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CHANGING AND UNCHANGING ON TADPOLE DOMINATION NUMBER IN $G-e$, $G+e$ GRAPHS

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Abstract. In this paper, the effect of "Tadpole domination" change is examined, which is one of the domination types, when a graph is modified, by deleting or adding an edge in G. This edge deletion may represent a cut in a network. The occurrence of any interruption in connections of this network may lead to the pause of data transmission in the network and thus affect the work of the entire network, as well as the need to create new necessary connections within the network or excluding others for the possibility of reducing the cost. Based on these criteria "Tadpole domination number" change has been examined. The increase, decrease, and non-increase or decrease was determined for this number, in case of deletion or addition, and we have proved some basic cases for this domination change.

Keywords: dominating set, tadpole graph, tadpole domination number, edge deletion, edge addition.

AMS Subject Classification: 05C72.

1. Introduction

Today advances of wireless networks have blurred the distinction between the network infrastructure and network clients. Sensor networks, for instance, contain one or more base stations and a big number of inexpensive nodes, which combine sensors and little power wireless radios. Due to restricted radio range and battery power, most nodes cannot communicate openly with a base station, but rather rely on their peers to forward messages to and from base stations. Similarly, in mobile ad hoc networks (is a self-configuring, infrastructure-less network of mobile devices connected wirelessly) the routing of messages is also done by usual nodes. The idea of virtual backbone routing for ad hoc wireless networks is to operate routing procedures over a virtual backbone. One purpose of virtual backbone routing is to alleviate the broadcast storm problem suffered by many exiting on-demand routing protocols for route detection. Thus building a virtual backbone is very significant. The mathematical model for the electronic network is an undirected or directed graph. Motivated from this the definition of dominating sets in graph is, A set $D \subseteq V$ of vertices in a graph G is called a dominating set, if every vertex $v \in V$

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is either an element of D or is adjacent to an element of D. The domination number of G is the minimum cardinality taken over all dominating sets [1] and [2]. For some applications of the dominating set in a computer network can be seen in [3]. Many kinds of domination given in the references [19, 20, 21, 22, 23, 24, 25, 26, 27]. The connected dominating set has been a common subject studied in graph theory since 1975. Since the 1990s, it has been found to have significant requests in communication networks, particularly in wireless networks, it shows a significant role in wireless networks where the communication channel is shared between each node and its neighbors, as an example, if we want to monitor the functions of each of the computers (in a computer network the computers are the vertices of the graph and the links between them represent the edges) by a small number of computers in such a way that each of these computers can control its neighbors. Connected dominating sets are typical choices for vertices to be used for data interchange in any type of network, see some type of connected domination given in [4, 5, 6, 7, 8, 9]. Let $G(V, E)$ be a finite, simple, connected, and undirected graph where V denotes its vertices set and E its edges set. A degree of a vertex of any graph is defined as the number of edges incident on (v), it is denoted by $deg(v)$. A maximal path in a graph G is a path P in G that is not contained in a longer path. The set of neighbors of a vertex v in G is $N(v) = \{u \in V : uv \in E\}$, that is called the neighborhood of v. If $X \subseteq V(G)$ and $u \in X$ then the private neighbor of u with respect to X is defined by $P_n[u, X]= \{\{v : N[v] \cap \mathrm{X}\} = \{u\}\}$. The got graph by joining a cycle to a path with a bridge called the Tadpole graph denoted by $T_{m,n}$. A subset D of the vertices of a non-trivial connected graph $V(G)$ is said to be a tadpole dominating set of G if D is a dominating set and the set of vertices of D forms a tadpole graph $T_{m,n}$, where $m \geq 3$, $n \geq 1$, this concept is introduced in [10]. It is basically made of two fragments, a cycle graph and path graph where one of the cycle vertices is used as a gateway to path graph. The tadpole dominating set is a valuable example of a connected dominating set within an electronic network that requires a getaway, where ($\gamma_{TP} - set$) represents a minimum set of processors that can communicate directly with all other processors in the system network. For any graph, the study of determining the effect of removal or addition of an edge from the graph has numerous significant applications in any network's work, for some type of domination number and change in domination number given in the references [11, 12, 13, 14, 15, 16, 17, 18].

2. Basic Results

Theorem 2.1. [10] *Connected graph $G \ncong C_m$ has a tadpole domination if and only if:*
i) There exist a maximal path P such that $V(P)$ dominates G.
ii) The maximal path P dominates a cycle in G such that there exists at most one path of order greater than 2, which is common with one vertex with this cycle.

Remark 2.2. *Let $G - e$, and $G + e$, denote the graph formed by removing an edge e, and adding an edge e, from G respectively. Let G has a tadpole dominating set D and the vertices of the set D are labeled as follows.*
The vertices of the path are $V(P_n) =\{ x_1, x_2, x_3, \ldots, x_n \}$, and the vertices of the cycle are $V(C_m)= \{ y_1, y_2, y_3, \ldots, y_m\}$, and the edge in tadpole graph joined P_n and C_m is $e = y_1x_1$. The edge set of G is partitioned into three subsets according to how their removal or addition affects the cardinality of $\gamma_{TP}(G)-$ set. So, if $G - e$, $e \in G$ has a tadpole dominating set, then an edge e belongs to $(E^0 \cup E^+ \cup E^-)$.

Also, if $G+e$, $e \in \bar{G}$ *has a tadpole dominating set, then an edge* e *belongs to* $(E^0 \cup E^-)$, *such that:*

- $E_*^0 = \{e \in E \;:\; \gamma_{TP}(G*e) = \gamma_{TP}(G)\}$,
- $E_*^+ = \{e \in E \;:\; \gamma_{TP}(G*e) > \gamma_{TP}(G)\}$, *and*
- $E_*^- = \{e \in E \;:\; \gamma_{TP}(G*e) < \gamma_{TP}(G)\}$

Where, $* = \{- \in G, + \in \bar{G}\}$, $E_+^+ = \emptyset$.

Example 2.3. *Consider the graph* G *in the Figure. 1,* G *has a minimum tadpole dominating set is* $D = \{v_4, v_3, v_6, v_9, v_{11}, v_{12}\}$, *with cardinality* $|D| = 6$, *then:*

1. If e is deleted from G then:
$G - e_4$, $D = \{v_1, v_4, v_3, v_6, v_9, v_{11}, v_{12}\}$, $|D| = 7$, $e_4 \in E_-^+$.
$G - e_1$, $D = \{v_4, v_3, v_6, v_9, v_{11}, v_{12}\}$, $|D| = 6$, $e_1 \in E_-^0$.
2. If e is added for G then:
$G + e$, $D = \{v_4, v_3, v_6, v_9, v_{11}, v_{12}\}$, $|D| = 6$, $e = v_2v_7 \in E_+^0$.
$G + e$, $D = \{v_4, v_3, v_6, v_{11}, v_{12}\}$, $|D| = 5$, $e = v_{10}v_{11} \in E_+^-$.

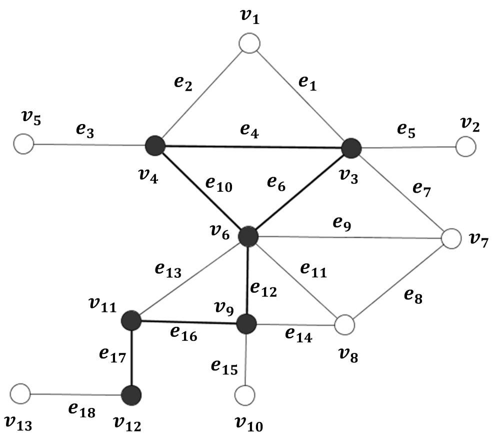

FIGURE 1. A minimum tadpole dominating set.

Observation 2.4. *If* G *is a graph has* $\gamma_{TP}(G) = 4$, *such that* $G-e$ *and* $G+e$ *has tadpole dominating set, then* $e \in E_-^0$ *and* $e \in E_+^0$, *respectively.*

Example 2.5. *For the Wheel graph* W_n, $n \geq 4$, $\gamma_{TP}(W_n - e) = \gamma_{TP}(W_n + e) = \gamma_{TP}(W_n) = 4$.

3. CHANGING AND UNCHANGING TADPOLE DOMINATION NUMBER WITH REGARD TO ADDING AND DELETING THE EDGE

Theorem 3.1. *Let* G *be a graph with* γ_{TP}*−set, if* e *is added to* $G[D]$, *then* $e \in E_+^0$, *if the following hold:*

1.

i. *If e is added between any two vertices, y_i, $y_j \in C_m$, such that, in both cycles of $C_m = C_{n_1} \cup C_{n_2}$ at least one vertex has private neighborhoods except y_i and y_j, or not dominated by D.*
ii. *If e is added between any two vertices, x_i, $x_j \in P_n - x_n$, $n \geq 3$, such that neither $pn\,[x_{i+1},\ D\,]$ and $pn\,[\,x_{j-1},\ D\,] \neq \emptyset$ nor at least one vertex between $\{x_{i+1}$ and $x_{j-1}\}$, is not dominated by D.*
iii. *If $e = x_i x_n \in P_n$ is added such that cycle C_n is formed and $D - V(\,C_n)$ has private neighborhoods from $V - D$, or if contain some vertices not dominated by C_n with no private neighborhoods from $V - D$.*
2. *If e is added between any vertex $y_i \in C_m$ and the vertex x_n, $m \geq 3$, $n \geq 1$, such that $pn\,[x_1, x_{n-1}, y_2(\text{or } y_m\,),\ y_{i-1},\ D\,] \neq \emptyset$.*
3.$|pn[u\ , D]| \geq 1$, *for all $u \in D$.*

Proof.
1)
i) When e is added between any two vertices y_i, $y_j \in C_m$, then $C_m = C_{n_1} \cup C_{n_2}$ if one cycle is excluded and the other regarded as a cycle of tadpole dominating set. According to assumption the vertices of this cycle are not dominated by D or the private neighborhoods of vertices in this cycle will not be dominated, therefore, $e \in E^0_+$.
ii) By the same proof in (i).
iii) When $e = x_i\, x_n$ is added then a cycle is formed say C_n. Then if C_n is regarded as a cycle of tadpole dominating set. Since the vertices of $D - V(\,C_n)$ have private neighborhoods from $V - D$, or they are not dominated by C_n, then any vertex from $D - V(\,C_n)$ is not excluded. Hence, $e \in E^0_+$.
2) If e is added between $y_i \in C_m$ and x_n, this forms a new cycle, say C_k, $V(C_k) = \{\,y_1,\ x_1,\ \ldots,\ x_n,\ y_i,\ \ldots,\ y_1\}$. If C_k, is chosen as a cycle of tadpole dominating set then $e \in E^0_+$, since these are all the remaining vertices of D that do not belong to C_k, will be the vertices of the path in tadpole dominating set in $G + e$. Also in the case that $pn\,[x_{i+1}\ ,\ x_{j-1},\ D\,] = \emptyset$, and at least one vertex between x_{i+1} and x_{j-1} is not dominated by D then D will remain as the tadpole dominating set.
3)Its clear. □

Example 3.2. *The following figures illustrate various cases discussed in Theorem 3.1 Such that for $G + e$, $e \in E^0_+$.*

1. In Figure 2 case1 (i) is illustrated. In (a) $\gamma_{TP}(G) = \gamma_{TP}(G + e) = 5$, and in(b) $\gamma_{TP}(G) = \gamma_{TP}(G + e) = 4$. In (a) when $e = y_2\, y_4$ is added there is no effect on the cardinality of D since y_3 has a private neighborhoods from $V - D$. But in (b) y_1 has no private neighborhoods and there is an edge between y_4 and x_1, if the cycle $\{y_4,\ y_2,\ y_3\}$ is chosen as a cycle of tadpole dominating, then $y_1 \notin D$.

2. Figure 3 illustrates case 1 (ii, iii). In (a) $\gamma_{TP}(G) = \gamma_{TP}(G{+}e) = 9$, and in (b) $\gamma_{TP}(G) = \gamma_{TP}(G + e) = 9$. In (c), the cycle of vertices $\{x_5,\ x_4,\ x_3,\ x_2,\ x_1,\ x_5\}$ is chosen and the path vertices y_1, y_2, after adding e=$x_5\, x_1$. Vertex (y_m) is excluded since it has no private neighborhoods from $V - D$, $\gamma_{TP}(G) = 8$, $\gamma_{TP}(G +\ e) = 7$.
3. Figure 4 illustrates case 2, no vertex can be excluded from y_2 to y_{i+1}, so $\gamma_{TP}(G) = \gamma_{TP}(G + e) = 13$.

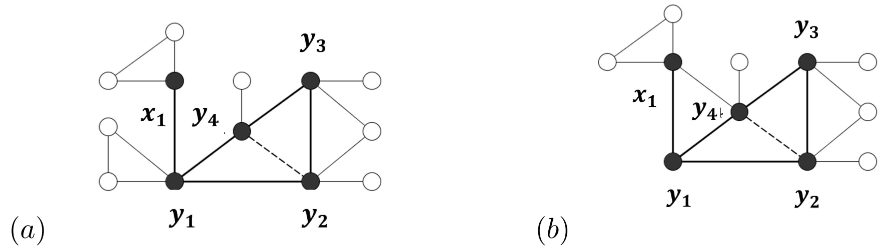

FIGURE 2. A minimum tadpole dominating sets in G.

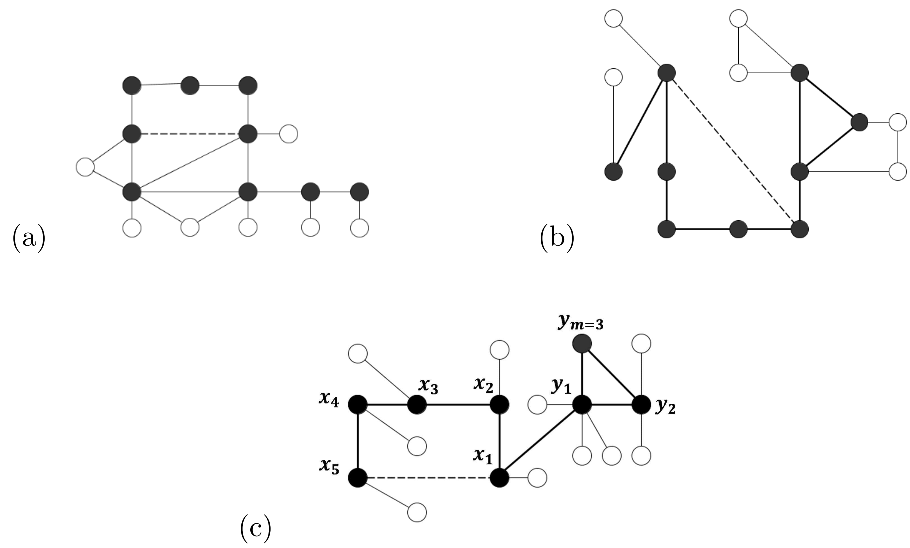

FIGURE 3. A minimum tadpole dominating sets in G.

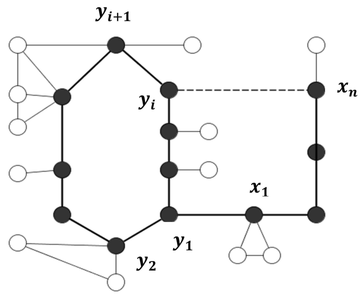

FIGURE 4. A minimum tadpole dominating set in G.

Observation 3.3. *Let G be a graph with minimum tadpole dominating set D. If e is added to $G\left[V-D\right]$ then $e \in E_{+}^{0}$, if:*

i) $|pn[v,\ D]|\ \geq 1$*, for all* $v \in D$*.*
ii) *For any e added between any two vertices from $V-D$ adjacent to v_k and v_l respectively such that the new cycle is formed by adding e will not satisfy case(ii) in Theorem 2.1 if at least one vertex between the two vertices v_k and v_l* $\{v_{k+1}, \ldots,\ v_{l-1}\}$

have privat neighborhoods from $V - D$ or is not dominated by other vertices from D only by its neighborhoods in the sequence of the path of D.
iii) *The new cycle formed by adding e satisfies case (ii) in Theorem 2.1 with the following:*
a. *If $e = v_i\ v_j$, v_i and v_j are adjacent to $V(P_n)$ such that the vertices which are adjacent to y_1 in C_m have private neighborhoods from $V - D$.*
b.*If $e = v_i\ v_j$, v_i and v_j are adjacent to y_k and y_l respectively, y_k, $y_l \in V(C_m)$, such that the two vertices that are adjacent to y_1 and the two vertices y_{k-1} and y_{l+1} have private neighborhoods from $V - D$.*
c. *If $e = v_i\ v_j$, v_i is adjacent to $x_k \in V(P_n)$ and v_j is adjacent to $y_l \in V(C_m)$, such that x_{k+1}, y_{l-1} and the two vertices that are adjacent to y_1 have private neighborhoods from $V - D$.*

Observation 3.4. *Let G be a graph with minimum tadpole dominating set D, and $G+e$, $e \in \bar{G}$, has a tadpole dominating set $D^{'}$ then $e \in E_{+}^{-}$ provided that the following hold:*

1). *If the cases in Observation 3.3 are not hold, then if neither we take the shorter path $\{v_k,\ v_i,\ v_j,\ v_l\}$, nor if we take the cycle formed by e to belong to $D^{'}$ without taking all the remaining vertices of D for the path of $D^{'}$.*
2). *If e is added between any two vertices of C_m, such that, $C_m = C_{n_1} \cup C_{n_2}$, the vertices (with no private neighborhood from $V - D$) of one of the two cycle is dominated by the other, or by some other vertices in D. The same condition holds if e is added between any two vertices of P_n, $m \geq 4$, $n \geq 3$, such that the vertices between them are dominated.*
3). *If $e = x_i\ y_j$, $x_i \in P_n$ and $y_j \in C_m$, then if neither we take for $D^{'}$ the shorter path $\{x_i,\ y_j\}$ without taking all the remaining vertices (have no private neighborhoods from $V - D$ or are dominated by some other vertices in D) of P_n, nor if we take the cycle (satisfies case (ii) in Theorem 2.1) formed by e such that at least one vertex (has no private neighborhoods from $V - D$) from D is dominated by this cycle.*
4). *As in the previous procedure, when $e = v_i\ v_j$ is added, $v_i \in D$ and $v_j \in V - D$.*

Example 3.5. *The following examples illustrates Observation 3.4 Such that for $G + e$, $e \in E_{+}^{-}$.*

1.Figure 5 illustrates the "Observation 3.4(2)", $\gamma_{TP}(G) = 16$, $\gamma_{TP}(G + y_i\, y_j) = 12$, $\gamma_{TP}(G + x_i\, x_j) = 13$.
2. "Observation 3.4.(3)" is illustrated in Figure 6 the cycle chosen with the path of vertices $\{y_{i+1},\ y_{i+2}\ \}$ in $D^{'}$ and the vertices $\{v_m,\ v_n,\ v_h,\ v_r\}$ are excluded because they have no private neighborhoods from the set $V - D$. $\gamma_{TP}(G)$= 14, $\gamma_{TP}(G+y_i\, x_n) = 10$.

Theorem 3.6. *For any graph G with $\gamma_{TP}-$set, if $G-e$, $e \in G$ has a tadpole dominating set $D^{'}$ then $e \in E_{-}^{0}$, if the following hold:*
a) *If any e is deleted from $G[V - D]$.*
b) *If any e which joins two vertices of D but does not belong to the set of edges of tadpole graph.*
c) *If $e = v_i\ v_{i+1}$ is deleted from D except $e = x_{n-1}\ x_n$, and there is a vertex say v_w adjacent to v_i and v_{i+2} (if $v_i = x_{n-1}$ then v_w adjacent to v_i) and to all the private neighborhoods of v_{i+1}, then we replace v_{i+1} by v_w in $D^{'}$.*

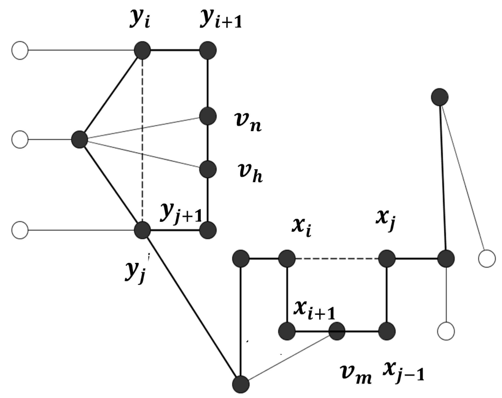

Figure 5. A minimum tadpole dominating set in G.

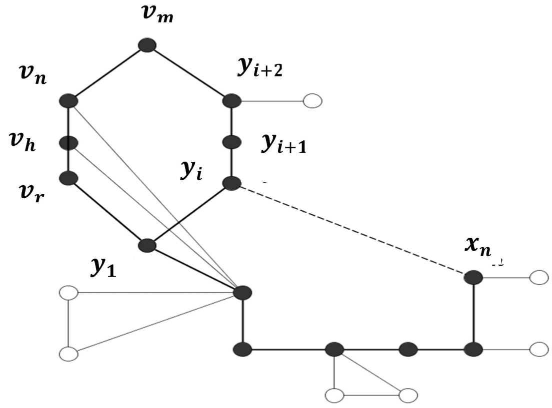

Figure 6. A minimum tadpole dominating set in G.

d) *If* $e = x_{n-1}\ x_n$ *is deleted from* P_n*, and there is a vertex* v_w *that is adjacent to* x_{n-1} *and adjacent to* x_n *and its neighborhoods, then we can take* v_w *in* D'.
e) *If* $e = v_k\ v_t$ *is deleted, where* v_k *belongs to* $V - D$ *is adjacent to* v_i*, and* v_i*,* v_t *belong to* D.

Example 3.7. *The following figures illustrate some cases discussed in Theorem 3.6 Such that for* $G - e$*,* $e \in E^0_-$.

1) Figure 7 illustrates the case of "Theorem 3.6(b)", $\gamma_{TP}(G) = \gamma_{TP}(G - e) = 6$.

2) Figure 8 illustrates the case of the "Theorem 3.6(d)", $\gamma_{TP}(G) = \gamma_{TP}(G - e) = 12$.

4. Conclusions

The tadpole domination number can be kept after changing the network links (deleting or adding the edge) and avoiding redetermined it in a new way a second time, the previous approach showed this. As well as the possibility of preserving the same set elements as possible to avoid an unnecessary and unhelpful change of network structure in the case

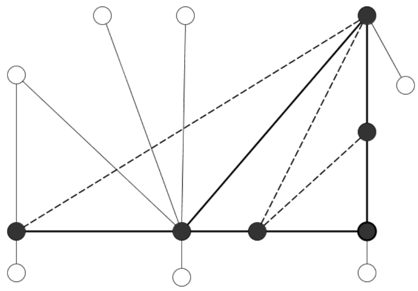

FIGURE 7. A minimum tadpole dominating set in G.

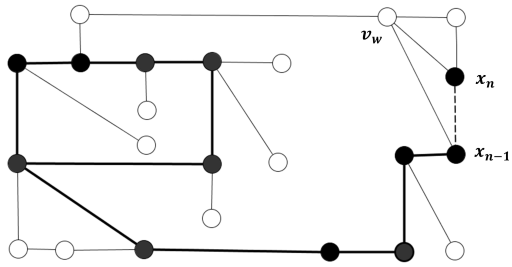

FIGURE 8. A minimum tadpole dominating set in G.

that the dominant set is replaced by another. It is also found that it is possible to reduce the number of members of these sets in some cases of addition.

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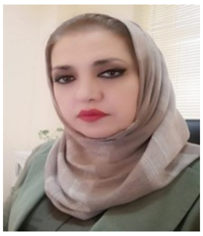

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