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FIXED SOFT POINT THEOREMS FOR GENERALIZED CONTRACTIVE MAPPING ON SOFT METRIC SPACES

K. ABD-RABOU[1], S. JAFARI[2], I. K. HALFA[1,3], K. BARAKAT[1,4], M. EL SAYED[5], §

Abstract. In this paper, we introduce new notions in a soft metric space. We study a fixed soft point under generalized contractive conditions without mappings continuity. Further, we prove some results related to our generalization. Moreover, we provide one example to present the application.

Keywords: Soft set, Soft metric space, contractive mapping, fixed soft point.

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1. Introduction

A soft set notion is introduced by Molodotsov [8] in 1999. He and others brought out applications for a soft set in different fields [1, 2, 6, 10, 11, 13]. In 2012, Das et al. [3] introduced the concepts of a realsoft set and a real soft number, also they studied some of their properties. The concept of a soft metric and some of its main characters are introduced by Das et al. [4]. The notion of a fixed point for a soft mapping was studied by Wardowski in [12]. In 2016, Hasan [5] introduced the notion of a soft metric on a soft set and investigated its properties. He studied the continuity of soft mappings on soft metric spaces. Moreover, the Banach contraction theorem on complete soft metric spaces was proved. In this paper, we create some notions of a soft metric space, and introduce the notion of a fixed soft point under generalized contractive conditions, also some properties of our generalization are explained and one example is given as an application.

[1] Department of Mathematics, Faculty of Science, Shaqra University, KSA.
e-mail: k_abdrabo@yahoo.com; ORCID: https://orcid.org/0000-0002-2527-8856.
[2] Department of Mathematics, College of Vestsjaelland South, Herrestraede 11, 4200 Slagelse, Denmark.
e-mail: jafaripersia@gmail.com; ORCID: https://orcid.org/0000-0001-5744-7354.
[3] Department of Mathematics and Theoritical Physics, NRC, Atomic Energy Authority, Cairo, Egypt.
e-mail: ihalfa@su.edu.sa.by; ORCID: https://orcid.org/0000-0002-1895-2734.
[4] Department of Mathematics, Faculty of Science, Helwan University, Egypt.
e-mail: khmohammed@su.edu.sa; ORCID: https://orcid.org/0000-0002-6440-7489.
[5] Department of Mathematics, College of Science and Arts, Najran University, KSA.
e-mail: mohammsed@yahoo.com; ORCID: https://orcid.org/0000-0003-1707-5327.
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2. Preliminaries

Definition 2.1 [8] A pair (F, E) is called a soft set over X, where F is a function given by $F : E \to P(X)$ and E is a set of parameters. In other words, a soft set over $X$is a parameterized family of subsets of the universe X. For any parameter $x \in E$, $F(x)$ may be considered as the set of x- approximate elements of the soft set (F, E)**.**

Definition 2.2 [7] Let $(F, E)$and (G, D) be two soft sets over X. We say that (F, E) is a sub soft set of $(G, D)$and denote it by $(F, E)\tilde{\subset}(G, D)$if :

(1) $E \subseteq D$, and
(2) $F(e) \subseteq G(e)$, $\forall\, e \in E$.

(F, E) is said to be a super soft set for(G, D), if (G, D) is a sub soft set of (F, E). We denote it by $(F, E) \supseteq (G, D)$.

Definition 2.3 [7] Let (F, E) be a soft set over X. Then

(1) (F, E) is said to be a null soft set denoted by $\tilde{\phi}$ if for every $e \in E$, $F(e) = \phi$.
(2) (F, E) is said to be an absolute soft set denoted by $\tilde{X}$, if for every $e \in E$, $F(e) = X$.

Definition 2.4 [5] Let $A \subseteq E$ be a set of parameters. The ordered pair (a, r), where $r \in R$and $a \in A$, is called a soft parametric scalar. The parametric scalar (a, r) is called nonnegative if $r \geq 0$. Let (a, r) and (b, r') be two soft parametric scalars, then (a, r) is called no less than (b, r') denoted by $(a, r) \geq (b, r')$, if $r \geq r'$.
Definition 2.5 [5] Let $A \subseteq E$ be a set of parameters,$\tilde{\pi} : E \times E \to E$ a parametric function and (a, r),(b, r') be two soft parametric scalars. Then an addition and a scalar multiplication of soft parametric scalars (a, r) and (b, r') are defined as follows:

$(a, r) \dot{+} (b, r') = (\tilde{\pi}(a, b), r + r')$ and $\lambda(a, r) = (a, \lambda r)$, for every $\lambda \in R$.

Definition 2.6 [5] Let (F, E) be a soft set over$X$. A function f on $(F, E)$is called soft parametric scalar valued, if there are functions $f_1 : E \to E$ and $f_2 : F(E) \to R$ such that $f(F, E) = (f_1, f_2)(E, F(E))$.

Similarly, as an extension of the above defined parametric scalar valued function,$f : (F,E) \times (F,E) \to (E, R)$ given by $f(E \times \mathrm{E}, F(E) \times F(E)) = (f_1, f_2)(E \times \mathrm{E}, F(E) \times F(E))$, where $f_1 : E \times E \to E$ and $f_2 : F(E) \times F(E) \to R$.

Definition 2.7 [4] Let (F, E) be a soft set over X and $\tilde{\pi} : E \times E \to E$ a parametric function. The parametric scalar valued function $\tilde{d} : (F,E) \times (F,E) \to (E, R^+\{0\})$ is called a soft meter on (F, E) if $\tilde{d}$ satisfies the following conditions:

(1) $\tilde{d}((a, F(a)), (b, F(b))) \geq (\tilde{\pi}(a, b), 0)$, and equality holds, whenever $a = b$.
(2) $\tilde{d}((a, F(a)), (b, F(b))) = \tilde{d}((b, F(b)), (a, F(a)))$, for all $a, b \in E$.
(3) $\tilde{d}((a, F(a)), (c, F(c))) \leq \tilde{d}((a, F(a)), (b, F(b))) \dot{+} \tilde{d}((b, F(b)), (c, F(c)))$, for all $a, b, c \in E$.

Definition 2.8 [5] Let (F, E) be a soft set over X. An element $(c, F(c)) \in (F, E)$ is called a soft point of (F, E), where $c \in E$. In general, if for some $c \in E$, $x \in F(c)$, then it is not necessary that (c, x) belongs to (F, E), otherwise, (c, x) is considered as a soft point.

Definition 2.9 [5] Let $(F,\, E)$ be a soft set over X. A soft sequence in $(F,\, E)$ is a function $f:\, N \to (F,\, E)$ given by $f(n) = (F_n,\, E)$, such that $(F_n,\, E)$ is a soft subset of $(F,\, E)$, for $n \in N$ and is denoted by $\{(F_n,\, E)\}_{n=1}^{\infty}$.

3. Soft metric spaces

In this section, we consider X as a universal set, E a set of parameters and A as a set valued function on $E, A:\, E \to P(X)$, where $(A,\, E)$ is a soft set over X. Also, a soft sequence in $(A,\, E)$, denoted by $\{(A_n,\, E)\}_{n=1}^{\infty}$ and $(\tilde{X},\, \tilde{d})$ is a soft metric space over X, where $\tilde{X} = (A,\, E)$ and $\tilde{d}$ is a soft metric on $\tilde{X}$.

Definition 3.1 Let $(\tilde{X},\, \tilde{d})$ be a soft metric space over X. A soft sequence $\{(A_n,\, E)\}_{n=1}^{\infty}$ in $(A,\, E)$ is called convergent to $(e, A(e))$ if $\lim_{n \to \infty} \tilde{d}((\,a,\, A_n(a)),\, (e,\, A(e))) = (b,\, 0)$, $\;$ forall$a,\;e,\, b \in E$.

Definition 3.2 Let $(\tilde{X},\, \tilde{d})$ be a soft metric space over X. A soft sequence $\{(A_n,\, E)\}_{n=1}^{\infty}$ in $(A,\, E)$ is called Cauchy if $\lim_{n,\, m \to \infty} \tilde{d}((\,a,\, A_n(a)),\, (a,\, A_m(a))) = \tilde{0} = (b,\, 0)$,
for all $a,\, b \in\, E,\;\, m,\, n \in\, N$.

Definition 3.3 Let $(F,\, E)$ be a soft set over X and $\tilde{d}$ be a soft metric on $(F,\, E)$. Then $(\tilde{X},\;\tilde{d})$ is called a complete soft metric space if every Cauchy soft sequence converges in $(F,\, E)$.

Definition 3.4 Let f be a soft mapping $f\, :(A,\, E) \to\, (A,\, E), (x,\, A(x)) \in\, (A,\, E)$ such that $f\,(x,\, A(x)) =\, (x,\, A(x))$.Then $(x,\, A(x))$ is called:

(1) a fixed soft element for f.
(2) a fixed soft point for f if $A(x)$ is singleton.

Note : Each fixed soft point and a fixed soft elementis a soft set and clearly, every a fixed soft point is a fixed soft element.

Definition 3.5 Let $(A,\, E)$ be a soft set over X. We define the distance between two soft elements $(a,\, A\,(a))\,,\, (b,\, A\,(b)) \in\, (A,\, E)$ as follows:

$$\begin{aligned}\tilde{D}\,((a,\, A\,(\,a),\, (b,\, A\,(\,b)) &= \inf_{x\in A(a),\, y\in A(b)} \tilde{d}\,((a,\, \{\,x\},\, (b,\, \{\,y\}) \\ &= (\min\{a,\, b\},\, \inf_{x\in A(a),\, y\in A(b)} d(x,y)), \quad a,\, b \in\, E,\end{aligned}$$

where d is a metric on X, it is clear that $\tilde{D}$ is a soft metric on $\tilde{X}$.
If $A(a)$ consists of a single point x, then the distance between $(a,\, \{x\})$and $(b,\, A(b))$ is $\tilde{D}\,((a,\, \{x\}),\, (b,\, A(b))) = (\min\{a,\, b\},\, \inf_{y\in A(b)} d(x,y))$.
And also

$$\begin{aligned}\tilde{\delta}\,((a,\, A\,(\,a),\, (b,\, A\,(\,b)) &= \sup_{x\in A(a),\, y\in A(b)} \tilde{d}\,((a,\, \{\,x\},\, (b,\, \{\,y\}) \\ &= (\max\{a,\, b\},\, \sup_{x\in A(a),\, y\in A(b)} d(x,y)), \quad a,\, b \in\, E.\end{aligned}$$

It is obvious that $\tilde{\delta}$ is a soft metric.

Theorem 3.6 Let $(\tilde{X}, \tilde{d})$ be a complete soft meric space, and $T : \tilde{X} \to \tilde{X}$ a soft mapping such that whenever $(a, A(a)), (b, A(b)) \in \tilde{X}$,

(1) $\tilde{d}(T(a, A(a)), T(b, A(b))) \leq \max\{\alpha\,\tilde{d}((a, A(a)), (b, A(b))), \alpha\,\tilde{d}((b, A(b)), T(b, A(b))),$
$\alpha\,\tilde{d}((a, A(a)), T(a, A(a))), \mu\,\tilde{d}((a, A(a)), T(b, A(b))) + \lambda\,\tilde{d}((b, A(b)), T(a, A(a)))\}$
where $0 \leq \alpha < 1, \mu \geq 0, \lambda \geq 0, \mu + \lambda < 1$, and $\alpha. \max\{\frac{\mu}{1-\mu}, \frac{\lambda}{1-\lambda}\} < 1$.

Then T has a unique fixed soft point.

Proof

Let $(A_0, E), (A_1, E)$ be two soft elements in $\tilde{X}$, such that $(A_1, E) = T(A_0, E)$. Then we are able to define a soft sequence $\{(A_{n+1}, E)\}_{n=0}^{\infty}$ of soft elements such that $(A_{n+1}, E) = T(A_n, E)$, for $n = 0, 1, 2, ...$

Since

$$\tilde{d}((A_{n+1}, E), (A_n, E)) = \tilde{d}(T(A_n, E), T(A_{n-1}, E))$$

by (1), we have

$$\begin{aligned}&\tilde{d}((A_{n+1}, E), (A_n, E)) = \tilde{d}(T(A_n, E), T(A_{n-1}, E))\\ &\leq \max\{\alpha\,\tilde{d}((A_n, E), (A_{n-1}, E)), \alpha\,\tilde{d}((A_{n-1}, E), T(A_{n-1}, E)),\\ &\quad \alpha\,\tilde{d}((A_n, E), T(A_n, E)), \mu\,\tilde{d}((A_n, E), T(A_{n-1}, E)) + \lambda\,\tilde{d}((A_{n-1}, E), T(A_n, E))\}\end{aligned}$$

$$\begin{aligned}&= \max\{\alpha\,\tilde{d}((A_n, E), (A_{n-1}, E)), \alpha\,\tilde{d}((A_{n-1}, E), (A_n, E)),\\ &\quad \alpha\,\tilde{d}((A_n, E), (A_{n+1}, E)), \mu\,\tilde{d}((A_n, E), (A_n, E)) + \lambda\,\tilde{d}((A_{n-1}, E), (A_{n+1}, E))\}\end{aligned}$$

Since $0 \leq \alpha < 1$, $0 \leq \lambda < 1$ and
if $\alpha > \lambda$ thus

$$\tilde{d}((A_{n+1}, E), (A_n, E)) = \alpha\tilde{d}((A_n, E), (A_{n-1}, E))$$

if $\lambda > \alpha$ thus
$\tilde{d}((A_{n+1}, E), (A_n, E)) = \lambda\tilde{d}((A_{n+1}, E), (A_{n-1}, E))$
$\leq \lambda[\tilde{d}((A_{n+1}, E), (A_n, E)) + \tilde{d}((A_n, E), (A_{n-1}, E))]$,

thus
$\tilde{d}((A_{n+1}, E), (A_n, E)) \leq \frac{\lambda}{1-\lambda}\tilde{d}((A_n, E), (A_{n-1}, E))$.

Then

$$\tilde{d}((A_{n+1}, E), (A_n, E)) \leq \max\{\alpha, \frac{\lambda}{1-\lambda}\}\,\tilde{d}((A_n, E), (A_{n-1}, E))$$

By the same taken, we have

$$\tilde{d}((A_{n+2}, E), (A_{n+1}, E)) \leq \max\{\alpha, \frac{\mu}{1-\mu}\}\,\tilde{d}((A_{n+1}, E), (A_n, E))$$

Let $c = (\max\{\alpha, \frac{\mu}{1-\mu}\}. \max\{\alpha, \frac{\lambda}{1-\lambda}\})$, then $0 \leq c < 1$.

Thus

$$\tilde{d}((A_{n+2}, E), (A_{n+1}, E)) \leq c\,\tilde{d}((A_n, E), (A_{n-1}, E)), n = 1, 2, ...$$

$$\begin{aligned}\tilde{d}\,((A_{n+2},\,E),\ \ (A_{n+1},\,E)) &\le c\,\tilde{d}\,((A_n,\,E),\ \ (A_{n-1},\,E))\\ &\le c^2\,\tilde{d}\,((A_{n-2},\,E),\ \ (A_{n-3},\,E))\\ &\ \vdots\\ &\le c^n\,\tilde{d}\,((A_1,\,E),\ \ (A_0,\,E))\end{aligned}$$

For any $m > n$, and by the definition of a soft metric,

$$\begin{aligned}\tilde{d}\,((A_n,\,E),\ \ (A_m,\,E)) &\le \tilde{d}\,((A_n,\,E),(A_{n+1},\,E)) + ... + \tilde{d}\,((A_{m-1},\,E),(A_m,\,E))\\ &\le c^n\tilde{d}\,((A_1,\,E),(A_0,\,E)) + ... + c^{m-1}\,\tilde{d}\,((A_1,\,E),(A_0,\,E))\\ &\le (c^n + c^{n+1} + ... + c^{m-1})\ \ \tilde{d}\,((A_1,\,E),(A_0,\,E))\end{aligned}$$

Since $0 \le c < 1, \tilde{d}\,((A_n,\,E),\ \ (A_m,\,E)) \to \tilde{0}$ as $n,\ m \to \infty$, consequently the soft sequence $\{(A_n,\,E)\}_{n=0}^{\infty}$ is Cauchy . Since $(\tilde{X},\,\tilde{d})$ i s a complete soft meric space, there is $(a,\,A(a))$in $(A,\,E)$ such that $\{(A_n,\,E)\}_{n=0}^{\infty}$ converges to$(a,\,A(a))$. Thus
$T(A_n,\,E) \to (a,\,A(a))$, as $n \to \infty$.

Since $\alpha + \lambda < 1$,

$$\begin{aligned}&\tilde{d}\,(T\,(A_n,\,E),T\,(a,\,A(a)))\\ &\le \max\{\,\alpha\,\tilde{d}((a,\,A(a)),\,(A_n,\,E))\,,\,\alpha\,\tilde{d}((A_n,\,E),T\,(A_n,\,E)),\\ &\alpha\,\tilde{d}((a,\,A(a)),T\,(a,\,A(a))),\ \mu\,\tilde{d}((a,\,A(a)),T\,(A_n,\,E)) + \lambda\,\tilde{d}((A_n,\,E),T\,(a,\,A(a)))\,\}\end{aligned}$$

As $n \to \infty$,

$$\begin{aligned}&\tilde{d}\,(T\,(A_n,\,E),T\,(a,\,A(a))) = \tilde{d}\,((a,\,A(a)),T\,(a,\,A(a)))\\ &\le \max\{\,\alpha\,\tilde{d}\,((a,\,A(a)),T\,(a,\,A(a)))\,,\ \lambda\,\tilde{d}\,((a,\,A(a)),T\,(a,\,A(a)))\,\}\\ &\le (\alpha + \lambda)\,\tilde{d}\,((a,\,A(a)),T\,(a,\,A(a)))\,.\end{aligned}$$

Hence $T(a,\,A(a)) = (a,\,A(a))$.

Now, we show that $(a,\,A(a))$ is a unique fixed soft point for T. Suppose $(x,\,A(x)) \neq (a,\,A(a))$ such that $T(a,\,A(a)) = (a,\,A(a)), T(x,\,A(x)) = (x,\,A(x))$. Then from condition (1),

$$\begin{aligned}&\tilde{d}\,((x,\,A(x)),\,(a,\,A(a))) = \tilde{d}\,(T\,(x,\,A(x)),T\,(a,\,A(a)))\\ &\le \max\{\,\alpha\,\tilde{d}((a,\,A(a)),\,(x,\,A(x)))\,,\,\alpha\,\tilde{d}((x,\,A(x)),T\,(x,\,A(x))),\\ &\alpha\,\tilde{d}((a,\,A(a)),T\,(a,\,A(a))),\ \mu\,\tilde{d}((a,\,A(a)),T\,(x,\,A(x))) + \lambda\,\tilde{d}((x,\,A(x)),T\,(a,\,A(a)))\,\}\\ &\le \max\{\,\alpha\,\tilde{d}((a,\,A(a)),\,(x,\,A(x)))\,,\ \mu\,\tilde{d}((a,\,A(a)),\ (x,\,A(x))) + \lambda\,\tilde{d}((x,\,A(x)),\ (a,\,A(a)))\,\}\\ &\le (\mu + \lambda)\,\tilde{d}((a,\,A(a)),\ (x,\,A(x)).\end{aligned}$$

Hence $(x,\,A(x))$ is a unique fixed soft point for T.

Theorem 3.7 Let $(\tilde{X},\,\tilde{d})$ be a complete soft metric space over$X$ and $T : \tilde{X} \to \tilde{X}$ a soft mapping such that whenever $(a,\,A(a)),\,(b,\,A(b)) \in \tilde{X}$,

$$\begin{aligned}&\tilde{d}\,(T(a,\,A(a)),T\,(b,\,A(b))) \le c\ \max\{\,\tilde{d}((a,\,A(a)),\,(b,\,A(b)))\,,\ \tilde{d}((b,\,A(b)),T\,(b,\,A(b))),\\ &\tilde{d}((a,\,A(a)),T\,(a,\,A(a))),\ \frac{(\tilde{d}((a,A(a)),T\,(b,A(b))) + \tilde{d}((b,A(b)),T\,(a,A(a))))}{2}\,\}\end{aligned}$$

where $0 \le c < 1$.

Then T has a unique fixed soft point.

Proof

By condition (1) in Theorem 3.6, we have
$\mu\,\tilde{d}((a,\,A(a)),\,T\,(b,\,A(b))) +\,\lambda\,\tilde{d}((b,\,A(b)),\,T\,(a,\,A(a))))$
$\leq\ \max\{\,\mu,\,\lambda\}(\,\tilde{d}((a,\,A(a)),\,T\,(b,\,A(b))) +\ \tilde{d}((b,\,A(b)),\,T\,(a,\,A(a)))))$
$=\ \max\{\,2\mu,\,2\lambda\}(\,\frac{\tilde{d}((a,\,A(a)),\,T\,(b,\,A(b)))+\,\tilde{d}((b,\,A(b)),\,T\,(a,\,A(a)))}{2})$
If $c\ =\ \max\{\,2\mu,\,2\lambda,\,\alpha\,\}$. Hence the proof as in Theorem 3.6.

Theorem 3.8 Let $(\tilde{X},\,\tilde{d})$ be a complete soft metric space over $\tilde{X}$, and $T\,:\,\tilde{X}\to\tilde{X}$ a soft mapping such that whenever $(a,\,A(a)),\,(b,\,A(b))\,\in\,\tilde{X}$,
$\tilde{d}\,(\,T(a,\,A(a)),\,T\,(b,\,A(b)))\leq\,c\ \max\{\,\tilde{d}((a,\,A(a)),\,(b,\,A(b)))\,,\ \tilde{d}((b,\,A(b)),\,T\,(b,\,A(b))),\,\}$.
$\tilde{d}((a,\,A(a)),\,T\,(a,\,A(a)))$ where $0\leq c<\,1,\ a,\,b\,\in\,E.$
Then T has a unique fixed soft point.
Proof
If we put $\lambda=\ \mu\ =0,\ \alpha\ =c$ in Theorem 3.6, then the proof is complete.

Remark 3.9 Theorem 3.8 is a generalization for Theorem 4.3 [5] and Theorem 2 [9].

Theorem 3.10 Let $(\tilde{X},\,\tilde{d})$ be a complete soft metric space over X, and T be a soft mapping $T\,:\,\tilde{X}\to\tilde{X}$, such that whenever $(a,\,A(a)),\ (b,\,A(b))\,\in\,\tilde{X}$,

$$\tilde{\delta}\,(\,T(a,\,A(a)),\,T\,(b,\,A(b)))\leq\,c_1\,\tilde{\delta}((a,\,A(a)),\,(b,\,A(b)))\ +c_2\,\tilde{\delta}((a,\,A(a)),\,T\,(a,\,A(a)))$$
$$+c_3\,\tilde{\delta}((b,\,A(b)),\ T\,(b,\,A(b))),$$
where $c_i\geq 0,\,\forall\,i,\ \sum_{i=1}^{3}c_i\,<1$.

Then T has a unique common fixed soft set. Proof If we put $\lambda\,=\,0,\ \mu\ =0,\ \alpha\ =\ c_1+\ c_2+\ c_3$ in Theorem 3.6, then we are done.
Obviously, Theorem 3.10 is a generalization for Theorem 3 [9].

Theorem 3.11 Let $(\tilde{X},\,\tilde{d})$ be a complete soft metric space over X, and $T\,:\,\tilde{X}\to\tilde{X}$ be a soft mapping such that whenever $(a,\,A(a)),\,(b,\,A(b))\,\in\,\tilde{X}$,

$$\tilde{d}\,(\,T^m(a,\,A(a)),\,T^m(b,\,A(b)))\leq\max\{\,\alpha\,\tilde{d}((a,\,A(a)),\,(b,\,A(b)))\,,\,\alpha\,\tilde{d}((b,\,A(b)),\,T^m\,(b,\,A(b))),$$
$$\alpha\,\tilde{d}((a,\,A(a)),\,T^m\,(a,\,A(a))),\ \mu\,\tilde{d}((a,\,A(a)),\,T^m\,(b,\,A(b)))+\lambda\,\tilde{d}((b,\,A(b)),\,T^m\,(a,\,A(a)))\,\},$$
where $0\leq\alpha<\,1,\,\mu\geq 0,\,\lambda\geq 0,\,\mu+\lambda<1$, and $\alpha.\,\max\{\,\frac{\mu}{1-\mu},\,\frac{\lambda}{1-\lambda}\}<1.$

Then T has a unique fixed soft point.
Proof
It follows from Theorem 3.6

$$T^m(a,\,A(a))=\,(a,\,A(a)),$$
$$T^m(T(a,\,A(a)))=T(T^m(a,\,A(a)))=T(a,\,A(a))$$

Thus $T(a,\,A(a))$and $(a,\,A(a))$ are fixed soft points for T^m.
Since
$\tilde{d}\,(T(a,\,A(a)),\,(a,\,A(a)))\,\leq\ \tilde{d}\,(T(a,\,A(a)),\,T^m\,(a,\,A(a)))+\tilde{d}\,(T^m(a,\,A(a)),\,(a,\,A(a))),$
$T(a,\,A(a))=(a,\,A(a)),\,(a,\,A(a))$ is a fixed soft point for T.

We shall give an example, as an application for Theorem 3.11.
Example 3.12

Let $X = [0, \infty)$, $\tilde{X} = (A, E)$, $E = Q^+ \bigcup\{0\}$, and for all $(a, A(a)) \in (A, E)$, we consider $(a, A(a)) = [0, a]$. Define a map $T : \tilde{X} \to \tilde{X}$ such that $T(a, A(a)) = (\frac{a}{3}, A(\frac{a}{3}))$.

Also, let $\tilde{d} : \tilde{X} \,\mathrm{x}\, \tilde{X} \to (E, R^+ \bigcup\{0\})$ be such that

$$\begin{aligned} \tilde{\delta}((a, A(a)), (b, A(b))) &= \sup_{x \in A(a), y \in A(b)} \tilde{d}((a, \{x\}), (b, \{y\})) \\ &= (\max\{a, b\}, \sup_{x \in A(a), y \in A(b)} \{|x - y|\}), \end{aligned}$$

then it is clearthat $(\tilde{X}, \tilde{d})$is a complete soft metric space.

$$\tilde{d}(T(a, A(a)), T(b, A(b))) = \tilde{d}((\frac{a}{3}, A(\frac{a}{3})), (\frac{b}{3}, A(\frac{b}{3}))) = (\max\{\frac{a}{3}, \frac{b}{3}\}, \max\{\frac{a}{3}, \frac{b}{3}\})$$

$$\begin{aligned} &= \tfrac{1}{3}(\max\{\tfrac{a}{3}, \tfrac{b}{3}\}, \max\{a, b\}) \\ &\leq \max\{\tfrac{1}{3}(\max\{a, b\}, \max\{a, b\}), \tfrac{1}{3}(\max\{a, \tfrac{a}{3}\}, \max\{a, \tfrac{a}{3}\}), \tfrac{1}{3}(\max\{b, \tfrac{b}{3}\}, \max\{b, \tfrac{b}{3}\})\} \\ &\leq \max\{\tfrac{1}{3}\tilde{d}((a, A(a)), (b, A(b))), \tfrac{1}{3}\tilde{d}((a, A(a)), (\tfrac{a}{3}, A(\tfrac{a}{3}))), \tfrac{1}{3}\tilde{d}((b, A(b)), (\tfrac{b}{3}, A(\tfrac{b}{3})))\} \\ &\leq \max\{\tfrac{1}{3}\tilde{d}((a, A(a)), (b, A(b))), \tfrac{1}{3}\tilde{d}((a, A(a)), T(a, A(a))), \tfrac{1}{3}\tilde{d}((b, A(b)), T(b, A(b)))\} \end{aligned}$$

$$\begin{aligned} \leq \max\{&\tfrac{1}{3}\tilde{d}((a, A(a)), (b, A(b))), \tfrac{1}{3}\tilde{d}((a, A(a)), T(a, A(a))), \tfrac{1}{3}\tilde{d}((b, A(b)), T(b, A(b))), \\ &\tfrac{1}{5}\tilde{d}((a, A(a)), T(b, A(b))) + \tfrac{1}{4}\tilde{d}(T(a, A(a)), (b, A(b)))\} \end{aligned}$$

If $\alpha = \frac{1}{3}$, $\mu = \frac{1}{5}$ *and* $\lambda = \frac{1}{4}$, then $(0, A(0)) = \tilde{0}$ is a unique fixed soft point for the mapping T.

4. Conclusion

In this article, we have inserted new conceptions in a soft metric space.We have discussed a fixed soft point under generalized contractive conditions without continuity of mappings. In addition, we have compared our own results with those reached by other authors [5, 9] in this field, and found out that our results are more comprehensive and general. Moreover, we have given one example as an application.

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Khaled Abd-Rabou is an associate professor of mathematics in the Faculty of Science at Shaqra University, Al-qawwiya, Kingdom of Saudi Arabia. He has his M.Sc. degree in mathematics from the Faculty of Science, Zagazig University, Egypt and his Ph.D. in mathematics from the Faculty of Science, Tanta University, Egypt. His research interests are Function analysis and Topology and especially Fixed point theorem and its applications.

Saeid Jafari for the photography and short autobiography, see the current volume, pp. 425-434.

I. K. Halfa received his Ph.D. degree in mathematics for science at Tanta university, Egypt. He works in the Department of Mathematics, Atomic Energy Authority, Cairo, Egypt. His research interests are in the areas of pure mathematics, namely in topology, fuzzy topology, rough set theory, fuzzy and soft metric spaces. Moreover, applications of topology in nuclear physics and radiation pollutions.

K. Barakat received his Ph. D degree in mathematics for science at Helwan university, Egypt. He works in the Department of Mathematics, Helwan university, Cairo, Egypt. His research interests are in the areas of pure mathematics, namely in topology, fuzzy topology, intutionistic fuzzy topology, fuzzy and soft metric spaces.

Mohamed El Sayed received his Ph.D. degree in Mathematics Science from Tanta University, Department of Mathematics, Faculty of Science in 2011. His research interests are general topology and its applications, soft set theory, soft topology, soft fuzzy topological structures and its applications and rough sets.