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FIXED POINT THEOREMS OF FUZZY PARTIAL METRIC SPACES

AANCHAL TEHLAN[1], VIJAY KUMAR[1], §

Abstract. Fuzzy metric space has been defined using fuzzy numbers by many researchers. Recently Xia and Guo [20] have introduced fuzzy metric space using fuzzy scalars which is similar to the classical metric space. In this work we have generalized a fuzzy metric space to a partial fuzzy metric space using fuzzy scalars and its fixed point theorem have been established.

Keywords: Fuzzy metric space, Fuzzy partial metric space, fixed point theorem.

AMS Subject Classification: 47H10, 03E72.

1. Introduction

Metric is a function which defines distance between two points of the set and metric space is a set together with a metric on that set. It generalized the notion of distance function in Euclidean n-dimensional space. Mathews [13] developed the theory of partial metric spaces, which helps in the generalization of metric spaces. Many researchers have used fuzzy set theory in various disciplines of research. Michalek and Kramosil [11] introduced the concept of fuzzy metric spaces, which are closely related to the class of probabilistic metric spaces. Kaleva [9] and Dia [4] used fuzzy numbers to define metric spaces for measuring distances in fuzzy sets. Felbin [5] investigated various fixed point theorems, some useful results and properties of these spaces were investigated by George [6] who utilized fuzzy numbers to define fuzzy metric space. Chaudhri [3], Diamond [4] and Boxer [2] used similar type of approaches in their work under different kind of fuzzy environment, which leaves unique impact in the development of the theory. Among various approaches to define these spaces, Xia [20] used fuzzy scalars to define fuzzy metric space, which became a stronger form in fuzzy topological spaces as fuzzy metric spaces. Related work in this direction is seen in the works of Mazaheri [14] and Khalik [10]. Recently, Wu [19] and Gregori [7] used fuzzy numbers to define partial metric space. In this work, Banach's fixed point theorem has been proved for fuzzy partial metric space by using fuzzy scalars.

[1] Department of Mathematics, Manav Rachna International Institute of Research and Studies, Faridabad-121004, Haryana, India.
e-mail: tehlanaanchal@gmail.com; ORCID: https://orcid.org/0000-0001-9762-8416.
e-mail: drvijaykumarsudan@gmail.com; ORCID: https://orcid.org/0000-0002-6905-5160.
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2. Preliminaries

Definition 2.1 (Fuzzy point). *Xia [20] defined fuzzy point. In this paper, we denote (x, λ) as fuzzy points and a set containing all the fuzzy points of X as $P_F(X)$. Also, when $X = \{x : x \text{ is a real number}\}$ are called fuzzy scalars, and is written as $S_F(R)$. For non-negative fuzzy scalars, set is defined as $S_F^+(R)$.*

Definition 2.2 (Fuzzy Scalars). *According to Singh [17], consider, (x, λ), (y, γ) as any two fuzzy scalars. Then:*

(2.2.1) $(a, \lambda) \geq (b, \gamma)$, *if $a \succ b$ or $(a, \lambda) = (b, \gamma)$.*
(2.2.2) (a, λ) *is said to be not less than (b, γ) if $a \geq b$, then $(a, \lambda) \succ (b, \gamma$ or $(b, \gamma) \prec (a, \lambda)$*
(2.2.3) (a, λ) *is non-negative if $a > 0$.*

Definition 2.3 (Partial Fuzzy Metric Space). *Let X be a non-empty set and let a mapping $p_F : P_F(X) \times P_F(X) \to S_F^+(R)$, then $(P_F(X), p_f)$ is said to be partial fuzzy metric space, if for every fuzzy scalars $(x, \lambda), (y, \gamma), (z, \rho) \subset (P_F(X,)p_F)$ satisfies the following conditions:*

(i) $0 \leq p_F((x, \lambda), (x, \lambda)) \leq p_F((x, \lambda), (y, \gamma))$
(ii) $p_F((x, \lambda), (x, \lambda)) = p_F((x, \lambda), (y, \gamma)) = p_F((y, \gamma), (y, \gamma))$ *then $x = y$ and $\lambda = \gamma = 1$*
(iii) $p_F((x, \lambda)(y, \gamma)) = p_F((y, \gamma), (x, \lambda))$
(iv) $p_F((x, \lambda), (z, \rho)) \leq p_F((x, \lambda)(y, \gamma)) + p_F((y, \gamma), (z, \rho)) - p_F((y, \gamma), (y, \gamma))$

Example 2.1. *Suppose (X, p) is a partial metric space, then the distance of (x, λ), (y, γ) in $P_F(X)$ is defined as*

$$p_F((x, \lambda), (y, \gamma)) = \{p(x, y), \min(\lambda, \gamma)\}$$

where $p(x, y)$ defines the distance between points x and y.
Here, we shall show that p_F satisfies all the conditions, given in (2.3).
Given that $p(x, y)$ is a partial metric and $p(x, y) \geq 0$.
We have $p_F((x, \lambda)(y, \gamma)) = \{p(x, y), \min(\lambda, \gamma)\}$, which is non-negative as $0 \leq p(x, x) \leq p(x, y)$.

Symmetric. Consider, $(x, \lambda)(y, \gamma) \subset P_F(X)$ we have:

$$p_F((x, \lambda)(y, \gamma)) = \{p(x, y), \min(\lambda, \gamma)\}$$
$$p_F((x, \lambda)(y, \gamma)) = \{p(y, x) = \min(\gamma, \lambda)\}$$

which gives,

$$p_F((x, \lambda)(y, \gamma)) = p_F((y, \gamma), (y, \gamma))$$

Triangularity. Consider, $(x, \lambda), (y, \gamma), (z, \rho) \subset P_F(X)$, hence

$$\begin{aligned} p_F(x, \lambda), (z, \rho) &= \{p(x, z), \min(\lambda, \rho)\} \\ p_F(x, \lambda), (z, \rho) &\prec \{p((x, y) + (y, z) - (y, y), \min(\lambda, \gamma, \rho))\} \\ &= \{p((x, y), \min(\lambda, \gamma)\} + \{p((y, z), \min(\gamma, \rho)\} - \{p((y, y), \min(\gamma, \gamma)\} \\ &= p_F(x, \lambda), (y, \gamma) + p_F(y, \gamma), (z, \rho) - p_F(y, \gamma), (y, \gamma) \end{aligned}$$

Definition 2.4. *Consider any sequence (x_n, λ_n) in some fuzzy partial metric space $(x, \lambda) \in P_F(X)$. Then, (x_n, λ_n) will converge to any fuzzy point $(x, \lambda) \in X$ if $\lim_{n\to\infty} p_F((x_n, \lambda_n), (x, \lambda)) = 0_\lambda \ \forall \ \gamma \in (0, 1]$, where $\lambda \leq \gamma$, (x, λ) is the limit of (x_n, λ_n) given as $\lim_{n\to\infty} (x_n, \lambda_n) = (x, \lambda)$.*

Definition 2.5. *Any sequence defined using fuzzy points $(x_n, \lambda_n) \in (P_F(X), p_F)$ is cauchy if for any $\lambda \in (0, 1]$, $\lim_{n\to\infty} p_F((x_{m+n}, \lambda_{m+n}), (x_n, \lambda_n)) = 0_\lambda$ for all $m \in N$.*

Example 2.2. *Consider,* $X = [0, \infty)$ *and* $(P_F(X), p_F) = \{p(x, y), \min(\lambda, \gamma)\}$ *where* $p(x, y) = \max(x, y)$ *which means* $(P_F(X), p_F)$ *is a partial fuzzy metric. Consider,* $\{x_n\} = \{1, 2, 1, 2, \ldots\}$. *Then sequence will converge for all* $x \geq 2$, *when*

$$\lim_{n\to\infty} p_F((x_n, \lambda_n), (x, \gamma)) = p_F((x, \lambda), (x, \gamma))$$

therefore $L(x_n) = \{x_n \in X : x_n \to x\} = [2, \infty)$.
But $\lim_{n\to\infty} p_F((x_{m+n}, \lambda_{m+n}), (x_n, \lambda_n))$ *does not exist and hence* (x_n, λ_n) *is not Cauchy.*

Lemma 2.1 (Masihaa [12]). *A partial fuzzy metric space is complete if for some cauchy sequence belonging to it there will exist some unique limit in the space.*
Moreover, $\lim_{n\to\infty} (x_n, \lambda_n) = 0$ *iff* $p_F((x, \lambda), (x, \lambda)) = \lim_{n\to\infty} p_F(x_n, \lambda_n), (x, \lambda) = \lim_{n\to\infty} p_F(x_n, \lambda_n), (x_m, \lambda_m)$
Since every cauchy sequence which has fuzzy points will have some unique limit point as fuzzy point, as in the classical case.

Definition 2.6 (Masihaa [12]). *Let* $\phi : [0, 1] \to [0, 1]$ *satisfying;*

(i) $\phi(0) = 0$ *and* $\phi(t) > 0$ *for each* $t > 0$.
(ii) ϕ *is right lower semi-continuous*
(iii) *For any sequence* $\{r_n\}$ *with* $\lim_{n\to\infty} r_n = 0$, *there exists,* $a \in (0, 1)$ *and* $n_o \in N$ *such that* $\phi(r_n) \geq ar_n$ *for each* $n \geq n_o$.

3. Main Results

Theorem 3.1. *Suppose we have* $(X, \leq)$ *as some set which is partially ordered and* p_F *on* $P_F(X)$ *is a fuzzy partial metric space which is complete. Let* $T : P_F(X) \to P_F(X)$ *be a map which is continuous and non-decreasing :*

$$p_F(T(x, \lambda), T(y, \gamma)) \leq p_F((x, \lambda), (y, \gamma)) - \varphi\{p_F((x, \lambda), (y, \gamma))\} \qquad (3.1.1)$$

$\forall\ x, y \in X$ *where* x, y *are comparable and* ϕ *satisfies Definition* 2.6.
Whenever, we have an $(x_0, y_0) \in P_F(X)$ *so that* $(x_0, y_0) \leq T(x_0, y_0)$, *then* T *will have a unique fixed point which means there will exist* $(x, \lambda) \in P_F(X)$ *and* $T(x, \lambda) = (x, \lambda)$.

Proof. Let $T(x_0, \lambda_0) = (x_0, \lambda_0)$ then there is nothing to prove.
Let $(x_0, \lambda_0) \neq T(x_0, \lambda_0)$. Also, let $(x_n, \lambda_n) = T((x_{n-1}, \lambda_{n-1})$.
We need to show that $(x_{n_0}, \lambda_{n_0}) = (x_{n_0+1}, \lambda_{n_0+1})$ which gives that (x_{n_0}, λ_{n_0}) is a fixed point of T.
Therefore, let $(x_{n+1}, \lambda_{n+1}) \neq (x_n, \lambda_n)$.
As, $(x_0, \lambda_0) \leq T((x_0, \lambda_0))$, and T is non-decreasing for every $n \in N$. Therefore

$$(x_{n_0}, \lambda_{n_0}) \leq (x_1, \lambda_1) \leq (x_2, \lambda_2) \leq \cdots \leq (x_n, \lambda_n) \leq ((x_{n+1}, \lambda_{n+1}))$$

As $(x_{n-1}, \lambda_{n-1}) \leq (x_n, \lambda_n)$ using inequality (3.1.1). Therefore

$$\begin{aligned} p_F(((x_{n+1}, \lambda_{n+1}), (x_n, \lambda_n)) &= p_F(T(x_n, \lambda_n)), T((x_{n-1}, \lambda_{n-1})) \\ &\leq p_F(x_n, \lambda_n), (x_{n-1}, \lambda_{n-1}))) - \phi(p_F(x_n, \lambda_n), ((x_{n-1}, \lambda_{n-1}))) \\ &< p_F(x_n, \lambda_n)(x_{n-1}, \lambda_{n-1})) \end{aligned}$$

Therefore, $p_{f_n} = \{(p_F(x_{n-1}, \lambda_{n-1}), (x_{n,}\lambda_n)\}$ is a non-negative and decreasing sequence.
Therefore, it has a limit p. Let $p > 0$ then $\exists\ n_0 \in N$ such that $\phi(p_{f_n}) \geq \phi(p) > 0$ for all $n > n_o$ and $p_{f_n} \leq p_{f_{n-1}} - \phi(p_{f_{n-1}}) \leq p_{f_{n-1}} - \phi(p)$.
Let $n \to \infty$ we get:

$$p \leq p - \phi(p) \leq p$$

which gives a contradiction that $p_F = 0$.

Now, we need to show that (x_n, λ_n) is a cauchy sequence.
Since, $\lim_{n\to\infty} p_F((x_n, \lambda_n), (x_{n-1}, \lambda_{n-1}) = 0$ from (iii) of ϕ there exists $0 < a < 1$ and $n_0 \in N$ such that $\phi(p_F((x_n, \lambda_n), (x_{n-1}, \lambda_{n-1})) \geq a(x_{n+1}, \lambda_{n+1}), (x_n, \lambda_n)$, for all $n > n_0$.
Also,

$$\begin{aligned} p_F((x_{n+1}, \lambda_{n+1}), (x_n, \lambda_n) &\leq p_F((x_n, \lambda_n), (x_{n-1}, \lambda_{n-1}) - \phi(p_F((x_n, \lambda_n), (x_{n-1}, \lambda_{n-1})) \\ &\leq (1-a)p_F((x_n, \lambda_n), (x_{n-1}, \lambda_{n-1}) \end{aligned} \tag{3.1.2}$$

By this we get:

$$\begin{aligned} p_F((x_{n+1}, \lambda_{n+1}), (x_n, \lambda_n) &\leq (1-a)p_F((x_{n+1}, \lambda_{n+1}), (x_n, \lambda_n) \leq \dots \\ &\leq (1-a)^n p_F((x_1, \lambda_1), (x_0, \lambda_0). \end{aligned} \tag{3.1.3}$$

We fix $K = (1-a)$, therefore,

$$\begin{aligned} p_F((x_{n+1}, \lambda_{n+1}), (x_n, \lambda_n) &= 2p_F((x_{n+1}, \lambda_{n+1}), (x_n, \lambda_n) - p_F((x_{n+1}, \lambda_{n+1}), (x_{n+1}, \lambda_{n+1}) \\ &\quad + p_F((x_n, \lambda_n), (x_n, \lambda_n) \\ &\leq 4K^n p_F((x_1, \lambda_1), (x_0, \lambda_0)) \end{aligned}$$

Therefore, $\lim_{n\to\infty} p_F(x_{n+1}, \lambda_{n+1}),\ ((x_{n,}\lambda_n)) = 0$ therefore (x_n, λ_n) is cauchy and hence $\lim_{n\to\infty} p_F(x_m, \lambda_m), (x_n, \lambda_n) = 0$.
Using Lemma 2.1 we get:

$$p_F((x, \lambda), (x, \lambda)) = \lim_{n\to\infty} p_F(x_n, \lambda_n), (x, \lambda) = \lim_{n\to\infty} p_F(x_n, \lambda_n), (x_m, \lambda_m) = 0 \tag{3.1.4}$$

Hence, (x_n, λ_n) is Cauchy.
Also, as $\lim_{n\to\infty} p_F((x_{n+1}, \lambda_{n+1})), T((x, \lambda))) = p_F((T(x, \lambda), T(x, \lambda))$, and

$$\begin{aligned} p_F((x, \lambda), T(x, \lambda)) &\leq p_F((x, \lambda), (x_{n+1}, \lambda_{n+1})) + p_F((x_{n+1}, \lambda_{n+1})), T(x, \lambda)) \\ &\quad - p_F((x_{n+1}, \lambda_{n+1})), ((x_{n+1}, \lambda_{n+1}))) \\ &\leq p_F((x_{n+1}, \lambda_{n+1})), (x, \lambda)) + p_F((x_{n+1}, \lambda_{n+1})), T(x, \lambda)). \end{aligned}$$

Letting $n \to \infty$ we get:

$$\begin{aligned} p_F((x, \lambda), T(x, \lambda)) &\leq p_F((T(x, \lambda), T(x, \lambda)) \\ &\leq p_F(x, \lambda), (x, \lambda)) - \phi(p_F(x, \lambda), (x, \lambda)) = 0. \end{aligned}$$

Therefore $p_F((x, \lambda), T(x, \lambda)) = 0$.
Hence, $T(x, \lambda) = (x, \lambda)$. □

Theorem 3.2. *Suppose* $(X, \leq)$ *and* $(P_F(X), p_F)$ *is the same as in Theorem* 3.1. *Let* $T : P_F(X) \to P_F(X)$ *be a non-decreasing map as below:*

$$p_F(T(x, \lambda), (y, \gamma)) \leq p_F((x, \lambda), (y, \gamma)) - \phi\{p_F((x, \lambda), (y, \gamma))\}$$

$\forall\ x, y \in X$ *where* x, y *are comparable and* ϕ *satisfies Definition* 2.6.
Let $\{(x_n, \lambda_n)\}$ *be increasing such that* $(x_n, \lambda_n) \to (x, \lambda)$ *in* $P_F(X)$ *which implies* $(x_n, \lambda_n) \leq (x, \lambda)$ $\forall\ n$. *If* $\exists\ (x_0, \lambda_0) \in P_F(X)$ *with* $(x_0, \lambda_0) \leq T(x_0, \lambda_0)$ *such that* $(x, \lambda) = T(x, \lambda)$

Proof. As in Theorem 3.1, we will be able to have a sequence (x_{n_0}, λ_{n_0}) in $P_F(X)$ by using $x_n = T(x_{n-1})$ such that

$$(x_0, \lambda_0) \leq (x_1, \lambda_1) \leq (x_1, \lambda_1) \leq (x_{2,}\lambda_2) \leq \dots \leq (x_{n,}\lambda_n) \leq (x_{n+1}, \lambda_{n+1})$$

Also, we will prove that $\{(x_n, \lambda_n)\}$ is cauchy $(P_F(X), p_F)$ and $\exists\ x \in X$ giving

$$p_F((x, \lambda), (x, \lambda)) = \lim_{n\to\infty} p_F((x_n, \lambda_n), (x, \lambda)) = \lim_{n\to\infty} p_F((x_n, \lambda_n), (x_m, \lambda_m) = 0$$

We claim that $T(x,\lambda) = (x,\lambda)$. Suppose $p_F((x,\lambda),T(x,\lambda)) > 0$ by conditions of Theorem 3.1:

$$\begin{aligned} p_F((x,\lambda),T(x,\lambda)) &\leq p_F((x,\lambda),(x_{n+1},\lambda_{n+1}))) + p_F((x_{n+1},\lambda_{n+1})),T(x,\lambda)) \\ &\quad - p_F((x_{n+1},\lambda_{n+1})),(x_{n+1},\lambda_{n+1})) \\ &\leq p_F((x_{n+1},\lambda_{n+1})),(x,\lambda)) + p_F((x_{n+1},\lambda_{n+1})),T(x,\lambda))) \end{aligned}$$

Letting $n \to \infty$ we have, $p_F((x,\lambda),T(x,\lambda)) = 0$. □

We will now give a condition which sufficiently proves the uniqueness of fixed point theorems in both Theorem 3.1 and 3.2. The condition is stated below:

$$\text{for every } (x,\lambda),(y,\gamma) \in p_F(X) \ \exists \text{ either some lower bound or upper bound.} \qquad (3.2.1)$$

By [12] this condition is same as the following condition:

$$\forall\ x,y \in X \text{ there shall exist some } z \in X \text{ comparable to } (x,\lambda) \text{ and } (y,\gamma). \qquad (3.2.2)$$

Theorem 3.3. *We now prove the uniqueness of T in Theorems 3.1 and 3.2 using the condition* (3.2.2)*.*

Proof. Let there exist (z,ρ) and $(y,\gamma) \in p_F(X)$ as the two different fixed points of T. Therefore, $p_F((z,\rho),(y,\gamma)) > 0$. Now, we will be considering two cases:
Case I: If (z,ρ) and (y,γ) are comparable then

$$T^n(z,\rho) = (z,\rho) \text{ and } T^n(y,\gamma)) = (y,\gamma) \text{ are comparable for } n = 0,1,2$$

By condition (3.1.1), we have

$$\begin{aligned} p_F((z,\rho),(y,\gamma)) &\leq p_F(T^n(z,\rho),T^n(y,\gamma)) \\ &\leq p_F(T^{n-1}(z,\rho),T^{n-1}(y,\gamma)) - \phi(T^{n-1}(z,\rho),T^{n-1}(y,\gamma)) \\ &= p_F((z,\rho),(y,\gamma)) - \phi(p_F((z,\rho),(y,\gamma)) \\ &< p_F((z,\rho),(y,\gamma)) \text{ which is a contradiction.} \end{aligned}$$

Case II: If (z,ρ) and (y,γ) are comparable then $\exists\ x \in X$ comparable to z and y. Since, T is increasing therefore $(T^n(x)$ is comparable to $T^n(z,\rho) = (z,\rho)$ and

$$T^n(y,\gamma) = (y,\gamma) \text{ for } n = 0,1,2$$

Therefore

$$\begin{aligned} p_F((z,\rho),T^n(x)) &= p_F(T^n(z,\rho),T^n(x,\lambda)) \\ &\leq p_F(T^{n-1}(z,\rho),T^{n-1}(x,\lambda)) - \phi(p_F(T^{n-1}(z,\rho),T^{n-1}(x,\lambda)) \\ &= p_F((z,\rho)\,,T^{n-1}((x,\lambda))) - \phi p_F((z,\rho),T^{n-1}((x,\lambda))) \\ &< p_F((z,\rho),T^{n-1}(x,\lambda)) \end{aligned}$$

Therefore, we have got that $p_F((z,\rho),T^n(x,\lambda))$ is a sequence which is non-negative and non-decreasing, having a limit let it be $b > 0$. By last inequality we get that:

$$b \leq b - \phi(b) < b$$

Therefore, $a = 0$, In the same manner we can show that $\lim\limits_{n\to\infty} p_F((y,\gamma),T^{n-1}(x,\lambda)) = 0$.

$$\begin{aligned} p_F(z,\rho),(y,\gamma) &\leq p_F(z,\rho),T^n(x,\lambda) - p_F(T^n(x,\lambda),T^n(x,\lambda) \\ &\leq p_F(z,\rho),T^n(x,\lambda) + p_F(T^n(x,\lambda),T^n(x,\lambda) \end{aligned}$$

taking limit $n \to \infty$ gives $p_F(z,\rho),(y,\gamma) = 0$ which is a contradiction as $p_F(z,\rho),(y,\gamma) > 0$. Hence, proved. □

4. Conclusion

We have generalized a fuzzy metric space to a partial fuzzy metric space using fuzzy scalars which have many practical applications.

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The contribution of each author is on an equal basis in this work. Also the final manuscript have been read and approved by all authors. No conflict of interest is declared by the authors.

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Aanchal Tehlan is a research scholar in the Department of Applied Sciences (Mathematics), Faculty of Engineering & Technology, Manav Rachna International Institute of Research & Studies, Faridabad. She is doing her research in fuzzy sets and its generalizations.

Dr. Vijay Kumar is working as a professor in the Department of Applied Sciences (Mathematics), Faculty of Engineering & Technology, Manav Rachna International Institute of Research & Studies, Faridabad. His research interest includes fuzzy sets and its generalizations, decision making.