ON BIVARIATE CREDIBILITY ESTIMATOR WITH GLM THEORY

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ABSTRACT. Credibility theory is one of the cornerstones of actuarial science as applied to casualty and property insurance, based on the concept of limiting the estimator of individual premium to the class of estimators that are linear with respect to all observations of the portfolio. This work deals with the bivariate data(number and amounts of claims of the contracts), we give the bivariate credibility estimator using exponential families and GLM theory. Just like in the case of classical credibility model we will obtain a credible solution in the form of a linear combination of the individual estimate and the collective estimate. And we add the proprieties on exact Bayes premium.

Keywords: generalized linear models, credibility theory, Bayes probability, exact credibility.

AMS Subject Classification: 83-02, 99A00.

1. INTRODUCTION

Credibility theory is one of the cornerstones of actuarial science as applied to casualty and property insurance, based on the concept of limiting the estimator of individual premium to the class of estimators that are linear with respect to all observations of the portfolio. ([3],[1]). It is a technique for pricing insurance coverages that is widely used by health, group term life, and property and casualty actuaries. In this work, we give the bivariate credibility estimator using exponential families and GLM theory. Just like in the case of the classical credibility model, we will obtain a credibility solution in the form of a linear combination of the individual estimate and the collective estimate.

Our first section is the preliminaries of theory of generalized linear models (GLM), after the hypotheses we get us the credibility estimator[10]. Second section is main results, we give the bivariate credibility estimator with GLM theory. In the third section, the exponential family is presented for give the theorem of exact Bayes probability. Finally, a conclusion is given.

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2. Preliminaries

We consider an homogeneous insurance portfolio consisting of $n \geq 2$ contracts. Now we have for each contract $i, i = 1, ..., n$ the set of random variable:

$$(\Theta_i, Z_i^{\tau}) = (\Theta_i, (Z_{i,s}^{\tau})_{s=1,...,S}), \tag{1}$$

$i = 1, \ldots, n$ and $(Z_{i,s}^{\tau})_{s=1,...,S}$ are no negative. For example, $Z_{i,s}^{\tau}$ can be interpreted as the amount or the number of claims of the contract i in the period s. The random variable of the whole model are denoted by

$$(Z^{\tau}) = (Z_i^{\tau})_{i=1,...,n}. \tag{2}$$

We make the following model assumptions :
- **(B1)** The random structure variables $(\Theta_i)_{i=1,...,n}$ have the identical distribution Pr_{Θ}.
- **(B2)** The families of random variables (Θ_i, Z_i^{τ}), $i = 1, \ldots, n$, are mutually non-correlated with $Z_{i,s}^{\tau} \in L^2$ for all $s = 1, ..., S$, $i = 1, \ldots, n$.
- **(B3)** For fixed i and conditioned by Θ_i, the random variable $(Z_{i,s}^{\tau})_{s=1,...,S}$ are mutually non-correlated with the first and second conditional moments depending only on Θ_i :

$$E[Z_{i,s}^{\tau}|\Theta_i] := \mu^{\tau}(\Theta_i), \tag{3}$$
$$Var[Z_{i,s}^{\tau}|\Theta_i] := \alpha^{\tau}(\Theta_i). \tag{4}$$

We suppose that $\mu^{\tau}(\Theta_i)$ are strictly crescent en Θ_i and For non-conditional moments we use the following notations :

$$\begin{aligned} E[\mu^{\tau}(\Theta_i)] &:= M^{\tau} \\ E[\alpha^{\tau}(\Theta_i)] &:= A^{\tau} \\ Var[\mu^{\tau}(\Theta_i)] &:= B^{\tau}. \end{aligned} \tag{5}$$

We suppose that $Z_{i,s}^{\tau}|\Theta_i$ is distributed according to an exponential family :

$$\Pr_{Z_{i,s}^{\tau}|\Theta_i} \rightsquigarrow exp\{\frac{Z_{i,s}^{\tau}\Theta_i - b(\Theta_i)}{Q} + c(Z_{i,s}^{\tau}, Q)\}.$$

where $b(.)$ and $c(.)$ are known functions, Θ_i is the natural and Q the scale parameter. Assume that Θ_i has a prior density which is the so-called natural conjugate prior, Then :

$$\Pr_{\Theta} \rightsquigarrow exp\{a_1\Theta_i - a_2 b(\Theta_i)\}$$

where Lebesgue measure is the base measure on $[0, \infty)$ and $\Theta_i \geq 0$,$a_1 < 0$. So, the posterior distribution of the $\Theta_i|Z_{i,s}^{\tau}$ is :

$$\Pr_{\Theta_i|Z_{i,s}^{\tau}} \alpha \prod_s \Pr_{Z_{i,s}^{\tau}|\Theta} . \Pr_{\Theta}$$

log-likelihood of $\Pr_{\Theta_i|Z_{i,s}^{\tau}} = h_i$ as :

$$h_i = \sum_s (\frac{Z_{i,s}^{\tau}\Theta_i - b(\Theta_i)}{Q}) + c(Z_{i,s}^{\tau}, Q) + a_1\Theta_i - a_2 b(\Theta_i).$$

log-likelihood of $\Pr_{\Theta|Z^{\tau}} = h$ as :

$$h = \sum_i \sum_s (\frac{Z_{i,s}^{\tau}\Theta_i - b(\Theta_i)}{Q}) + c(Z_{i,s}^{\tau}, Q) + a_1\Theta_i - a_2 b(\Theta_i).$$

we want to estimate $\widehat{\mu^\tau}(\Theta_i)$, we have :

$$\frac{\partial b(\Theta_i)}{\partial \Theta_i} = \mu^\tau(\Theta_i)\frac{\partial h}{\partial \Theta_i} = \sum_s \left(\frac{Z_{i,s}^\tau - \mu^\tau(\Theta_i)}{Q}\right) + a_1 - a_2\mu^\tau(\Theta_i),$$

we have $\frac{\partial h}{\partial \Theta_i} = 0$. So,

$$\sum_s Z_{i,s}^\tau - S\widehat{\mu^\tau}(\Theta_i) + Qa_1 - Qa_2\widehat{\mu^\tau}(\Theta_i) = 0 \implies \widehat{\mu^\tau}(\Theta_i) = \xi\overline{Z}_i + (1-\xi)M.$$

Where : $\overline{Z}_i = \frac{1}{S}\sum_s Z_{i,s}^\tau$ and $\xi = \frac{S}{S+Qa_2}$, $M = a_1/a_2$.
Now, the density of Θ_i is proportional :

$$e^{\{a_1\Theta_i - a_2 b(\Theta_i)\}}.$$

$$\frac{\partial e^{\{a_1\Theta_i - a_2 b(\Theta_i)\}}}{\partial \Theta_i} \quad = \quad (a_1 - a_2\mu^\tau(\Theta_i))e^{\{a_1\Theta_i - a_2 b(\Theta_i)\}} \tag{6}$$

Integrating over the natural range of Θ_i and assuming $e^{\{a_1\Theta_i - a_2 b(\Theta_i)\}}$ is zero at the end points, we have

$$a_1 - a_2 E(\mu^\tau(\Theta_i)) = 0$$
$$M = \frac{a_1}{a_2}. \tag{7}$$

3. Bivariate Credibility and GLM Theory

Let $(Z_i^{(1)}, Z_i^{(2)})$ variables can be interpreted as respectively the amount and the number of claims of the contract i and (θ_i^1, θ_i^2) structure variables we have, $\tau = 1, 2$:

$$E[Z_{i,s}^\tau | \theta_i^1, \theta_i^2] \quad := \quad \mu^\tau(\theta_i^1, \theta_i^2), \tag{8}$$
$$Var[Z_{i,s}^\tau | \theta_i^1, \theta_i^2] \quad := \quad \alpha^\tau(\theta_i^1, \theta_i^2), \tag{9}$$
$$Cov\left(Z_{i,s}^\tau, Z_{i,s}^{3-\tau} | \theta_i^\tau, \theta_i^{3-\tau}\right) \quad := \quad \kappa(\theta_i^\tau, \theta_i^{3-\tau}). \tag{10}$$

For non-conditional moments we use the following notations:

$$E[\mu^\tau(\theta_i^1, \theta_i^2)] \quad := \quad M^\tau \tag{11}$$
$$E[\alpha^\tau(\theta_i^1, \theta_i^2)] \quad := \quad A^\tau \tag{12}$$
$$Var[\mu^\tau(\theta_i^1, \theta_i^2)] \quad := \quad B^\tau \tag{13}$$
$$E[\kappa(\theta_i^1, \theta_i^2)] \quad := \quad K \tag{14}$$
$$Cov[\mu^\tau(\theta_i^1, \theta_i^2), \mu^{3-\tau}(\theta_i^1, \theta_i^2)] \quad := \quad L. \tag{15}$$

we suppose that $Z_{i,s}^\tau, Z_{i,s}^{3-\tau} | \theta_i^1, \theta_i^2$ is distributed according to an exponential family :

$$P_{Z_{i,s}^\tau, Z_{i,s}^{3-\tau} | \theta_i^1, \theta_i^2} \rightsquigarrow exp\{\frac{Z_{i,s}^\tau \theta_i^1 + Z_{i,s}^{3-\tau}\theta_i^2 - b(\theta_i^1, \theta_i^2)}{Q} + c(Z_{i,s}^\tau, Z_{i,s}^{3-\tau}, Q)\}.$$

Assume that θ_i^1, θ_i^2 has a prior density which is the so-called natural conjugate prior,

$$\Pr_{\theta_i^1, \theta_i^2} \rightsquigarrow exp\{a_1\theta_i^1 + a_2\theta_i^2 - b(\theta_i^1, \theta_i^2)\}.$$

So, the posterior distribution of the $\theta_i^1, \theta_i^2 | Z_{i,s}^\tau, Z_{i,s}^{3-\tau}$ is :

$$\Pr_{\theta_i^1, \theta_i^2 | Z_{i,s}^\tau, Z_{i,s}^{3-\tau}} \alpha \prod_s \Pr_{Z_{i,s}^\tau, Z_{i,s}^{3-\tau} | \theta_i^1, \theta_i^2} \cdot \Pr_{\theta_i^1, \theta_i^2}.$$

log-likelihood of $\Pr_{\theta_i^1,\theta_i^2 \big| Z_{i,s}^\tau, Z_{i,s}^{3-\tau}} = h_i$ as :

$$h_i = \sum_s \left(\frac{Z_{i,s}^\tau \theta_i^1 + Z_{i,s}^{3-\tau}\theta_i^2 - b(\theta_i^1,\theta_i^2)}{Q} + c(Z_{i,s}^\tau, Z_{i,s}^{3-\tau}, Q) \right) + a_1\theta_i^1 + a_2\theta_i^2 - b(\theta_i^1,\theta_i^2).$$

log-likelihood of $\Pr_{\theta^1,\theta^2 \big| Z^\tau, Z^{3-\tau}} = h$ as :

$$h = \sum_{i,s} \left(\frac{Z_{i,s}^\tau \theta_i^1 + Z_{i,s}^{3-\tau}\theta_i^2 - b(\theta_i^1,\theta_i^2)}{Q} + c(Z_{i,s}^\tau, Z_{i,s}^{3-\tau}, Q) \right) + a_1\theta_i^1 + a_2\theta_i^2 - b(\theta_i^1,\theta_i^2).$$

we want to estimate $\widehat{\mu^{(1)}}(\theta_i^1,\theta_i^2), \widehat{\mu^{(2)}}(\theta_i^1,\theta_i^2)$, we have :

$$\frac{\partial b(\theta_i^1,\theta_i^2)}{\partial \theta_i^\tau} = \mu^\tau(\theta_i^1,\theta_i^2)$$

$$\frac{\partial^2 b(\theta_i^1,\theta_i^2)}{\partial^2 \theta_i^\tau}Q = \alpha^\tau(\theta_i^1,\theta_i^2)$$

$$\frac{\partial^2 b(\theta_i^\tau,\theta_i^2)}{\partial \theta_i^\tau \partial \theta_i^{3-\tau}}Q = \kappa(\theta_i^1,\theta_i^2). \tag{16}$$

we have $\frac{\partial h}{\partial \Theta_i} = 0$. So,

$$\begin{pmatrix} \frac{\partial h}{\partial \theta_i^1} = 0 \\ \frac{\partial h}{\partial \theta_i^2} = 0 \end{pmatrix} = \begin{pmatrix} \sum_s Z_{i,s}^\tau - S\widehat{\mu^\tau}(\theta_i^1,\theta_i^2) + Qa_1 - Q\widehat{\mu^\tau}(\theta_i^1,\theta_i^2) = 0 \\ \sum_s Z_{i,s}^{3-\tau} - S\widehat{\mu^{3-\tau}}(\theta_i^1,\theta_i^2) + Qa_2 - Q\widehat{\mu^{3-\tau}}(\theta_i^1,\theta_i^2) = 0 \end{pmatrix}$$

so,

$$\begin{pmatrix} \widehat{\mu^{(1)}}(\theta_i^1,\theta_i^2) \\ \widehat{\mu^{(2)}}(\theta_i^1,\theta_i^2) \end{pmatrix} = \begin{pmatrix} \frac{S}{S+Q}\sum_s Z_{i,s}^{(1)} + (1 - \frac{S}{S+Q})a_1 \\ \frac{S}{S+Q}\sum_s Z_{i,s}^{(2)} + (1 - \frac{S}{S+Q})a_2 \end{pmatrix}.$$

Now, the density of Θ_i is proportional, equal to :

$$e^{\{a_1\theta_i^1 + a_2\theta_i^2 - b(\theta_i^1,\theta_i^2)\}}.$$

We derive, we obtain :

$$\frac{\partial e^{\{a_1\theta_i^1 + a_2\theta_i^2 - b(\theta_i^1,\theta_i^2)\}}}{\partial \theta_i^1} = (a_1 - \mu^{(1)}(\theta_i^1,\theta_i^2))e^{\{a_1\theta_i^1 + a_2\theta_i^2 - b(\theta_i^1,\theta_i^2)\}} -$$

$$\frac{\partial e^{\{a_1\theta_i^1 + a_2\theta_i^2 - b(\theta_i^1,\theta_i^2)\}}}{\partial \theta_i^2}$$

$$= (a_2 - \mu^{(1)}(\theta_i^1,\theta_i^2))e^{\{a_1\theta_i^1 + a_2\theta_i^2 - b(\theta_i^1,\theta_i^2)\}}. \tag{17}$$

Integrating over the natural range of θ_i^1, θ_i^2 and assuming $e^{a_1\theta_i^1 + a_2\theta_i^2 - b(\theta_i^1,\theta_i^2)}$ is zero at the end points, we have

$$a_1 - E(\mu^{(1)}(\theta_i^1,\theta_i^2)) = 0 \tag{18}$$

$$M^1 = a_1 \tag{19}$$

$$a_2 - E(\mu^{(2)}(\theta_i^1,\theta_i^2)) = 0 \tag{20}$$

$$M^2 = a_2. \tag{21}$$

4. Some results based on exponential family

Exponential families presented here are also the basis of the theory of generalized linear models (GLM).

Definition 4.1. *Let $\mathbb{P}$ be a probability on a open convex set C in R^d (more generally in a vector space E. locally convex, which the dual is E').*

(1) *We define D' as the dual convex set defined by :*

$$D' := \left\{ v \in R^d \Big| \int e^{uv} \mathbb{P}(du) < +\infty \right\}^0.$$ (22)

where $(A)^0$ means the interior of the set A. We assume that

$$D' \neq \emptyset.$$ (23)

The dual domain D' is the convex set on which the cumulant function $\psi : C' \to R$ is defined as

$$\psi(v) := \ln \int e^{uv} \mathbb{P}(du).$$ (24)

(2) *Let $\chi : D \to D'$ be a continuous function defined on an open set D with values in D'. χ is called the link function.*
Thus, we can define the exponential family of $\mathbb{P}$ under χ as the set $\mathfrak{P}$ of probabilities $\mathbb{P}_t$ absolutely continuous with respect to $\mathbb{P}$:

$$\mathfrak{P} := \left\{ \mathbb{P}_t(du) := exp \left\{ u\chi(t) - \psi(\chi(t)) \right\} \mathbb{P}(u) \Big| t \in D \right\}.$$ (25)

(3) *Now let $\mathfrak{Q}$ be a probability on D satisfying*

$$\int exp \left\{ \frac{u\chi(t) - \psi(\chi(t))}{\eta} \right\} \mathfrak{Q}(dt) < +\infty.$$ (26)

(4) *The condition (26) allows us to define the dispersed cumulate function $\varphi(u,\eta)$ of $\mathfrak{Q}$ on the space $C \times (0,\infty)$ as:*

$$\varphi(u,\eta) := \ln \int exp \left\{ \frac{u\chi(t) - \psi(\chi(t))}{\eta} \right\} \mathfrak{Q}(dt).$$ (27)

(5) *As the dual exponential family with respect to $\mathfrak{P}$ we define the exponential family $\mathcal{Q}$ as:*

$$\mathcal{Q} = \left\{ \mathfrak{Q}_{u,\eta}(dt) := exp \left\{ \frac{u\chi(t) - \psi(\chi(t))}{\eta} - \varphi(u,\eta) \right\} \mathfrak{Q}(dt) \Big| (u,\eta) \in C \times (0,\infty) \right\}.$$ (28)

Definition 4.2. *The family $\boldsymbol{B}$ of exact Bayes probabilities is given by:*

$$\boldsymbol{B} := \left\{ \mathbb{B}_{\widehat{u}_0,\eta,l}(\prod_{i \in I} du_i) := \int \mathfrak{Q}_{\widehat{u}_0,\eta}(dt) \prod_{i \in I} \mathbb{P}_t(du_i) | \widehat{u}_0 \in C, \eta > 0, I \supset N, I \, is \, finite \right\}.$$ (29)

Theorem 4.1. *The family $\boldsymbol{B}$ of exact probability of Bayes is closed under the conditions $(|u_j = \widehat{u}_j, j \in J)$ for finite subsets $j \subseteq J$, i.e. for $\prod_{j \in J} \mathbb{P}_t(du_j)$ p-s. for all $(\widehat{u}_j)_j \in J$*

$$\mathbb{B}_{\widehat{u}_0,\eta,l}(\prod_{i \in I} du_i | u_j = \widehat{u}_j, j \in J) = \mathbb{B}_{\widetilde{u},\widetilde{\eta},l\setminus J}(\prod_{i \in I\setminus J} du_i) \prod_{i \in J} \delta_{\widehat{u_j}}(du_i).$$ (30)

Where $\widetilde{\eta} = [1 \ \eta + |J|]^{-1}$ and $\widetilde{u} = \widetilde{\eta}(\frac{\widehat{u}_0}{\widetilde{\eta}} + \Sigma_{j \in J}\widehat{u}_j)$.

Proof.

$$\mathbb{B}_{\widehat{u}_0,\eta,l}(\prod_{i\in I}du_i|u_j=\widehat{u}_j,j\in J)=\frac{\mathbb{B}_{\widehat{u}_0,\eta,l}(\prod_{j\in J}(du_i\bigcap\{\widehat{u}_j\})\times\prod_{i\in I\setminus J}du_i)}{\mathbb{B}_{\widehat{u}_0,\eta,l}(\prod_{j\in J}\{\widehat{u}_j\})}$$

$$=\frac{\int\mathfrak{Q}_{\widehat{u}_0,\eta}(dt)\prod_{j\in J}exp\{u_j.\chi(t)-\psi(\chi(t))\}\delta_{\widehat{u_j}}(du_j)\times\prod_{i\in I\setminus J}\mathbb{P}_t(du_i)}{\int\mathfrak{Q}_{\widehat{u}_0,\eta}(dt)\prod_{j\in J}exp\{\widehat{u}_j.\chi(t)-\psi(\chi(t))\}}$$

$$=\frac{\int\mathfrak{Q}(dt)exp\{(\widehat{u}_0/\eta+\sum_{j\in J}\widehat{u}_j).\chi(t)-\psi(\chi(t))/\widetilde{\eta}-\varphi(\widehat{u}_0,\eta)\}\times\prod_{i\in I\setminus J}\mathbb{P}_t(du_i)}{\int\mathfrak{Q}(dt)exp\{(\widehat{u}_0/\eta+\sum_{j\in J}\widehat{u}_j).\chi(t)-\psi(\chi(t))/\widetilde{\eta}-\varphi(\widehat{u}_0,\eta)\}}\times$$

$$\prod_{j\in J}\delta_{\widehat{u}_j}(du_j)$$

$$=\frac{\int\mathfrak{Q}(dt)exp\{(\widetilde{u}.\chi(t)-\Psi(\chi(t)))/\widetilde{\eta}\}\times\prod_{i\in I\setminus J}\mathbb{P}_t(du_i)}{\int\mathfrak{Q}(dt)exp\{(\widetilde{u}.\chi(t)-\Psi(\chi(t)))/\widetilde{\eta}\}}\prod_{j\in J}\delta_{\widehat{u_j}}(du_j)$$

$$=\int\mathfrak{Q}(dt)exp\{(\widetilde{u}.\chi(t)-\Psi(\chi(t)))/\widetilde{\eta}-\varphi(\widetilde{u},\widetilde{\eta})\}\times\prod_{i\in I\setminus J}\mathbb{P}_t(du_i)\prod_{j\in J}\delta_{\widehat{u}_j}(du_j)$$

$$=\int\mathfrak{Q}_{\widetilde{u},\widetilde{\eta}}(dt)\prod_{i\in I\setminus J}\mathbb{P}_t(du_i)\prod_{j\in J}\delta_{\widehat{u_j}}(du_j)$$

$$=\mathbb{B}_{\widehat{u},\eta,l\setminus J}(\prod_{i\in I\setminus J}du_i)\prod_{i\in I}\delta_{\widehat{u_j}}(du_j).$$

$\square$

Theorem 4.2. *If for every $u_0\in C,\eta>0$ we have:*

$$\mathfrak{Q}_{\widehat{u}_0,\eta}(dt)\mathbb{P}_t(d\widetilde{u})\prod_{i\in I}(du_i):=\mathbb{B}(dt,d\widetilde{u}\times\prod_{i\in I}du_i). \tag{31}$$

Where

$$E_{\mathbb{B}}(dt,d\widetilde{u}\times\prod_{i\in I}du_i|u_i=\widehat{u}_i,i\in I) \tag{32}$$

is not linear in $\widehat{u}_i$
The solution of the problem is :
$u^*=(1-\zeta)m+\frac{\zeta}{I}\Sigma_i u_i$.
Where

$$\zeta:=\frac{Ivar(E(u_i|\chi(t)))}{E(var(u_i|\chi(t)))+Ivar(E(u_i|\chi(t)))}. \tag{33}$$

And

$$m=E(u_i) \tag{34}$$

Proof. Obviously, the credibility problem

$$\Gamma:=\underset{\beta_0,\beta_{1i},i\in I}{Min}\{E_{\mathbb{B}(dt,d\widetilde{u}\times\prod_{i\in I}du_i|u_i=\widehat{u}_i,i\in I)}[(\widetilde{u}-\beta_0-\Sigma_i\beta_{1i}\widehat{u}_i)^2]\}. \tag{35}$$

can be separated. As necessary conditions of optimality, we have for i

$$0 \;=\; \frac{\partial \Gamma}{\partial \beta_0} \tag{36}$$

$$0 \;=\; m - \beta_0 - m\Sigma_i \beta_{1i} \tag{37}$$

$$\beta_0 \;=\; m - m\Sigma_i \beta_{1i} \tag{38}$$

$$0 \;=\; \frac{\partial \Gamma}{\partial \beta_{1i'}} \tag{39}$$

$$=\; E(\widehat{u}_{i'}(\widetilde{u} - \beta_0 - \Sigma_i \beta_{1i}\widehat{u}_i)) \tag{40}$$

$$cov(\widetilde{u}, \widehat{u}_i) \;=\; \Sigma_i \beta_{1i} cov(\widehat{u}_i, \widehat{u}_{i'}). \tag{41}$$

After some easy calculations (not presented here) show that :

$$\beta_{1i} \;=\; \frac{var(E(\widehat{u}_i|\chi(t)))}{E(var(\widehat{u}_i|\chi(t))) + Ivar(E(\widehat{u}_i|\chi(t)))} \tag{42}$$

$$\beta_0 \;=\; (1 - \zeta)m. \tag{43}$$

$\square$

Remark 4.1. *In standard notation of exponential family of $Z_i|\Theta$ we noted by u_i and v as canonical parameter.*
The $\psi(v)$ represents $b(\Theta)$, where $(b')^{-1}$ represented by the continuous function of an open set D in C' is : So, ρ is the distribution of $Z_i|\Theta$

$$E(u) \;=\; \psi(\chi(t))' \tag{44}$$

$$Var(u) \;=\; \psi(\chi(t))''. \tag{45}$$

We derive the credibility formula for exponential families of distributions. Denote the data by $Z_i|\Theta_i$ for $i = 1, ..., n$,The assumptions of the model of Bühlmann (1967) are :

- *The risks,Θ_i are independently, identically distributed.*
- *$Z_i|\Theta_i$ are independently, identically distributed.*

let $Z_i|\Theta_i \approx u_i$ So

$$E[Z_i|\Theta_i] = \nabla\psi(\chi(t)). \tag{46}$$

The canonical parameter of $Z_i|\Theta_i$ do not depend j, So :

$$\chi(t)' = \chi(\nabla\psi(\chi(t))). \tag{47}$$

5. Conclusions

In conclusion, this paper gives the bivariate credibility estimator using exponentials families and GLM theory. We obtain a credibility solution in the form of a linear combination of the individual estimate and the collective estimate. The first section of this paper are the preliminaries of theory of generalized linear models (GLM), and the credibility estimator. Second and third sections is main results, we give the bivariate credibility estimator with GLM theory, and the exponential family is presented for give the theorem of exact Bayes probability. Thus, the future prospects are at first Quadratic Bivariate Credibility, it's generalization of this work. Next, we work with the robust credibility (Kremer's Robust Regression Model).

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