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SOME INCOMPLETE RIEMANN-LIOUVILLE FRACTIONAL DERIVATIVE OPERATORS

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ABSTRACT. Several investigations were done by many researchers on extended fractional derivatives operators using different kinds of extended Beta functions (see, e.g.,([4],[7], [8],[18],[20]), and references therein). In this sequel, we develop some new properties on incomplete extended Riemann-Liouville fractional derivative operators with the help of new incomplete extended Beta functions. Other than this, we also present the above defined new incomplete extended Beta function, graphically within a wide range of assumed parameters.

Keywords: Incomplete gamma function, Incomplete generalized hypergeometric functions, Incomplete Pochhammer ratio, Incomplete beta function, Riemann-Liouville fractional derivative.

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1. Introduction

In the last four decades, many extensions of the renowned special functions and fractional calculus have been considered by sundry authors ([2],[9],[12],[14]). Because of the effectiveness and great importance of the fractional calculus and special functions, the authors develop a generalized incomplete form of the fractional derivative operator along with confluent hypergeometric function. These thoughts have led various workers in the field of special functions to explore the possible extensions and applications of the extended fractional derivative operators. Our present study is highly motivated by the usefulness of the above extensions.

Srivastava et al.[19] developed extended beta function and Gauss hypergeometric function defined as

$$B_h{}^{\zeta,\eta;k,u}(x,y) := \int_0^1 t^{x-1}(1-t)^{y-1}{}_1F_1\left(\zeta;\eta;\frac{-h}{t^k(1-t)^u}\right)dt, \tag{1}$$

$$min(Re(\zeta), Re(\eta), Re(k), Re(u)) > 0, Re(x) > -Re(k\zeta), Re(h) \geq 0, Re(y) > -Re(u\zeta),$$

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and

$$F_h{}^{(\zeta,\eta;k,u)}(a,b;c;z) := \sum_{n=0}^{\infty}(a)_n \frac{B_h{}^{(\zeta,\eta;k,u)}(b+n,c-b)}{B(b,c-b)}\frac{z^n}{n!}, \tag{2}$$

$$|z|<1, min(Re(\zeta), Re(\eta), Re(k), Re(u)) > 0, Re(c) > Re(b) > 0, Re(h) \geq 0.$$

Which is reducible to the generalized Beta type function defined by Parmar ([17]) when $k = u$, immediately, he studied some fundamental properties and characteristics of this generalized Beta type function

$$B_h{}^{\zeta,\eta;u}(x,y) := \int_0^1 t^{x-1}(1-t)^{y-1}{}_1F_1\Big(\zeta;\eta;\frac{-h}{t^u(1-t)^u}\Big)dt, \tag{3}$$

where $Re(h) \geq 0, min(Re(x), Re(y), Re(\zeta), Re(\eta), Re(u)) > 0$.
Equation (3) is reduces into the special case $B_h{}^{\zeta,\eta}(x,y)$ when $u = 1$, further $B_h{}^{\zeta,\eta}(x,y)$ reduces into to $B_h(x,y)$ at $\zeta = \eta$ and $B(x,y)$ at $h = 1$, $\zeta = \eta$(see, for details, ([5],[6],[15],[16])).
Parmar ([17]) also defined the Gauss hypergeometric function as

$$F_h{}^{(\zeta,\eta;u)}(a,b;c;z) := \sum_{n=0}^{\infty}(a)_n \frac{B_h{}^{(\zeta,\eta;u)}(b+n,c-b)}{B(b,c-b)}\frac{z^n}{n!}, \tag{4}$$

where $|z| < 1$, $Re(h) \geq 0, min(Re(\zeta), Re(\eta), Re(u)) > 0, Re(c) > Re(b) > 0, Re(a) > 0$.
Further, extension of extended Gauss hypergeometric function was defined by Praveen Agrawal et. al.[1] by using extended beta function which was developed by Srivastava such as

$$F_{h;k,u}(a,b;c;z;\Theta) := \sum_{n=0}^{\infty}\frac{(a)_n(b)_n}{(c)_n}\frac{B_h{}^{\zeta,\eta;k,u}(b+n,c-b+\Theta)}{B(b+n,c-b+\Theta)}\frac{z^n}{n!}, \tag{5}$$

where $(\Theta < Re(b) < Re(c); |z| < 1, Re(k) > 0, Re(u) > 0), Re(h) \geq 0, \Theta \in N$.
Extended Riemann-Liouville fractional derivative f(z) of order v defined by them as

$$D_z^{(v;h;k;u)}f(z) := \frac{1}{\Gamma(-v)}\int_0^z (z-t)^{-v-1}f(t){}_1F_1\Big(\zeta;\eta;-\frac{hz^{k+u}}{t^k(z-t)^u}\Big)dt, \tag{6}$$

where $(Re(v) < 0; Re(h) \geq 0; Re(k) > 0; Re(u) > 0)$.
When $Re(v) \geq 0$, $\Theta \in N$ such that $\Theta - 1 \leq R(v) < \Theta$. Then the extended Riemann-Liouville fractional derivative of $f(z)$ of order v was defined by

$$D_z^{(v;h;k;u)}f(z) := \frac{d^\Theta}{dz^\Theta}D_z^{(v-\Theta;h;k;u)}f(z)$$

$$:= \frac{d^\Theta}{dz^\Theta}\Bigg\{\frac{1}{\Gamma(\Theta-v)}\int_0^z (z-t)^{\Theta-v-1}f(t){}_1F_1\Big(\zeta;\eta;\frac{hz^{k+u}}{t^k(z-t)^u}\Big)dt\Bigg\} \tag{7}$$

$$(Re(h) \geq 0; Re(k) > 0; Re(u) > 0).$$

- when $h = 0$, the above results become classical Riemann-Liouville fractional derivtive operator.
- when $h > 0$ with $\zeta = \eta$ and $k = u = 1$, it gives similar results defined by [14].

Özarslan et al.[13] defined incomplete Riemann-Liouville fractional derivatives $D_z{}^v[f(z); y]$ and $D_z{}^v\{f(z); y\}$ as

$$D_z{}^v[f(z), y] := \frac{z^{-v}}{\Gamma(-v)}\int_0^y f(tz)(1-t)^{-v-1}dt, \tag{8}$$

where $Re(v) < 0$,
and its counterpart is by

$$D_z{}^v\{f(z), y\} := \frac{z^{-v}}{\Gamma(-v)} \int_0^{1-y} f((1-t)z)t^{-v-1}dt, \tag{9}$$

where $Re(v) < 0$.

Our present study is highly motivated by the usefulness of the above extended Riemann-Liouville fractional derivative operator by extending the work of some more researchers (see, e.g.,[3],[10],[11]). This paper is organized in three sections. The first section contain the properties based on Incomplete Riemann-Liouville fractional derivative. In the second section we discuss about Incomplete beta function graphically with different perameters. In the third section we define the conclusion.

2. Incomplete extended Riemann-Liouville fractional derivative

In this section, we introduce and investigate Incomplete extended beta function, Incomplete extended Gauss hypergeometric function, and Incomplete extended Riemann-Liouville fractional derivative, with $Re(h) \geq 0$ as :

$$B_{h,s}^{\zeta,\eta,k,u}(x,y) = \int_0^s t^{x-1}(1-t)^{y-1}{}_1F_1\Big(\zeta;\eta;-\frac{h}{t^k(1-t)^u}\Big)dt, \tag{10}$$

and its counterpart

$$B_{h,1-s}^{\zeta,\beta,k,u}(y,x) = \int_0^{1-s} t^{y-1}(1-t)^{x-1}{}_1F_1\Big(\zeta;\eta;-\frac{h}{t^k(1-t)^u}\Big)dt,$$

$$min\{Re(\zeta), Re(\eta), Re(k), Re(u) > 0\} > 0, Re(x) > -Re(k\zeta), Re(y) > -Re(u\zeta).$$

The Incomplete extended beta function satisfies the relation

$$B_{h,s}^{\zeta,\eta,k,u}(x,y) + B_{h,1-s}^{\zeta,\eta,k,u}(y,x) = B_h^{\zeta,\eta,k,u}(x,y), \quad 0 \leq s \leq 1.$$

The Incomplete extended Gauss hypergeometric function defined as

$$F_{h,s}^{(\zeta,\eta,k,u)}(a,b;c;z) = \sum_{n=0}^{\infty}(a)_n \frac{B_{h,s}^{(\zeta,\eta;k,u)}(b+n,c-b)}{B(b,c-b)}\frac{z^n}{n!}, \tag{11}$$

where $B(u,v)$ is the familiar Beta function defined by

$$B(u,v) = \begin{cases} \int_0^1 t^{u-1}(1-t)^{v-1}dt, & (Re(u) > 0; Re(v) > 0) \\ \frac{\Gamma(u)\Gamma(v)}{\Gamma(u+v)}, & (u,v \in \mathbb{C} \text{ other than } \mathbb{Z}_0{}^-) \end{cases} \tag{12}$$

Now we define the incomplete extended Riemann-Liouville fractional derivative operators $D_z^{(v;h;k;u)}[f(z);y]$ and $D_z^{(v;h;k;u)}\{f(z);y\}$ as

- when $Re(v) < 0$

$$D_z^{(v;h;k;u)}[f(z);y] := \frac{z^{-v}}{\Gamma(-v)} \int_0^y f(v_1 z)(1-v_1)^{-v-1}{}_1F_1\left(\zeta;\eta;\frac{-h}{{v_1}^k(1-v_1)^u}\right)dv_1, \tag{13}$$

and its counterpart is given by

$$D_z^{(v;h;k;u)}\{f(z);y\} := \frac{z^{-v}}{\Gamma(-v)}\int_y^1 f(v_1 z)(1-v_1)^{-v-1}{}_1F_1\left(\zeta;\eta;\frac{-h}{{v_1}^k(1-v_1)^u}\right)dv_1, \tag{14}$$

where

$$D_z^{(v;h;k;u)}[f(z);y] + D_z^{(v;h;k;u)}\{f(z);y\} := D_z^{(v;h;k;u)}f(z), \qquad (Re(v)<0). \tag{15}$$

- When $Re(v) \geq 0$
  $\Theta \in N$ with the condition $\Theta - 1 \leq Re(v) < \Theta$,

$$D_z^{(v;h;k;u)}[f(z);y] := \frac{d^\Theta}{dz^\Theta} D_z^{(v-\Theta;h;k;u)}[f(z);y]$$

$$:= \frac{d^\Theta}{dz^\Theta}\left\{\frac{z^{\Theta-v}}{\Gamma(\Theta-v)}\int_0^y (1-v_1)^{\Theta-v-1} f(zv_1)_1F_1\Big(\zeta;\eta;\frac{-h}{{v_1}^k(1-v_1)^u}\Big)dv_1\right\}, \tag{16}$$

where $(Re(h) \geq 0; Re(k) > 0; Re(u) > 0)$
and

$$D_z^{(v;h;k;u)}\{f(z),y\} := \frac{d^\Theta}{dz^\Theta} D_z^{(v-\Theta;h;k;u)}\{f(z);y\}$$

$$:= \frac{d^\Theta}{dz^\Theta}\left\{\frac{z^{\Theta-v}}{\Gamma(\Theta-v)}\int_0^{1-y} (t)^{\Theta-v-1} f((1-t)z)_1F_1\Big(\zeta;\eta;\frac{-h}{(1-t)^k(t)^u}\Big)dt\right\}, \tag{17}$$

where $(Re(h) \geq 0; Re(k) > 0; Re(u) > 0)$.

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  - when $h = 0$, Equation (15) converts into classical Riemann-Liouville fractional derivative operator.
  - when $h > 0$ with $\zeta = \eta$ and $k = u = 1$, equation (15) give similar result defined by [14].

Theorem 2.1. *Let if $\Theta - 1 \leq Re(v) < \Theta$ for some $\Theta \in N$ and $Re(v) < Re(\lambda)$ $Re(h) \geq 0; Re(k) > 0; Re(u) > 0$, then*

$$D_z^{(v;h;k;u)}[z^\lambda;y] := \frac{\Gamma(\lambda+1)B_{h,y}^{\zeta,\eta;k,u}(\lambda+1,\Theta-v)}{\Gamma(\lambda-v+1)B(\lambda+1,\Theta-v)} z^{\lambda-v}. \tag{18}$$

Proof. Applying equation (16) by substituting $f(z) = z^\lambda$, we have

$$D_z^{(v;h;k;u)}[z^\lambda;y] := \frac{d^\Theta}{dz^\Theta}\left\{\frac{z^{\Theta-v}}{\Gamma(\Theta-v)}\int_0^y (1-v_1)^{\Theta-v-1}(v_1 z)^\lambda{}_1F_1\Big(\zeta;\eta;\frac{-h}{{v_1}^k(1-v_1)^u}\Big)dv_1\right\},$$

$$:= \frac{d^\Theta}{dz^\Theta} z^{\Theta-v+\lambda}\left\{\frac{1}{\Gamma(\Theta-v)}\int_0^y (1-v_1)^{\Theta-v-1}(v_1)^\lambda{}_1F_1\Big(\zeta;\eta;\frac{-h}{{v_1}^k(1-v_1)^u}\Big)dv_1\right\}, \tag{19}$$

using the property

$$\frac{d^\Theta}{dz^\Theta} z^{\Theta+\lambda-v} := \frac{\Gamma(1+\lambda-v+\Theta)}{\Gamma(1+\lambda-v)} z^{\lambda-v} \tag{20}$$

using property (20) and (10) in equation (19), we led to the desired result.

□

Theorem 2.2. *Let if* $\Theta - 1 \leq Re(v) < \Theta$ *for some* $\Theta \in N$ *and* $Re(v) < Re(\lambda)$, $Re(h) \geq 0; Re(k) > 0; Re(u) > 0$, *then*

$$D_z^{(v;h;k;u)}\{z^\lambda; y\} := \frac{\Gamma(\lambda+1) B_{h,1-y}^{\zeta,\eta;k,u}(\lambda+1, \Theta - v)}{\Gamma(\lambda - v + 1) B(\lambda+1, \Theta - v)} z^{\lambda - v}. \tag{21}$$

Proof. Applying equation (17) by substituting $f(z) = z^\lambda$, we have

$$D_z^{(v;h;k;u)}\{z^\lambda; y\} := \frac{d^\Theta}{dz^\Theta}\left\{\frac{z^{\Theta-v+\lambda}}{\Gamma(\Theta - v)} \int_0^{1-y} (t)^{\Theta-v-1}(1-t)^\lambda {}_1F_1\Big(\zeta;\eta;\frac{-h}{(1-t)^k (t)^u}\Big) dt\right\},$$

$$D_z^{(v;h;k;u)}\{z^\lambda; y\} := \frac{d^\Theta}{dz^\Theta} z^{\Theta-v+\lambda}\left\{\frac{1}{\Gamma(\Theta - v)} \int_0^{1-y} (t)^{\Theta-v-1}(1-t)^\lambda {}_1F_1\Big(\zeta;\eta;\frac{-h}{(1-t)^k (t)^u}\Big) dt\right\}, \tag{22}$$

using property (20) and counterpart of equation (10) in equation (22), we led to the desired result.

□

We use the extended Riemann-Liouville fractional derivative to a function $f(z)$ analytic at the origin.

Theorem 2.3. *Let if* $\Theta - 1 \leq Re(v) < \Theta$ *for some* $\Theta \in N$, $Re(h) \geq 0; Re(k) > 0; Re(u) > 0$. *Suppose a function* $f(z)$ *is analytic at the origin with its Maclaurin's expansion given by* $f(z) = \sum_{n=0}^{\infty} a_n z^n (|z| < \rho_1)$ *for some* $\rho_1 \in R^+$. *Then, we have*

$$D_z^{(v;h;k;u)}[f(z); y] := \sum_{n=0}^{\infty} a_n D_z^{(v;h;k;u)}[z^n; y].$$

Proof. Applying equation (16) to the function $f(z)$ with its series expansion, we have

$$D_z^{(v;h;k;u)}[f(z), y] := \frac{d^\Theta}{dz^\Theta}\left\{\frac{z^{\Theta-v}}{\Gamma\Theta - v} \int_0^{y} (1-v_1)^{\Theta-v-1} {}_1F_1\Big(\zeta;\eta;\frac{-h}{{v_1}^k (1-v_1)^u}\Big) \sum_{n=0}^{\infty} a_n (v_1 z)^n dv_1\right\}. \tag{23}$$

For any closed disk centered at the origin with its radius smaller then ρ_1, the power series converges uniformly. The series on the line segment from 0 to a fixed z for $|z| < \rho_1$, changing the order of summation and integration, we have

$$D_z^{(v;h;k;u)}[f(z); y] := \sum_{n=0}^{\infty} a_n \frac{d^\Theta}{dz^\Theta}\left\{\frac{z^{\Theta-v}}{\Gamma(\Theta - v)} \int_0^{y} (1-v_1)^{\Theta-v-1} {}_1F_1\Big(\zeta;\eta;\frac{-h}{{v_1}^k (1-v_1)^u}\Big) (v_1 z)^n dv_1\right\},$$

$$D_z^{(v;h;k;u)}[f(z), y] := \sum_{n=0}^{\infty} a_n D_z^{(v;h;k;u)}[z^n, y]. \tag{24}$$

□

Theorem 2.4. *Let* $\Theta-1 \leq Re(v) < \Theta$ *for some* $\Theta \in N$, $Re(h) \geq 0; Re(k) > 0; Re(u) > 0$. *Suppose a function f(z) is analytic at the origin with its Maclaurin's expansion given by* $f(z) = \sum_{n=0}^{\infty} a_n z^n (|z| < \rho_1)$ *for some* $\rho_1 \in R^+$*.Then, we have*

$$D_z^{(v;h;k;u)}\{f(z), y\} := \sum_{n=0}^{\infty} a_n D_z^{(v;h;k;u)}\{z^n, y\}.$$

Proof. Applying equation (17) to the function $f(z)$ with its series expansion, we have

$$D_z^{(v;h;k;u)}\{f(z), y\}$$

$$:= \frac{d^\Theta}{dz^\Theta}\left\{\frac{z^{\Theta-v}}{\Gamma(\Theta-v)}\int_0^{1-y}(t)^{\Theta-v-1}{}_1F_1\Big(\zeta;\eta;\frac{-h}{(1-t)^k(t)^u}\Big)\sum_{n=0}^{\infty}a_n((1-t)z)^n dt\right\}. \quad (25)$$

For any closed disk centered at the origin with its radius smaller then ρ_1, the power series converges uniformly. The series on the line segment from 0 to a fixed z for $|z| < \rho$, changing order of summation and integration, we have

$$D_z^{(v;h;k;u)}\{f(z), y\}$$

$$:= \sum_{n=0}^{\infty} a_n \frac{d^\Theta}{dz^\Theta}\left\{\frac{z^{\Theta-v}}{\Gamma(\Theta-v)}\int_0^{1-y}(t)^{\Theta-v-1}{}_1F_1\Big(\zeta;\eta;\frac{-h}{(1-t)^k(t)^u}\Big)((1-t)z)^n dt\right\},$$

$$D_z^{(v;h;k;u)}\{f(z), y\} := \sum_{n=0}^{\infty} a_n D_z^{(v;h;k;u)}\{z^n, y\}. \quad (26)$$

□

Theorem 2.5. *Let* $\Theta - 1 \leq Re(v) < \Theta < Re(\lambda)$ *for some* $\Theta \in N$, $Re(h) \geq 0; Re(k) > 0; Re(u) > 0$. *Suppose a function f(z) is analytic at the origin with its Maclaurin expansion given by* $f(z) = \sum_{n=0}^{\infty} a_n z^n (|z| < \rho_1)$ *for some* $\rho_1 \in R^+$. *Then, we have*

$$D_z^{(v;h;k;u)}[z^{\lambda-1}f(z), y] := \sum_{n=0}^{\infty} a_n D_z^{(v;h;k;u)}[z^{\lambda+n-1}, y],$$

$$:= \frac{\Gamma(\lambda)z^{\lambda-v-1}}{\Gamma(\lambda-v)}\sum_{n=0}^{\infty} a_n \frac{(\lambda)_n}{(\lambda-v)_n}\frac{B_{h,y}^{\zeta,\eta;k;u}(\lambda+n,\Theta-v)z^n}{B(\lambda+n,\Theta-v)}. \quad (27)$$

and its counterpart is given by

$$D_z^{(v;h;k;u)}\{z^{\lambda-1}f(z), y\} := \sum_{n=0}^{\infty} a_n D_z^{(v;h;k;u)}\{z^{\lambda+n-1}, y\},$$

$$:= \frac{\Gamma(\lambda)z^{\lambda-v-1}}{\Gamma(\lambda-v)}\sum_{n=0}^{\infty} a_n \frac{(\lambda)_n}{(\lambda-v)_n}\frac{B_{h,1-y}^{\zeta,\eta;k;u}(\lambda+n,\Theta-v)z^n}{B(\lambda+n,\Theta-v)}. \quad (28)$$

Proof. To prove theorem 2.5, If we use $f(z) = \sum_{n=0}^{\infty} a_n z^n$ in equation (16) and (17) and applying results of theorems 2.1 and 2.2, after simplifying them we get the desired result. □

Here we present two subsequent theorems which may be useful to find certain generating functions and their relations.

Theorem 2.6. *Let* $\Theta - 1 \leq Re(\lambda - v) < \Theta < Re(\lambda)$ *for some* $\Theta \in N$, $Re(h) \geq 0; Re(k) > 0; Re(u) > 0$. *Then we have*

$$D_z^{(\lambda-v;h;k;u)}[z^{\lambda-1}(1-z)^{-\zeta}, y]$$

$$:= \frac{\Gamma(\lambda) z^{v-1}}{\Gamma(v)} \sum_{n=0}^{\infty} \frac{(\zeta)_n (\lambda)_n}{(v)_n} \frac{B_{h,y}^{\zeta,\eta;k,u}(\lambda+n, v-\lambda+\Theta)}{B(\lambda+n, v-\lambda+\Theta)} \frac{z^n}{n!}. \tag{29}$$

Proof. Using the binomial theorem:

$$(1-z)^{-\zeta} := \sum_{n=0}^{\infty} \frac{(\zeta)_n}{n!} z^n, \quad (|z| < 1; \zeta \in C),$$

we may write

$$D_z^{(\lambda-v;h;k,u)}[z^{\lambda-1}(1-z)^{-\zeta}, y] := D_z^{(\lambda-v;h;k,u)}[z^{\lambda-1} \sum_{n=0}^{\infty} (\zeta)_n \frac{z^n}{n!}, y]$$

$$:= \sum_{n=0}^{\infty} \frac{(\zeta)_n}{n!} D_z^{(\lambda-v;h;k,u)}[z^{\lambda+n-1}, y] \tag{30}$$

using equation (18) in equation (30), after some simplification we get the desired result
□

Theorem 2.7. *Let if* $\Theta - 1 \leq Re(\lambda - v) < \Theta < Re(\lambda)$ *for some* $\Theta \in N$, $Re(h) \geq 0; Re(k) > 0; Re(u) > 0$. *Then*

$$D_z^{(\lambda-v;h;k;u)}\{z^{\lambda-1}(1-z)^{-\zeta}, y\}$$

$$:= \frac{\Gamma(\lambda) z^{v-1}}{\Gamma(v)} \sum_{n=0}^{\infty} \frac{(\zeta)_n (\lambda)_n}{(v)_n} \frac{B_{h,1-y}^{\zeta,\eta;k,u}(\lambda+n, v-\lambda+\Theta)}{B(\lambda+n, v-\lambda+\Theta)} \frac{z^n}{n!}. \tag{31}$$

Proof. To prove this theorem, we follow the process defined in theorem (2.6) and we obtain the required result.

□

Theorem 2.8. *Let* $\Theta - 1 \leq Re(\lambda - v) < \Theta < Re(\lambda)$ *for some* $\Theta \in \mathbb{N}$, $Re(h) \geq 0; Re(k) > 0; Re(u) > 0$. *Then, we have*

$$D_z^{\lambda-v;h;k,u}[z^{\lambda-1}(1-az)^{-\zeta}(1-bz)^{-\eta}, y]$$

$$:= \frac{\Gamma(\lambda)}{\Gamma(v)} z^{v-1} \sum_{n,p=0}^{\infty} \frac{(\zeta)_n (\lambda)_{n+p} (\eta)_p}{(v)_{n+p}} \frac{B_{h,y}^{\zeta,\eta;k,u}(\lambda+n+p, \Theta-\lambda+v)}{B(\lambda+n+p, \Theta-\lambda+v)} \frac{(az)^n}{n!} \frac{(bz)^p}{p!} \tag{32}$$

$$(|az| < 1, |bz| < 1; a, b, \zeta, \eta \in \mathbb{C})$$

and its counterpart

$$D_z^{\lambda-v;h;k,u}\{z^{\lambda-1}(1-az)^{-\zeta}(1-bz)^{-\eta}, y\}$$

$$:= \frac{\Gamma(\lambda)}{\Gamma(v)} z^{v-1} \sum_{n,p=0}^{\infty} \frac{(\zeta)_n(\lambda)_{n+p}(\eta)_p}{(v)_{n+p}} \frac{B_{h,1-y}^{\zeta,\eta;k,u}(\lambda+n+p, \Theta-\lambda+v)}{B(\lambda+n+p, \Theta-\lambda+v)} \frac{(az)^n}{n!} \frac{(bz)^p}{p!}, \tag{33}$$

$$(|az| < 1, |bz| < 1; a, b, \zeta, \eta \in \mathbb{C})$$

Proof. Using the binomial theorem for

$$(1-az)^{-\zeta} := \sum_{n=0}^{\infty} \frac{(\zeta)_n (az)^n}{n!},$$

and

$$(1-bz)^{-\eta} := \sum_{p=0}^{\infty} \frac{(\eta)_p (bz)^p}{p!},$$

we have

$$D_z^{\lambda-v;h;k,u}[z^{\lambda-1}(1-az)^{-\zeta}(1-bz)^{-\eta}, y] := D_z^{\lambda-v;h;k,u}[z^{\lambda-1} \sum_{n,p=0}^{\infty} \frac{(\zeta)_n(\eta)_p a^n b^p z^{n+p}}{n!p!}, y],$$

changing the order of integration and summation, we get

$$:= \sum_{n,p=0}^{\infty} \frac{(\zeta)_n(\eta)_p a^n b^p}{n!p!} D_z^{\lambda-v;h;k,u}[z^{\lambda+n+p-1}; y],$$

using equation (18) with some simplifications, we yield the desired result. □

Theorem 2.9. *Let* $\Theta - 1 \leq Re(\lambda - v) < \Theta < Re(\lambda)$ *for some* $\Theta \in N$, $Re(h) \geq 0; Re(k) > 0; Re(u) > 0$. *Then, we have*

$$D_z^{\lambda-v;h;k,u}[z^{\lambda-1}(1-az)^{-\zeta}(1-bz)^{-\eta}(1-cz)^{-\gamma}, y]$$

$$:= \frac{\Gamma(\lambda)}{\Gamma(v)} z^{v-1} \sum_{p,q,r=0}^{\infty} \frac{(\lambda)_{p+q+r}(\zeta)_p(\eta)_q(\gamma)_r}{(v)_{p+q+r}} \frac{B_{h,y}^{\zeta,\eta,;k,u}(\lambda+p+q+r, v-\lambda+\Theta)}{B(\lambda+p+q+r, v-\lambda+\Theta)} \frac{(az)^p}{p!} \frac{(bz)^q}{q!} \frac{(cz)^r}{r!}, \tag{34}$$

and its counterpart is

$$D_z^{\lambda-v;h;k,u}\{z^{\lambda-1}(1-az)^{-\zeta}(1-bz)^{-\eta}(1-cz)^{-\gamma}, y\}$$

$$:= \frac{\Gamma(\lambda)}{\Gamma(v)} z^{v-1} \sum_{p,q,r=0}^{\infty} \frac{(\lambda)_{p+q+r}(\zeta)_p(\eta)_q(\gamma)_r}{(v)_{p+q+r}} \frac{B_{h,1-y}^{\zeta,\eta,;k,u}(\lambda+p+q+r, v-\lambda+\Theta)}{B(\lambda+p+q+r, v-\lambda+\Theta)} \frac{(az)^p}{p!} \frac{(bz)^q}{q!} \frac{(cz)^r}{r!}, \tag{35}$$

$$(|az| < 1; |bz| < 1; |cz| < 1; a, b, \zeta, \eta, \gamma \in \mathbb{C}).$$

Proof. we prove the above theorem by using the binomial property and changing the order of integration and summation, Finally using equation (18), we get the desired result. □

Theorem 2.10. *Let $\Theta - 1 \leq Re(\lambda - v) < \Theta < Re(\lambda)$ and $\Theta < Re(\eta) < Re(\gamma)$, for some $\Theta \in N$, $Re(h) \geq 0; Re(k) > 0; Re(u) > 0$. Then we have*

$$D_z^{\lambda-v;h;k,u}[z^{\lambda-1}(1-z)^{-\zeta}F_{h;k,u}(\zeta,\eta;\gamma;\frac{x}{1-z};\Theta),y]$$

$$:= \frac{\Gamma(\lambda)}{\Gamma(u)}z^{v-1}\sum_{n,p=0}^{\infty}\frac{(\zeta)_{n+p}(\eta)_n(\lambda)_p}{(\gamma)_n(v)_p}\frac{B_h^{\zeta,\eta,;k,u}(\eta+n,\gamma-\eta+\Theta)}{B(\eta+n,\gamma-\eta+\Theta)}\frac{B_{h,y}^{\zeta,\eta,;k,u}(\lambda+p,v-\lambda+\Theta)}{B(\lambda+p,v-\lambda+\Theta)}\frac{x^n}{n!}\frac{z^p}{p!}. \tag{36}$$

Proof. Using the binomial theorem for $(1-z)^{-\zeta}$ and applying the equation (5) for $F_{h;k,u}(-)$ we get

$$D_z^{\lambda-v;h;k,u}[z^{\lambda-1}(1-z)^{-\zeta}F_{h;k,u}(\zeta,\eta;\gamma;\frac{x}{1-z};\Theta),y]$$

$$:= D_z^{\lambda-v;h;k,u}[z^{\lambda-1}(1-z)^{-\zeta}\sum_{n=0}^{\infty}\frac{(\zeta)_n(\eta)_n}{(\gamma)_n n!}\frac{B_h{}^{\zeta,\eta,;k,u}(\eta+n,\gamma-\eta+\Theta)}{B(\eta+n,\gamma-\eta+\Theta)}(\frac{x}{1-z})^n,y],$$

changing the order of integration and summation, we have

$$:= \sum_{n=0}^{\infty}\frac{(\zeta)_n(\eta)_n}{(\gamma)_n}\frac{B_h{}^{\zeta,\eta,;k,u}(\eta+n,\gamma-\eta+\Theta)}{B(\eta+n,\gamma-\eta+\Theta)}\frac{x^n}{n!}D_z^{\lambda-v;h;k,u}[z^{\lambda-1}(1-z)^{-\zeta-n},y]. \tag{37}$$

Finally, using equation (29), in the above expression, we obtain the result. □

Theorem 2.11. *Let if $\Theta - 1 \leq Re(\lambda - v) < \Theta < Re(\lambda)$ and $\Theta < Re(\eta) < Re(\gamma)$ for some $\Theta \in N$, $Re(h) \geq 0; Re(k) > 0; Re(u) > 0$. Then*

$$D_z^{\lambda-v;h;k,u}\{z^{\lambda-1}(1-z)^{-\zeta}F_{h;k,u}(\zeta,\eta;\gamma;\frac{x}{1-z};\Theta),y\}$$

$$:= \frac{\Gamma(\lambda)}{\Gamma(u)}z^{v-1}\sum_{n,p=0}^{\infty}\frac{(\zeta)_{n+p}(\eta)_n(\lambda)_p}{(\gamma)_n(v)_p}\frac{B_h^{\zeta,\eta,;k,u}(\eta+n,\gamma-\eta+\Theta)}{B(\eta+n,\gamma-\eta+\Theta)}\frac{B_{h,1-y}^{\zeta,\eta,;k,u}(\lambda+p,v-\lambda+\Theta)}{B(\lambda+p,v-\lambda+\Theta)}\frac{x^n}{n!}\frac{z^p}{p!}. \tag{38}$$

Proof. Using the binomial theorem for $(1-z)^{-\zeta}$ and applying the equation (5) for $F_{h;k,u}(-)$ we get

$$D_z^{\lambda-v;h;k,u}\{z^{\lambda-1}(1-z)^{-\zeta}F_{h;k,u}(\zeta,\eta;\gamma;\frac{x}{1-z};\Theta),y\}$$

$$:= D_z^{\lambda-v;h;k,u}\{z^{\lambda-1}(1-z)^{-\zeta}\sum_{n=0}^{\infty}\frac{(\zeta)_n(\eta)_n}{(\gamma)_n n!}\frac{B_h{}^{\zeta,\eta,;k,u}(\eta+n,\gamma-\eta+\Theta)}{B(\eta+n,\gamma-\eta+\Theta)}(\frac{x}{1-z})^n,y\},$$

$$:= \sum_{n=0}^{\infty}\frac{(\zeta)_n(\eta)_n}{(\gamma)_n}\frac{B_h{}^{\zeta,\eta,;k,u}(\beta+n,\gamma-\eta+\Theta)}{B(\eta+n,\gamma-\eta+\Theta)}\frac{x^n}{n!}D_z^{\lambda-v;h;k,u}\{z^{\lambda-1}(1-z)^{-\zeta-n},y\}. \tag{39}$$

Now, using equation(31), we obtain the desired result. □

2.1. **special cases.**

- By combining the result of theorems (2.1) & (2.2), (2.3) & (2.4), (2.6) & (2.7) , (2.10) & (2.11), we get the simlilar properties established by Agrawal et al.[1].
- The results of theorems (2.5), (2.8), (2.9), together with their counterparts are also converted into the properties defined by Agrawal et al.[1].

3. Graphical representation and discussion

In this section, we discuss the changes of results in incomplete extended beta function with assumed parameters graphically.
The representation of incomplete beta functions $B_{h,s}$ and $B_{h,1-s}$ with a fixed s value as 0.25 for various values of h, k and u is shown in the figure 1 to 6 and table 1 to 6. Here we have chosen a fixed range for ζ between 1 to 5 and η between 0.5 to 1 and fixed values of x=0.5, y=0.2 $\{x, y > 0\}$.

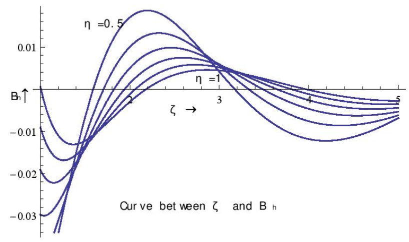

FIGURE 1. t from 0 to 0.25

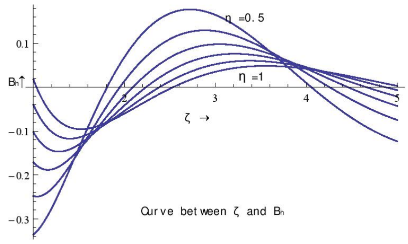

FIGURE 2. t from 0.25 to 1

(Curve between zeta and B_h with x=0.5, y=0.2, u=k=h=1)

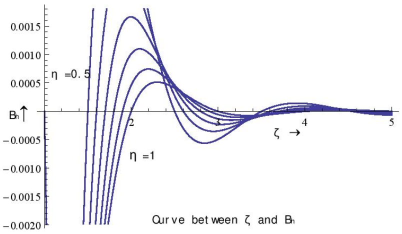

FIGURE 3. t from 0 to 0.25

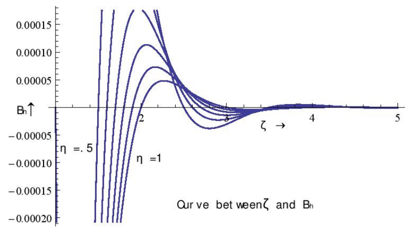

FIGURE 4. t from 0.25 to 1

(Curve between zeta and B_h with x=0.5, y=0.2, u=k=1, h=5)

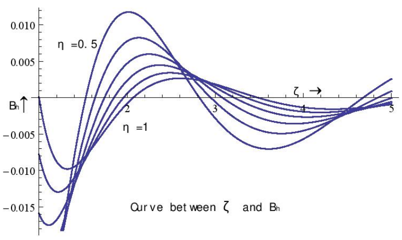

FIGURE 5. t from 0 to 0.25

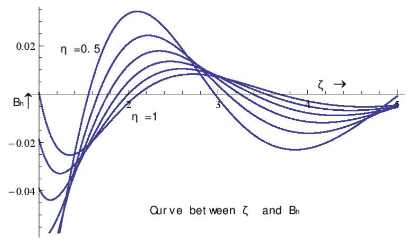

FIGURE 6. t from 0.25 to 1

(Curve between zeta and B_h with x=0.5,y=0.2,u=2,k=h=1)

	$\zeta = 1$		$\zeta = 2$		$\zeta = 3$		$\zeta = 4$		$\zeta = 5$	
η	${}_1F_1$	B_h	${}_1F_1$	B_h	${}_1F_1$	B_h	${}_1F_1$	B_h	${}_1F_1$	B_h
0.5	-0.0532	-0.0529	0.0120	0.0170	-0.0057	0.004	0.0007	-0.0118	0.0043	-0.0068
0.6	-0.0420	-0.0409	0.0085	0.0100	-0.0033	0.0053	-0.0004	-0.0075	0.00325	-0.0061
0.7	-0.0311	-0.0297	0.0056	0.0050	-0.0017	0.0059	-0.0010	-0.0046	0.0023	-0.0053
0.8	-0.02058	-0.0191	0.0032	0.0014	-0.0005	0.0056	-0.0012	-0.0026	0.0016	-0.0044
0.9	-0.0101	-0.0090	0.0013	-0.0011	0.0001	0.0051	-0.0012	-0.0011	0.0011	-0.0035
1.0	0.0000	0.0005	-0.0001	-0.0029	0.0006	0.0044	-0.0011	-0.0001	0.0007	-0.0027

Table of Figure-1

$\zeta = 1$			$\zeta = 2$		$\zeta = 3$		$\zeta = 4$		$\zeta = 5$	
η	${}_1F_1$	B_h	${}_1F_1$	B_h	${}_1F_1$	B_h	${}_1F_1$	B_h	${}_1F_1$	B_h
0.5	-0.1440	-0.3360	0.0521	0.0687	0.0602	0.1649	-0.0401	0.0098	-0.0744	-0.1233
0.6	-0.1076	-0.2479	0.0235	0.0141	0.0526	0.1281	-0.0176	0.0323	-0.0538	-0.0747
0.7	-0.0755	-0.1712	0.0046	-0.0202	0.0443	0.0979	-0.0040	0.0425	-0.0386	-0.0426
0.8	-0.0464	-0.1024	-0.0079	-0.0415	0.0363	0.0729	0.0041	0.0457	-0.0271	-0.0211
0.9	-0.0198	-0.0398	-0.0160	-0.0540	0.0288	0.0523	0.0089	0.0450	-0.0183	-0.0065
1.0	0.0048	0.0178	-0.0209	-0.0602	0.0219	0.0351	0.0115	0.0419	-0.0117	0.0031

Table of Figure-2

$\zeta = 1$			$\zeta = 2$		$\zeta = 3$		$\zeta = 4$		$\zeta = 5$	
η	${}_1F_1$	B_h	${}_1F_1$	B_h	${}_1F_1$	B_h	${}_1F_1$	B_h	${}_1F_1$	B_h
0.5	-0.0095	-0.0083	0.0002	0.0003	-0.0000	-0.00003	9.8*10−7	$4.4*10^{-6}$	$-9.75*10^{-8}$	$-1.04*10^{-6}$
0.6	-0.007	-0.0066	0.0001	0.0002	$-9.55*10^{-6}$	-0.00002	$6.79*10^{-7}$	$3.05*10^{-6}$	$-6.56*10^{-8}$	$-6.87*10^{-7}$
0.7	-0.005	-0.005	0.0001	0.00017	$-6.33*10^{-6}$	-0.00001	$4.36*10^{-7}$	$1.94*10^{-6}$	$-4.11*10^{-8}$	$-4.22*10^{-7}$
0.8	-0.0036	-0.0033	0.00008	0.0001	$-3.70*10^{-6}$	$-8.22*10^{-6}$	$2.47*10^{-7}$	$1.08*10^{-6}$	$-2.26*10^{-8}$	$-2.28*10^{-7}$
0.9	-0.001	-0.0016	0.0000	0.000	$-1.6*10^{-6}$	$-3.5*10^{-6}$	$1.0*10^{-7}$	$4.5*10^{-7}$	$-9.2*10^{-9}$	$-9.1*10^{-8}$
1.0	$7.4*10^{-25}$	$8.3*10^{-14}$	$-4.0*10^{-23}$	$-2.2*10^{-12}$	$1.0*10^{-21}$	$2.7*10^{-11}$	$-1.7*10^{-20}$	$-2.0*10^{-10}$	$2.1*10^{-19}$	$1.0*10^{-9}$

Table of Figure-3

$\zeta = 1$			$\zeta = 2$		$\zeta = 3$		$\zeta = 4$		$\zeta = 5$	
η	${}_1F_1$	B_h	${}_1F_1$	B_h	${}_1F_1$	B_h	${}_1F_1$	B_h	${}_1F_1$	B_h
0.5	-0.0199	-0.0474	0.0013	0.0032	-0.0001	-0.0004	0.00003	0.0001	$-9.38*10^{-6}$	-0.00006
0.6	-0.015	-0.0378	0.0009	0.0024	-0.0001	-0.0003	0.00002	0.00009	$-6.13*10^{-6}$	-0.00003
0.7	-0.0118	-0.0282	0.0006	0.0016	-0.00007	-0.0002	0.00001	0.00005	$-3.72*10^{-6}$	-0.00002
0.8	-0.0078	-0.0187	0.0004	0.0010	-0.00004	-0.0001	$7.42*10^{-6}$	0.00003	$-1.99*10^{-6}$	$-9.46*10^{-6}$
0.9	-0.0039	-0.0093	0.0001	0.0004	-0.00001	-0.00005	$3.07*10^{-6}$	0.00001	$-7.78*10^{-7}$	$-1.83*10^{-6}$
1.0	$2.62*10^{-12}$	$9.86*10^{-10}$	$-6.73*10^{-11}$	$-1.92*10^{-8}$	$7.95*10^{-10}$	$1.67*10^{-7}$	$-5.69*10^{-9}$	$-8.55*10^{-7}$	$2.74*10^{-8}$	$2.77*10^{-6}$

Table of Figure-4

$\zeta = 1$			$\zeta = 2$		$\zeta = 3$		$\zeta = 4$		$\zeta = 5$	
η	${}_1F_1$	B_h	${}_1F_1$	B_h	${}_1F_1$	B_h	${}_1F_1$	B_h	${}_1F_1$	B_h
0.5	-0.0469	-0.0420	0.0088	0.0117	-0.0038	-0.0028	0.0014	-0.0056	0.0016	0.0025
0.6	-0.0371	-0.0330	0.0063	0.0077	-0.0023	-0.0005	0.0004	-0.0045	0.0014	0.0009
0.7	-0.0275	-0.0242	0.0042	0.0047	-0.0013	0.0007	-0.00007	-0.0035	0.0012	-0.00003
0.8	-0.0181	-0.0158	0.0025	0.0023	-0.00062	0.00143	-0.0003	-0.0026	0.0009	-0.0005
0.9	-0.0089	-0.0077	0.0010	0.0005	-0.0001	0.0017	0.0004	-0.0019	0.0007	-0.0008
1.0	$4.34*10^{-6}$	0.0001	-0.00004	-0.0007	0.0002	0.0018	-0.0005	-0.0013	0.0005	-0.0009

Table of Figure-5

$\zeta = 1$			$\zeta = 2$		$\zeta = 3$		$\zeta = 4$		$\zeta = 5$	
η	${}_1F_1$	B_h	${}_1F_1$	B_h	${}_1F_1$	B_h	${}_1F_1$	B_h	${}_1F_1$	B_h
0.5	-0.0961	-0.105	0.0393	0.0332	0.001	-0.0017	-0.0334	-0.0225	-0.0041	-0.0008
0.6	-0.0742	-0.0818	0.0240	0.0209	0.0072734	0.0035	-0.0234	-0.0162	-0.0078	-0.0039
0.7	-0.0538	-0.0599	0.0130	0.0118	0.0101	0.0062	-0.016	-0.011	-0.0091	-0.0051
0.8	-0.0346	-0.0389	0.0050	0.0051	0.0110	0.0073	-0.010	-0.0078	-0.009	-0.0054
0.9	-0.0164	-0.0188	-0.0008	0.00009	0.0107	0.007	-0.0066	-0.0050	-0.0083	-0.0051
1.0	0.0008	0.0005	-0.0049	-0.0035	0.0098	0.0074	-0.0036	-0.0029	-0.0072	-0.0046

Table of Figure-6

- In the above figures, it can easily be seen that the variations in incomplete beta function are very high for small values of η and the graph shows flat behaviour as Zeta increases.
- For less h values, beta function will attain similar values after ζ greater than 10.
- For high h values, beta function has high variations between ζ 1 to 3 and shows flat behaviour after ζ greater than 3.
- In last curves 5 and 6, we have chosen $u = 2$ which shows the similar behavior of incomplete beta functions as the above curves.

4. Conclusions

In this following investigation, we establish a new form of incomplete extended Riemann Liouville fractional integral and derivative operator with the help of incomplete extended beta function in terms of confluent hypergeometric function as its kernal. Further we developed some properties like power function, generating function for this new function. In viewing the last set of curves, it can easily be seen that the variations in incomplete beta functions will be approximately similar, whether we vary the parameter in the power of t or the parameter in the power of (1-t) in the kernel of incomplete extended beta function. The variation in x and y parameters does not make any sense as incomplete beta functions behave like they do in the range of x and y between (0, 1).

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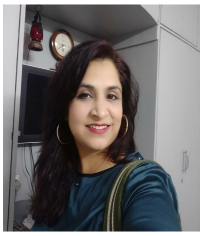

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