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EXPONENTIAL FRACTION INDEX OF CERTAIN GRAPHS

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Abstract. Topological indices play a great role in Mathematical chemistry. Many graph theorists as well as chemists attracted towards these molecular descriptors. The aim of this paper is to introduce and investigate the Exponential Fraction index (a degree based Topological index). It is defined as follows.

$$EF(G) = \sum_{uv \in E(G)} e^{\frac{d_u}{d_v}}.$$

Here d_u and d_v are the maximum and minimum degree respectively. In this paper, we calculate the Exponential Fraction index of double graphs, subdivision graphs and complements of some standard graphs. Also we compute the index for chemical structures Graphene and Carbon nanocones.

Keywords: Exponential Fraction index, double graphs, subdivision graphs, k-complement, k(i)-complement.

AMS Subject Classification: 83-02, 99A00.

1. Introduction

Throughout the paper, we have considered the graph structures which are simple, undirected, loop free, mark free and signless graphs. Topological indices play a vital role in the field of Mathematical Chemistry. It is a numerical quantity of a molecule that is evaluated from the structural graph of a molecule. These molecular descriptors can be used in modelling physical, pharmacological, biological and other properties of chemical compounds. Here we are introducing a new molecular descriptor, that is the Exponential Fraction index which is defined in terms of the vertex degrees as follows:

$$EF(G) = \sum_{uv \in E(G)} e^{\frac{d_u}{d_v}}$$

Here d_u and d_v are the maximum and minimum degrees respectively. Here we give few definitions for the different graph structures which we used in the later sections.

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Definition 1.1. [8] *The complement of a graph G is a graph $\overline{G}$ having the same vertex set such that two distinct vertices of $\overline{G}$ are adjacent if and only if they are not adjacent in G.*

Definition 1.2. [8] *Let G be a graph and $P_k = \{V_1,\ V_2,\ \cdots,\ V_k\}$ be a partition of its vertex set V. Similarly to the complement of a graph, the k-complement of G is defined as follows: For all V_i and V_j in P_k for $i \neq j$, remove the edges between V_i and V_j and add the edges between the vertices of V_i and V_j which are not in G and is denoted by $\overline{G_k}$.*

Definition 1.3. [8] *Let G be a graph and $P_k = \{V_1,\ V_2,\ \cdots,\ V_k\}$ be a partition of its vertex set V. Then the $k(i)$-complement of G, yet another type of the complement of a graph, is obtained as follows: For each partition set V_r in P_k, remove the edges of G joining the vertices in V_r and add the edges of $\overline{G}$ (complement of G) joining the vertices of V_r, and is denoted by $\overline{G_{k(i)}}$.*

Definition 1.4. *The subgraph or subdivision graph $S(G)$ of a simple graph G is defined as the new graph obtained by adding an extra vertex into each edge of G.*

The different toplogical indices for subgraphs have been studied in literature [7], [13], [9], and [6].

Definition 1.5. [2] *For a graph G with vertex set $V(G) = \{v_1,\ v_2,\ \cdots,\ v_n\ \}$, we take another copy of G with vertices labelled by $\{v_1,\ v_2,\ \cdots,\ v_n\}$, this time, where v_i corresponds to v_i for each i. If we connect v_i to the neighbours of v_i for each i, we obtain a new graph called the double graph of G. It is denoted by $D(G)$.*

Togan, M. et.al., studied different topological indices for double graph, subdivision graphs, subdivision graph of double graphs and r-subdivision graphs of double graphs in [9],[10],[11] and [12]. In this paper we study the Exponential Fraction index for different graph structures. We also find this molecular descriptor for Graphene and Carbon nanocones.

In Graphene each and every atom is available for chemical reaction from two sides. It is of the honeycomb lattice structure formed by the carbon atoms. Due to it's property of electron mobility, it can be used in electronic devices. Here we obtained the Exponential Fraction index for the structure Graphene.

Carbon nanocones are very important structures whose bases are attached to the graphite. those of the laboratory made nanocones. In gas sensors, biosensors and to store the energy, these nano structures will be used.

Here following notations are used to represent different graphs: P_n represents Path graph, S_n^0 represents Crown graph, $K_{n\times 2}$ represents cocktail party graph, C_n is Cycle graph,K_n represents Complete graph, $K_{1,n-1}$ represents Star graph, F_n^3 represents Friendship graph, D_4^n represents Dutch windmill graph, W_n represents Wheel graph, $T_(n, m)$ represents Tadpole graph, $D(G)$ represents Double graph, $(\overline{G})$ represents Complement of a graph, $S(P_n)$ represents Subdivision graphs, $S(D(P_n))$ represents Subdivision of Double graph, $D(S(P_n))$ represents Double graph of subdivision graph.

2. Exponential Fraction index of some standard graphs

Theorem 2.1.

$$EF(G) = \begin{cases} (n-3)e + 2e^2 & \textit{if } G = P_n \\ n(n-1)e & \textit{if } G = S_n^0 \\ n(n+1)e & \textit{if } G = K_{n\times 2} \\ ne & \textit{if } G = C_n \\ \frac{n(n-1)}{2}e & \textit{if } G = K_n \\ (n-1)e^{n-1} & \textit{if } G = K_{1,n-1} \\ 2ne^n + ne & \textit{if } G = F_n^3 \\ 2n(e+e^n) & \textit{if } G = D_4^n \\ (n-1)e^{\frac{n-1}{3}} + (n-1)e & \textit{if } G = W_n \\ (m+n)e + e^2 + 3e^{\frac{3}{2}} - 4e & \textit{if } G = T_(n,m) \end{cases}$$

Proof. We prove the theorem for Friendship graphs. Similar methods can be used for others. Let G be a Friendship graph F_n^3. Here we have $2n$ vertices with degree 2 and $2n$ in the centers of Friendship graph. We have n edges between the vertices of degree 2. There are $2n$ edges betwen the central vertex and remaining vertices of degree 2.
According to the definition of Exponential Fraction index,

$$\begin{aligned} EF(F_n^3) &= ne^{\frac{2}{2}} + 2ne^{\frac{2n}{2}} \\ &= ne + 2ne^n. \end{aligned}$$

□

3. Exponent Fraction index of double graphs

Theorem 3.1.

$$EF(D(G)) = \begin{cases} 8e^2 + 4(n-3)e & \textit{if } G = P_n, n \geq 3 \\ 4ne & \textit{if } G = C_n, n \\ 2n(n-1)e & \textit{if } G = K_n \\ 4n(n-1)e^{n-1} & \textit{if } G = S_n^0 \\ 8n(n-1)e & \textit{if } G = K_{n\times 2} \\ 4ne + 8ne^n & \textit{if } G = F_n^3 \\ 7ne + 9ne^n & \textit{if } G = D_4^n \\ 4(n-1)e^{n-1} & \textit{if } G = K_{1,n-1} \\ 4(n-1)e + 4(n-1)e^{\frac{n-1}{3}} & \textit{if } G = W_n \end{cases}$$

Proof. We prove the theorem for Double graph of Cycle. Similar methods can be used for others. Let G be a Double graph of Cycle $D(C_n)$. Here all the vertices are having the degree 4. Here we have $4n$ edges.
By the definition of Exponential Fraction index,

$$\begin{aligned} EF(D(C_n)) &= 4ne^{\frac{4}{4}} \\ &= 4ne. \end{aligned}$$

□

4. Exponential Fraction index of complements

Theorem 4.1.

$$EF(G)=\begin{cases}\frac{1}{2}(n-1)(n-2)e & \textit{if } G=\overline{K_{1,n-1}}\\ ne & \textit{if } G=\overline{K_{n\times 2}}\\ \frac{1}{2}n(n-3)e & \textit{if } G=\overline{C_n}\\ n^2e & \textit{if } G=S_n^0\\ 2n(n-1)e & \textit{if } G=F_n^3\\ \frac{1}{2}(n-1)(n-4)e & \textit{if } G=W_n\\ 0 & \textit{if } G=K_n\end{cases}$$

Proof. We prove the theorem for complement of Star graphs. Similar methods can be used for others. Let G be a complement of Star graph $\overline{K_{(1,n-1)}}$. Here we have $n-1$ vertices with degree $n-2$ and one isolated vertex. We have $\frac{(n-1)(n-2)}{2}$ edges.
According to the definition of Exponent Fraction index,

$$\begin{aligned}EF(\overline{K_{(1,n-1)}}) &= \frac{(n-1)(n-2)}{2}e^{\frac{(n-2)}{(n-2)}}\\ &= \frac{1}{2}(n-1)(n-2)e.\end{aligned}$$

□

5. Exponential Fraction index of subdivision graphs

Theorem 5.1.

$$EF[(S(G))]=\begin{cases}2e^2+2(n-2)e & \textit{if } G=P_n\\ 2n(n-1)e^{\frac{n-1}{2}} & \textit{if } G=S_n^0\\ 2n(n-1)e^{\frac{n+1}{2}} & \textit{if } G=K_{n\times 2}\\ 2ne & \textit{if } G=C_n\\ n(n-1)e^{\frac{n-1}{2}} & \textit{if } G=K_n\\ (n-1)e^2+(n-1)e^{\frac{n-1}{2}} & \textit{if } G=K_{1,n-1}\\ 4ne+2ne^n & \textit{if } G=F_n^3\\ 3ne+2ne^n & \textit{if } G=D_4^n\\ (n-1)e^{\frac{n-1}{2}}+3(n-1)e^{\frac{3}{2}} & \textit{if } G=W_n\end{cases}$$

Proof. We prove the theorem for subdivision graph of path graph. Similar methods can be used for others. Let G be a subdivision graph of Path graph $S(P_n)$. Here we have $2n-3$ vertices with degree 2, two pendant vertices. We have $2n-4$ edges between the vertices of degree 2. There are two edges between pendant vertex to the vertex of degree 2.
By the definition of Exponential Fraction index,

$$\begin{aligned}EF(S(P_n)) &= 2e^{\frac{2}{1}}+(2n-4)e^{\frac{2}{2}}\\ &= 2e^2+2(n-2)e.\end{aligned}$$

□

6. Exponential Fraction index of subdivision of double graphs

Theorem 6.1.

$$EF[S(D(G))] = \begin{cases} 8e + 8(n-2)e^2 & \text{if } G = P_n \\ 4n(n-1)e^{n-1} & \text{if } G = K_n \\ 8ne^2 & \text{if } G = c_n \\ 4(n-1)e + 4(n-1)e^{n-1} & \text{if } G = K_{1,n-1} \\ 8n(n-1)e^{n-1} & \text{if } G = S_n^0, n \\ 16n(n-1)e^{2(n-1)} & \text{if } G = K_{n\times 2} \\ 16ne^2 + 8ne^{2n} & \text{if } G = F_n^3 \\ 23ne^2 + 9ne^{2n} & \text{if } G = D_4^n \\ 12(n-1)e^3 + 4(n-1)e^{n-1} & \text{if } G = W_n \end{cases}$$

Proof. We prove the theorem for subdivision graph of double graph of Path. Similar methods can be used for others. Let G be a subdivision graph of double graph of Path $S(D(P_n))$. Here we have $4n$ vertices with degree 2, $2n-4$ vertices with degree 4. We have 8 edges between the vertices of degree 2. There are $8n-16$ edges between the vertices of degree 4 and 2.
By the definition of Exponential Fraction index,

$$\begin{aligned} EF(S(D(P_n))) &= 8e^{\frac{2}{2}} + (8n-16)e^{\frac{4}{2}} \\ &= 8e + 8(n-2)e^2 \end{aligned}$$

□

7. Exponential Fraction index of double graph of subdivision graph

Theorem 7.1.

$$EF[D(S(G))] = \begin{cases} 8e^2 + 8(n-2)e & \text{if } G = P_n \\ 8ne & \text{if } G = C_n \end{cases}$$

Proof. We prove the theorem for double graph of subdivision graph of Path. Similar methods can be used for others. Let G be a double graph of subdivision graph of Path $D(S(P_n))$. Here we have 4 vertices with degree 2, $4n-6$ vertices with degree 4. We have 8 edges between the vertices of degree 2 and 4. There are $8n-16$ edges between the vertices of degree 4.
By the definition of Exponential Fraction index,

$$\begin{aligned} EF(D(S(P_n))) &= 8e^{\frac{4}{2}} + (8n-16)e^{\frac{4}{4}} \\ &= 8e^2 + 8(n-2)e \end{aligned}$$

□

8. Exponential Fraction index of Graphene and Carbon Nanocones

Theorem 8.1. *Exponential Fraction index of Graphene with a rows of benzene rings and b benzene rings in each row is given by*

$$EF(Graphene) = \begin{cases} (2a + 4b - 4)e^{\frac{3}{2}} + (3ab - 2b + 3)e & \text{if } a \neq 1 \\ (4b-4)e^{\frac{3}{2}} + (b+5)e & \text{if } a = 1 \end{cases}$$

Proof. The chemical structure Graphene is having a rows and b benzene rings in each row. Graphene contains $(a+4)$ edges between the vertices of degree 2 each, $2a+4b-4$ edges between the vertices with degree 2 and 3. $3ab-2b-a-1$ edges exist between the vertices with common degree 3. Hence,

Case 1 Exponential Fraction index of Graphene for $x \neq 1$ is

$$\begin{aligned} EF(Graphene) &= (a+4)e^{\frac{2}{2}} + (2a+4b-4)e^{\frac{3}{2}} + (3ab-2b-a-1)e^{\frac{3}{3}} \\ &= (2a+4b-4)e^{\frac{3}{2}} + (3ab-2b+3)e \end{aligned}$$

Case 2 For $a=1$, there would be 6 edges between the vertices with degree 2 each, also $(4b-4)$ edges between the vertices of degrees 2 and 3 and $(b-1)$ edges connected between the vertices of degree 3 each. Thus,

$$\begin{aligned} EF(Graphene) &= (6)e^{\frac{2}{2}} + (4b-4)e^{\frac{3}{2}} + (b-1)e^{\frac{3}{3}} \\ &= (4b-4)e^{\frac{3}{2}} + (b+5)e \end{aligned}$$

□

Theorem 8.2. *For $[CNC_k[n]]$ nanocones with $k \geq 3$ and $n = 1, 2, 3, ...,$*

$$EF[CNC_k[n]] = k[(2n-2)e^{\frac{3}{2}} + e[1.5n^2 - 2.5n + 2]$$

Proof. In $[CNC_k[n]]$ nanocones, there are k edges between the vertices of degree 2 each. There exist $2k$ edges between the vertices with degree $(3,2)$. It have $0.5k(3n^2-5n+2)$ edges between the vertices with degree 3 each. Applying the EF index, one can get

$$EF[CNC_k[n]] = k[e + (2n-2)e^{\frac{3}{2}} + 0.5e[3n^2 - 5n + 2]$$
$$EF[CNC_k[n]] = k[(2n-2)e^{\frac{3}{2}} + e[1.5n^2 - 2.5n + 2]$$

□

Definition 8.1. [5] *The operator $Q(G)$ is the graph obtained from G by inserting a new vertex into each edge of G and by joining edges to new vertices which lie on adjacent edges of G.*

Theorem 8.3. *For $Q[CNC_k[n]]$ nanocones with $k \geq 3$ and $n = 1, 2, 3, ...,$*

$$EF(Q[CNC_k[n]]) = 19.04k + 54.419kn + 30.322kn^2$$

Proof. In $Q[CNC_k[n]]$ nanocones, there are $2k$ edges between the vertices of degrees $(2,4)$. There are $2kn$ edges between the vertices with degree $(2,5)$. There are $2kn$ edges between the vertices with degree $(3,5)$. There are $kn(3n+1)$ edges between the vertices with degree $(3,6)$. There are $2k$ edges between the vertices with degree $(4,5)$. There are $k(2n-1)$ edges between the vertices with degree $(5,5)$. There are $2kn$ edges between the vertices with degree $(5,6)$. There are $3kn^2$ edges edges between the vertices with degree $(6,6)$. Applying the EF index, one can get

$$EF(Q[CNC_k[n]]) = 19.04k + 54.419kn + 30.322kn^2$$

□

Definition 8.2. [6] *The line graph of the graph G, written $L(G)$, is the simple graph whose vertices are the edges of G, with $xy \in E(L(G))$ when x and y have a common end point in G.*

Theorem 8.4. *For the line graph of Carbon Nanocones CNCk[n]*

$$EF[L(CNCk[n])] = 2.7182kn^2 + 13.0237kn + 6.2451k$$

Proof. In $L(CNCk[n])$ nanocones, there are $2k$ edges between the vertices of degrees $(3, 2)$. There are $2kn$ edges between the vertices with degree $(4, 3)$. There are $3kn^2$ edges between the vertices with degree $(4, 4)$. There are $k(2n-1)$ edges between the vertices with degree $(3, 3)$. Applying the EF index, one can get

$$EF(L(CNCk[n])) = 2.7182kn^2 + 13.0237kn + 6.2451$$

□

9. Conclusions

As new molecules and more complex graphs evolve, it may be difficult to study them using old indices, hence proposal of new indices plays crucial role for future research work.

In this paper, we calculate the Exponential Fraction index of chemical structures like Graphene and Carbon Nanocones, and also double graphs, subdivision graphs, Subdivision of Double graphs, Double graph of subdivision graphs and complements of some standard graphs. As the Exponential Fraction index is a new index proposed by us, we are looking forward for its contribution towards the field of graph theory.

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K. N. Prakasha for the photography and short autobiography, see TWMS J. App. and Eng. Math. V.9, N.4.

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