n > 0$ is a positive integer. Then

$$AX_1A = 2A = (\langle a_{ij}, a'_{ij}{}^2 \rangle), AX_1AX_1A = 3A = (\langle a_{ij}, a'_{ij}{}^3 \rangle)$$

$$\text{In general, } nA = \underbrace{AX_1 \dots X_1 A}_n = (\langle a_{ij}, a'_{ij}{}^n \rangle).$$

Similarly, we can get $A^n = \overbrace{AX_2 \dots X_2 A}^n = (\langle a_{ij}^n, a'_{ij} \rangle)$.

The results of nA and A^n are also IFMs.

Proposition 3.3. For any IFM A , $n > 0$ is a positive integer, then we have

$$(i) A^n = (nA^C)^C,$$

$$(ii) nA = ((A^C)^n)^C.$$

Proof : (i) Let $A = (\langle a_{ij}, a'_{ij} \rangle)$ be an IFM and $n > 0$ is a positive integer. Then

$$nA^C = (\langle a'_{ij}, a_{ij}^n \rangle)$$

$$(nA^C)^C = (\langle a_{ij}^n, a'_{ij} \rangle) = A^n.$$

Hence, $A^n = (nA^C)^C$.

(ii) The proof of (ii) is similar to that of (i).

Proposition 3.4. For any two IFMs A and B of the same size, $m > 0$ and $n > 0$ be positive integers. Then, the following desirable properties can be obtained algebraically:

$$(i) mAX_1nA = (m+n)A,$$

(ii) $nAX_1nB = n(AX_1B)$.

Proof : (i) Let A and B be two IFMs of the same size, $m > 0$ and $n > 0$ be positive integers. Then, we have $mA = \left(\left\langle a_{ij}, a'_{ij}{}^m \right\rangle\right)$ and $nA = \left(\left\langle a_{ij}, a'_{ij}{}^n \right\rangle\right)$.

Hence, $mAX_1nA = \left(\left\langle a_{ij}, a'_{ij}{}^{m+n} \right\rangle\right) = (m+n)A$.

(ii) By Definition 3.1, $nAX_1nB = \left(\left\langle \max(a_{ij}, b_{ij}), a'_{ij}{}^n b'_{ij}{}^n \right\rangle\right) = n(AX_1B)$.

Hence, $nAX_1nB = n(AX_1B)$.

Proposition 3.5. For any two IFMs A and B of the same size, $m > 0$ and $n > 0$ be positive integers. Then, we have

- (i) $A^m X_2 A^n = A^{m+n}$,
- (ii) $A^n X_2 B^n = (AX_2 B)^n$.

Proof : (i) From Proposition 3.2, Proposition 3.3 and the Proposition 3.4 (i), then we have

$$\begin{aligned} A^m X_2 A^n &= (mA^C)^C X_2 (nA^C)^C \\ &= (mA^C X_1 nA^C)^C \\ &= ((m+n)A^C)^C. \end{aligned}$$

Also, $A^{m+n} = ((m+n)A^C)^C$.

Hence, $A^m X_2 A^n = A^{m+n}$.

(ii) From Proposition 3.2, Proposition 3.3 and the Proposition 3.4 (ii), then we have

$$\begin{aligned} A^n X_2 B^n &= (nA^C)^C X_2 (nB^C)^C \\ &= (nA^C X_1 nB^C)^C \\ &= (n(A^C X_1 B^C))^C. \end{aligned}$$

Also, $(AX_2 B)^n = (n(AX_2 B)^C)^C = (n(A^C X_1 B^C))^C$.

Hence, $A^n X_2 B^n = (AX_2 B)^n$.

Proposition 3.6. For any two IFMs A and B of the same size, $n > 0$ is a positive integer. Then, we have

- (i) $nAX_2nB = n(AX_2B)$,
- (ii) $A^n X_1 B^n = (AX_1B)^n$.

Proof : (i) By Definition 3.1, $nAX_2nB = \left(\left\langle a_{ij} b_{ij}, \max(a'_{ij}{}^n, b'_{ij}{}^n) \right\rangle\right) = n(AX_2B)$.

Hence, $nAX_2nB = n(AX_2B)$.

(ii) From Proposition 3.2, Proposition 3.3 and the Proposition 3.4 (ii), then we have

$$\begin{aligned} A^n X_1 B^n &= (nA^C)^C X_1 (nB^C)^C \\ &= (nA^C X_2 nB^C)^C \\ &= (n(A^C X_2 B^C))^C. \end{aligned}$$

Also, $(AX_1 B)^n = (n(AX_1 B)^C)^C = (n(A^C X_2 B^C))^C$.

Hence, $A^n X_1 B^n = (AX_1 B)^n$.

Proposition 3.7. For any IFM A , $m > 0$ and $n > 0$ be positive integers. Then, we have

- (i) $m(nA) = (mn)A$,
- (ii) $(A^m)^n = A^{mn}$.

Proof : (i) $m(nA) = m \left(\left\langle a_{ij}, a'_{ij}{}^n \right\rangle\right) = \left(\left\langle a_{ij}, a'_{ij}{}^{nm} \right\rangle\right)$

$$\begin{aligned} &= \left(\left\langle a_{ij}, a'_{ij}{}^{mn} \right\rangle \right) \\ &= (mn)A. \end{aligned}$$

Hence, $m(nA) = (mn)A$.

(ii) The proof of (ii) is similar to that of (i).

Proposition 3.8. *For any two IFMs A and B of the same size, $n > 0$ is a positive integer. Then, we have*

- (i) $nA \geq nB$,
(ii) $A^n \geq B^n$.

Proof : (i) Let A and B be any two IFMs of the same size, $n > 0$ is a positive integer.

By Remark 3.1, $nA = \left(\left\langle a_{ij}, a'_{ij}{}^n \right\rangle \right)$ and $nB = \left(\left\langle b_{ij}, b'_{ij}{}^n \right\rangle \right)$.

Since, $a_{ij} \geq b_{ij}$ and $a'_{ij} \leq b'_{ij}$, we have $a'_{ij}{}^n \leq b'_{ij}{}^n$.

Hence, $nA \geq nB$.

(ii) The proof of (ii) is similar to that of (i).

4. RESULTS ON nA AND A^n COMBINED WITH MAX-MIN AND MIN-MAX COMPOSITIONS OF IFMS

In this section, we discuss some results on nA and A^n combined with max-min and min-max compositions of intuitionistic fuzzy matrices.

Proposition 4.1. *For any two IFMs A and B of the same size, $n > 0$ is a positive integer. Then, we have*

- (i) $n(A \vee B) = nA \vee nB$,
(ii) $n(A \wedge B) = nA \wedge nB$.

Proof : (i) $n(A \vee B) = \left(\left\langle \max(a_{ij}, b_{ij}), \min(a'_{ij}{}^n, b'_{ij}{}^n) \right\rangle \right)$
 $= \left(\left\langle (a_{ij}, a'_{ij}{}^n) \right\rangle \vee \left(\left\langle b_{ij}, b'_{ij}{}^n \right\rangle \right) \right)$
 $= nA \vee nB$.

Hence, $n(A \vee B) = nA \vee nB$.

(ii) The proof of (ii) is similar to that of (i).

Proposition 4.2. *For any two IFMs A and B of the same size, $n > 0$ is a positive integer. Then, we have*

- (i) $(A \vee B)^n = A^n \vee B^n$,
(ii) $(A \wedge B)^n = A^n \wedge B^n$.

Proof : (i) $(A \vee B)^n = \left(\left\langle \max(a_{ij}^n, b_{ij}^n), \min(a'_{ij}, b'_{ij}) \right\rangle \right)$
 $= \left(\left\langle (a_{ij}^n, a'_{ij}) \right\rangle \vee \left(\left\langle b_{ij}^n, b'_{ij} \right\rangle \right) \right)$
 $= A^n \vee B^n$.

Hence, $(A \vee B)^n = A^n \vee B^n$.

(ii) The proof of (ii) is similar to that of (i).

Proposition 4.3. *For any two IFMs A and B of the same size, $n > 0$ is a positive integer. Then, we have*

- (i) $nA \wedge (nAX_1nB) = nA$,
(ii) $A^n \vee (A^nX_2B^n) = A^n$.

Proof : (i) $nA \wedge (nAX_1nB) = \left(\left\langle a_{ij}, a'_{ij}{}^n \right\rangle \right) \wedge \left(\left\langle \max(a_{ij}, b_{ij}), a'_{ij}{}^n b'_{ij}{}^n \right\rangle \right)$
 $= \left(\left\langle \min(a_{ij}, \max(a_{ij}, b_{ij})), \max(a'_{ij}{}^n, a'_{ij}{}^n b'_{ij}{}^n) \right\rangle \right)$
 $= \left(\left\langle a_{ij}, a'_{ij}{}^n \right\rangle \right)$
 $= nA.$

Hence, $nA \wedge (nAX_1nB) = nA.$

(ii) The proof of (ii) is similar to that of (i).

Proposition 4.4. *For any two IFMs A and B of the same size, n > 0 is a positive integer. Then, we have*

- (i) $n(AX_1B) \geq n(A \wedge B),$
- (ii) $n(AX_2B) \leq n(A \vee B),$
- (iii) $n(AX_2B) \leq n(A \wedge B),$
- (iv) $n(AX_1B) \leq n(A \vee B).$

Proof : (i) $n(AX_1B) = \left(\left\langle \max(a_{ij}, b_{ij}), a'_{ij}{}^n b'_{ij}{}^n \right\rangle \right)$ and

$$n(A \wedge B) = \left(\left\langle \min(a_{ij}, b_{ij}), \max(a'_{ij}{}^n, b'_{ij}{}^n) \right\rangle \right).$$

Since, $\max(a_{ij}, b_{ij}) \geq \min(a_{ij}, b_{ij})$ and $a'_{ij}{}^n b'_{ij}{}^n \leq \max(a'_{ij}{}^n, b'_{ij}{}^n)$, for all $i, j.$

Hence, $n(AX_1B) \geq n(A \wedge B),$

(ii) $n(AX_2B) = \left(\left\langle a_{ij} b_{ij}, \max(a'_{ij}{}^n, b'_{ij}{}^n) \right\rangle \right)$ and

$$n(A \vee B) = \left(\left\langle \max(a_{ij}, b_{ij}), \min(a'_{ij}{}^n, b'_{ij}{}^n) \right\rangle \right).$$

Since $a_{ij} b_{ij} \leq \max(a_{ij}, b_{ij})$ and $\max(a'_{ij}{}^n, b'_{ij}{}^n) \geq \min(a'_{ij}{}^n, b'_{ij}{}^n)$, for all $i, j.$

Hence, $n(AX_2B) \leq n(A \vee B),$

(iii) Since, $a_{ij} b_{ij} \leq \min(a_{ij}, b_{ij})$, for all $i, j.$

Hence, $n(AX_2B) \leq n(A \wedge B),$

(iv), Since $a'_{ij}{}^n b'_{ij}{}^n \leq \min(a'_{ij}{}^n, b'_{ij}{}^n)$, for all $i, j.$

Hence, $n(AX_1B) \leq n(A \vee B).$

Similarly, we can prove the following property.

Proposition 4.5. *For any two IFMs A and B of the same size, n > 0 is a positive integer. Then, we have*

- (i) $(AX_1B)^n \geq (A \wedge B)^n,$
- (ii) $(AX_2B)^n \leq (A \vee B)^n,$
- (iii) $(AX_2B)^n \leq (A \wedge B)^n,$
- (iv) $(AX_1B)^n \leq (A \vee B)^n.$

Proposition 4.6. *For any two IFMs A and B of the same size, n > 0 is a positive integer. Then, we have*

- (i) $n(A \vee B) \vee n(AX_1B) = n(AX_1B),$
- (ii) $n(A \wedge B) \wedge n(AX_2B) = n(AX_2B).$

Proof : (i) $n(A \vee B) \vee n(AX_1B)$
 $= \left(\left\langle \max(a_{ij}, b_{ij}), \min(a'_{ij}{}^n, b'_{ij}{}^n) \right\rangle \right) \vee \left(\left\langle \max(a_{ij}, b_{ij}), (a'_{ij}{}^n b'_{ij}{}^n) \right\rangle \right)$
 $= \left(\left\langle \max(\max(a_{ij}, b_{ij}), \max(a_{ij}, b_{ij})), \min(\min(a'_{ij}{}^n, b'_{ij}{}^n), a'_{ij}{}^n b'_{ij}{}^n) \right\rangle \right)$
 $= \left(\left\langle \max(a_{ij}, b_{ij}), a'_{ij}{}^n b'_{ij}{}^n \right\rangle \right)$
 $= n(AX_1B).$

Hence, $n(A \vee B) \vee n(AX_1B) = n(AX_1B)$.

(ii) The proof of (ii) is similar to that of (i).

Proposition 4.7. For any two IFMs A and B of the same size, $n > 0$ is a positive integer. Then, we have

$$(i)(A \vee B)^n \vee (AX_1B)^n = (AX_1B)^n,$$

$$(ii)(A \wedge B)^n \wedge (AX_2B)^n = (AX_2B)^n.$$

Proof : (i) $(A \vee B)^n \vee (AX_1B)^n$

$$\begin{aligned} &= \left(\left\langle \max(a_{ij}^n, b_{ij}^n), \min(a'_{ij}, b'_{ij}) \right\rangle \right) \vee \left(\left\langle \max(a_{ij}^n, b_{ij}^n), a'_{ij} b'_{ij} \right\rangle \right) \\ &= \left(\left\langle \max(\max(a_{ij}^n, b_{ij}^n), \max(a'_{ij}, b'_{ij})), \min(\min(a'_{ij}, b'_{ij}), a'_{ij} b'_{ij}) \right\rangle \right) \\ &= \left(\left\langle \max(a_{ij}^n, b_{ij}^n), a'_{ij} b'_{ij} \right\rangle \right) \\ &= (AX_1B)^n. \end{aligned}$$

Hence, $(A \vee B)^n \vee (AX_1B)^n = (AX_1B)^n$.

(ii) The proof of (ii) is similar to that of (i).

Proposition 4.8. For any two IFMs A and B of the same size, $n > 0$ is a positive integer. Then, we have

$$(i)n(A \wedge B)X_1n(A \vee B) = n(AX_1B),$$

$$(ii)n(A \wedge B)X_2n(A \vee B) = n(AX_2B).$$

Proof : (i) $n(A \wedge B)X_1n(A \vee B)$

$$\begin{aligned} &= \left(\left\langle \min(a_{ij}, b_{ij}), \max(a'_{ij}{}^n, b'_{ij}{}^n) \right\rangle \right) X_1 \left(\left\langle \max(a_{ij}, b_{ij}), \min(a'_{ij}{}^n, b'_{ij}{}^n) \right\rangle \right) \\ &= \left(\left\langle \max(\min(a_{ij}, b_{ij}), \max(a_{ij}, b_{ij})), \max(a'_{ij}{}^n, b'_{ij}{}^n) \min(a'_{ij}{}^n, b'_{ij}{}^n) \right\rangle \right) \\ &= \left(\left\langle \max(a_{ij}, b_{ij}), a'_{ij}{}^n b'_{ij}{}^n \right\rangle \right) \\ &= n(AX_1B). \end{aligned}$$

Hence, $n(A \wedge B)X_1n(A \vee B) = n(AX_1B)$.

(ii) The proof of (ii) is similar to that of (i).

Proposition 4.9. For any two IFMs A and B of the same size, $n > 0$ is a positive integer. Then, we have

$$(i)(A \wedge B)^n X_1(A \vee B)^n = (AX_1B)^n,$$

$$(ii)(A \wedge B)^n X_2(A \vee B)^n = (AX_2B)^n.$$

Proof : (i) $n(A \wedge B)X_1n(A \vee B)$

$$\begin{aligned} &= \left(\left\langle \min(a_{ij}^n, b_{ij}^n), \max(a'_{ij}, b'_{ij}) \right\rangle \right) X_1 \left(\left\langle \max(a_{ij}^n, b_{ij}^n), \min(a'_{ij}, b'_{ij}) \right\rangle \right) \\ &= \left(\left\langle \max(\min(a_{ij}^n, b_{ij}^n), \max(a_{ij}^n, b_{ij}^n)), \max(a'_{ij}, b'_{ij}) \min(a'_{ij}, b'_{ij}) \right\rangle \right) \\ &= \left(\left\langle \max(a_{ij}^n, b_{ij}^n), a'_{ij} b'_{ij} \right\rangle \right) \\ &= (AX_1B)^n. \end{aligned}$$

Hence, $(A \wedge B)^n X_1(A \vee B)^n = (AX_1B)^n$.

(ii) The proof of (ii) is similar to that of (i).

5. CONCLUSIONS

In this work, using the definition of the multiplicative operations X_1 and X_2 of intuitionistic fuzzy matrices, we constructed nA and A^n of an intuitionistic fuzzy matrix A

and investigated their algebraic properties. Some results on nA and A^n combined with max-min and min-max compositions of intuitionistic fuzzy matrices are presented.

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