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CUBIC $(1, 2)$-IDEALS ON SEMIGROUPS

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Abstract. In this paper we introduce the concept of cubic $(1, 2)$-ideals on semigroups and we study basic properties of cubic $(1, 2)$-ideals. In particular, we find condition cubic bi-ideal is cubic $(1, 2)$-ideal coincide. Finally we can show that the images or inverse images of a cubic $(1, 2)$-ideal of a semigroup become a cubic $(1, 2)$-ideal.

Keywords: Cubic set, Cubic ideal, Cubic $(1, 2)$-ideal, Cubic bi-ideal.

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1. Introduction

A semigroup is an algebraic structure consisting of a non-empty set S together with an associative binary operation. Semigroups are important in many areas of mathematics, for example, coding and language theory, automata theory, combinatorics and mathematical analysis.

In 1965, Zadeh [24] introduced the concept of fuzzy sets. Since fuzzy set has been applied to many branches in mathematics. The fuzzification of algebraic structures was initiated by Rosenfeld [14], and he introduced the notion of fuzzy subgroups. In 1981, Kuroki [8] introduced and studied the concepts of fuzzy ideal and fuzzy bi-ideals on semigroups. In 1975, the concept of interval valued fuzzy sets was introduced by Zadeh [25], as a generalization of the notion of fuzzy sets. In 2006, Narayanan and Manikanran [13] initiated the notion of interval valued fuzzy ideal in semigroup. In 2012, Jun et al. [5] introduced a new notion, called a cubic set, and investigated several properties and introduced cubic subsemigroups and cubic left (right) ideals of semigroups. In 2015 Sadaf et al. [15], discussed cubic bi-ideal of a semigroup. Recently, Khamrod and Deetae [7] studied Q-cubic bi-quasi ideals of semigroups. Moreover, the concept of cubic sets has been discussed in other research and fields such as cubic soft ideals in BCK/BCI-algebras [6], cubic soft sets with applications in BCK/BCI-algebras [9], subalgebras of BCK/BCI-Algebras based on cubic soft sets [10], stable cubic sets [11], cubic intuitionistic structures applied to ideals of BCI-algebras [16], neutrosophic cubic set theory applied to UP-algebras [22], and the concept of cubic sets is more relevant in mathematics [17, 20, 21].

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In this paper we have studied the concept study cubic $(1,2)$-ideal of a semigroup and we study basic properties of cubic $(1,2)$-ideal and study the cubic bi-ideal is a cubic $(1,2)$-ideal coincidence. Furthermore we can show that the images or inverse images of a cubic $(1,2)$-ideal of a semigroup become a cubic $(1,2)$-ideal.

2. Preliminaries

In this topic, we review some definitions and results which are used in the next section.

A semigroup S is said to be *regular* if for each element $a \in S$, there exists an element $x \in S$ such that $a = axa$. A non-empty subset A of a semigroup S is a subsemigroup of S if it $A^2 \subseteq A$. A non-empty subset A of a semigroup S is called a *left*(right)ideal of S if $SA \subseteq A(AS \subseteq A)$. An ideal A of S is a nonempty subset which is both a left ideal and a right ideal of S. A non-empty subset K of a semigroup S is called a *generalized bi-ideal* of S if $KSK \subseteq K$. A subsemigroup A of a semigroup S is called a *bi-ideal* of S if $ASA \subseteq A$. A subsemigroup A of S is called a $(1,2)$-ideal of S if $ASA^2 \subseteq A$. We note here that every bi-ideal of a semigroup S is a $(1,2)$-ideal of S [12].

Definition 2.1. [24] A *fuzzy subset* f of a non-empty subset T is a function $T \to [0,1]$.

For any $\eta_1, \eta_2 \in [0,1]$, we have

$$\eta_1 \vee \eta_2 = \max\{\eta_1, \eta_2\} \quad \text{and} \quad \eta_1 \wedge \eta_2 = \min\{\eta_1, \eta_2\}.$$

More generally, if $\{\eta_i : i \in \mathcal{J}\}$ is a collection of fuzzy sets of T, then

$$\underset{i\in\mathcal{J}}{\vee} \eta_i := \sup_{i\in\mathcal{J}}\{\eta_i\} \quad \text{and} \quad \underset{i\in\mathcal{J}}{\wedge} \eta_i := \inf_{i\in\mathcal{J}}\{\eta_i\}.$$

Definition 2.2. [12] Let S be a semigroup. A fuzzy subset f of S is said to be

(1) a *fuzzy subsemigroup* of S if $f(uv) \geq f(u) \wedge f(v)$, for all $u, v \in S$,
(2) a *fuzzy left*(right) ideal of S if $f(uv) \geq f(v)(f(uv) \geq f(u))$, for all $u, v \in S$,
(3) a *fuzzy ideal* of S if it is a fuzzy left ideal and a fuzzy right ideal of S,
(4) a *fuzzy generalized bi-ideal* of S if $f(uvw) \geq f(u) \wedge f(w)$ for all $u, v, w \in S$,
(5) a *fuzzy bi-ideal* of S if if f is a fuzzy subsemigroup of S and $f(uvw) \geq f(u) \wedge f(w)$ for all $u, v, w \in S$,
(6) a *fuzzy (1,2)-ideal* of S if f is a fuzzy subsemigroup of S and $f(ua(vw)) \geq f(u) \wedge f(v) \wedge f(w)$ for all $u, a, v, w \in S$.

Suppose $C[0,1]$ to denote the set of all closed subintervals of $[0,1]$, i.e.,

$$C[0,1] = \{\overline{\eta} = [\eta^-, \eta^+] \mid 0 \leq \eta^- \leq \eta^+ \leq 1\}.$$

Note that $[\eta, \eta] = \{\eta\}$ for all $\eta \in [0,1]$. For $\eta = 0$ or 1 we shall denote $[0,0]$ by $\overline{0}$ and $[1,1]$ by $\overline{1}$.

Definition 2.3. [23] For each interval $\overline{\eta} = [\eta^-, \eta^+]$ and $\overline{\vartheta} = [\vartheta^-, \vartheta^+]$ in $C[0,1]$, define the operations "$\succeq$", "$\preceq$", "$=$", "$\curlywedge$" "$\curlyvee$ as follows:

(1) $\overline{\eta} \succeq \overline{\vartheta}$ if and only if $\eta^- \geq \vartheta^-$ and $\eta^+ \geq \vartheta^+$,
(2) $\overline{\eta} \preceq \overline{\vartheta}$ if and only if $\eta^- \leq \vartheta^-$ and $\eta^+ \leq \vartheta^+$,
(3) $\overline{\eta} = \overline{\vartheta}$ if and only if $\eta^- = \vartheta^-$ and $\eta^+ = \vartheta^+$,
(4) $\overline{\eta} \curlywedge \overline{b} = [(\eta^- \wedge \vartheta^-), (\eta^+ \wedge \vartheta^+)]$,
(5) $\overline{\eta} \curlyvee \overline{b} = [(\eta^- \vee \vartheta^-), (\eta^+ \vee \vartheta^+)]$.

We write $\overline{\eta} \succeq \overline{\vartheta}$ whenever $\overline{\vartheta} \preceq \overline{\eta}$.

Proposition 2.4. [1] *For $\overline{\eta}, \overline{\vartheta}, \overline{\omega} \in C[0,1]$, the following properties are true:*
(1) $\overline{\eta} \curlywedge \overline{\eta} = \overline{\eta}$ *and* $\overline{\eta} \curlyvee \overline{\eta} = \overline{\eta}$,
(2) $\overline{\eta} \curlywedge \overline{\vartheta} = \overline{\vartheta} \curlywedge \overline{\eta}$ *and* $\overline{\eta} \curlyvee \overline{\vartheta} = \overline{\vartheta} \curlyvee \overline{\eta}$,
(3) $(\overline{\eta} \curlywedge \overline{\vartheta}) \curlywedge \overline{\omega} = \overline{\eta} \curlywedge (\overline{\vartheta} \curlywedge \overline{\omega})$ *and* $(\overline{\eta} \curlyvee \overline{\vartheta}) \curlyvee \overline{\omega} = \overline{\eta} \curlyvee (\overline{\vartheta} \curlyvee \overline{\omega})$,
(4) $(\overline{\eta} \curlywedge \overline{\vartheta}) \curlyvee \overline{\omega} = (\overline{\eta} \curlyvee \overline{\omega}) \curlywedge (\overline{\vartheta} \curlyvee \overline{r})$ *and* $(\overline{\eta} \curlyvee \overline{\vartheta}) \curlywedge \overline{\omega} = (\overline{\eta} \curlywedge \overline{\omega}) \curlyvee (\overline{\vartheta} \curlywedge \overline{\omega})$,
(5) *If* $\overline{\eta} \preceq \overline{\vartheta}$, *then* $\overline{\eta} \curlywedge \overline{\omega} \preceq \overline{\vartheta} \curlywedge \overline{\omega}$ *and* $\overline{\eta} \curlyvee \overline{\omega} \preceq \overline{\vartheta} \curlyvee \overline{\omega}$.

Definition 2.5. [4] For each interval $\overline{\eta}_i = [\eta_i^-, \eta_i^+] \in C[0,1]$, $i \in \mathcal{J}$ where $\mathcal{J}$ is an index set, define

$$\underset{i\in\mathcal{J}}{\curlywedge} \overline{\eta}_i = [\underset{i\in\mathcal{J}}{\wedge} \eta_i^-, \underset{i\in\mathcal{J}}{\wedge} \eta_i^+] \quad \text{and} \quad \underset{i\in\mathcal{J}}{\curlyvee} \overline{\eta}_i = [\underset{i\in\mathcal{J}}{\vee} \eta_i^-, \underset{i\in\mathcal{J}}{\vee} \eta_i^+].$$

Definition 2.6. [23] Let T be a non-empty set. Then the function $\overline{f} : T \to C[0,1]$ is called an *interval valued fuzzy subset* (shortly, IVF subset) of T.

Definition 2.7. [13, 3] An IVF subset $\overline{f}$ of a semigroup S is said to be
(1) an *IVF subsemigroup* of S if $\overline{\mu}(uv) \succeq \overline{\mu}(u) \curlywedge \overline{\mu}(v)$ for all $u, v \in S$,
(2) an *IVF left (right) ideal* of S if $\overline{\mu}(uv) \succeq \overline{\mu}(v)(\overline{\mu}(uv) \succeq \overline{\mu}(u))$ for all $u, v \in S$. An IVF subset $\overline{\mu}$ of S is called an *IVF ideal* of S if it is both an IVF left ideal and an IVF right ideal of S,
(3) an *IVF generalized bi-ideal* of S if $\overline{\mu}(uvw) \succeq \overline{\mu}(u) \curlywedge \overline{\mu}(w)$ for all $u, v, w \in S$,
(4) an *IVF bi-ideal* of S if $\overline{\mu}$ is an IVF subsemigorup and $\overline{\mu}(uvw) \succeq \overline{\mu}(u) \curlywedge \overline{\mu}(w)$ for all $u, v, w \in S$,
(5) an *IVF* $(1,2)$*-ideal* of S if $\overline{\mu}$ is an IVF subsemigorup and $\overline{\mu}(ua(vw)) \succeq \overline{\mu}(u) \curlywedge \overline{\mu}(v) \curlywedge \overline{\mu}(w)\}$ for all $a, u, v, w \in S$.

Definition 2.8. [4] Let T be a non-empty set. A *cubic set* $\mathcal{A}$ in T is a structure of the form

$$\mathcal{A} = \{\langle x, \overline{\mu}(x), f(x)\rangle : x \in T\}$$

and denoted by $\mathcal{A} = \langle \overline{\mu}, f\rangle$ where $\overline{\mu} = [\mu^-, \mu^+]$ is an interval-valued fuzzy set (briefly. IVF) in X and f is a fuzzy set in T. In this case we will use

$$\mathcal{A}(x) = \langle \overline{\mu}(x), f(x)\rangle = \langle [\mu^-(x), \mu^+(x)], f(x)\rangle$$

For all $x \in T$. Note that a cubic set is a generalization of an intuitionistic fuzzy set.

Definition 2.9. Let S be a semigroup. Then cubic set characteristic function $\chi_A = \langle \overline{\mu}_{\chi_A}, f_{\chi_A}\rangle$ of is defined as

$$\overline{\mu}_{\chi_A}(x) = \begin{cases} \overline{1}, & \text{if} \quad x \in A, \\ \overline{0}, & \text{if} \quad x \notin A. \end{cases}$$

and

$$f_{\chi_A}(x) = \begin{cases} 0, & \text{if} \quad x \in A, \\ 1, & \text{if} \quad x \notin A. \end{cases}$$

Definition 2.10. [4] The whole cubic set $\mathcal{S}$ in a semigroup S is defined to be a structure

$$\mathcal{S} = \{\langle x, 1_S(x), 0_S(x)\rangle : x \in S\}$$

with $1_S(x) = \overline{1}$ and $0_S(x) = \overline{0}$ for all $x \in S$. It will briefly denoted by $\mathcal{S} = \langle 1_S, 0_S\rangle$.

Definition 2.11. [4] For two cubic set $\mathcal{A} = \langle \overline{\mu}, f\rangle$ and $\mathcal{B} = \langle \overline{\lambda}, g\rangle$ in a semigroup S, we define

$$\mathcal{A} \sqsubseteq \mathcal{B} \Leftrightarrow \overline{\mu} \preceq \overline{\lambda}, f \geq g$$

Definition 2.12. [4] Let $\mathcal{A} = \langle \overline{\mu}, f\rangle$ and $\mathcal{B} = \langle \overline{\lambda}, g\rangle$ be two cubic set in a semigroup S. Then the cubic product of $\mathcal{A}$ and $\mathcal{B}$ is a structure

$$\mathcal{A} \odot \mathcal{B} = \{\langle x, (\overline{\mu} \circ \overline{\lambda})(x), (f \circ g)(x)\rangle : x \in S\}$$

which is briefly denoted by $\mathcal{A} \odot \mathcal{B} = \langle \overline{\mu} \circ \overline{\lambda}, f \circ g\rangle$ where $\overline{\mu} \circ \overline{\lambda}$ and $f \circ g$ are defined as follows, respectively:

$$(\overline{\mu} \circ \overline{\lambda})(x) = \begin{cases} \bigsqcup\limits_{(y,z)\in F_x} [\overline{\mu}(y) \curlywedge \overline{\lambda}(z)\}] & \text{if } \quad x = yz, \\ \overline{0}, & \text{otherwise,} \end{cases}$$

and

$$(f \circ g)(x) = \begin{cases} \bigwedge\limits_{(y,z)\in F_x} [f(y) \vee g(z)] & \text{if } \quad x = yz \\ 1, & \text{otherwise,} \end{cases}$$

for all $x \in S$.

Definition 2.13. [4] Let $\mathcal{A} = \langle \overline{\mu}, f\rangle$ and $\mathcal{B} = \langle \overline{\lambda}, f_B\rangle$ be two cubic set in a semigroup S. Then the intersection of $\mathcal{A}$ and $\mathcal{B}$ denoted by $\mathcal{A}\overline{\sqcap}\mathcal{B}$ is the cubic set

$$\mathcal{A}\overline{\sqcap}\mathcal{B} = \langle \overline{\mu} \sqcap \overline{\lambda}, f \vee g\rangle$$

where $(\overline{\mu} \sqcap \overline{\lambda})(x) = \overline{\mu}(x) \curlywedge \overline{\lambda}(x)$ and $(f \vee g)(x) = f(x) \vee g(x)$ for all $x \in S$. And union of $\mathcal{A}$ and $\mathcal{B}$ denoted by $\mathcal{A}\underline{\sqcup}\mathcal{B}$ is the cubic set

$$\mathcal{A}\underline{\sqcup}\mathcal{B} = \langle \overline{\mu} \sqcup \overline{\lambda}, f \wedge g\rangle$$

where $(\overline{\mu} \sqcup \overline{\lambda})(x) = \overline{\mu}(x) \curlyvee \overline{\lambda}(x)$ and $(f \wedge g)(x) = f(x) \wedge g(x)$ for all $x \in S$.

Proposition 2.14. [4] *For any cubic set $\mathcal{A} = \langle \overline{\mu}, f\rangle, \mathcal{B} = \langle \overline{\lambda}, g\rangle$ and $\mathcal{C} = \langle \overline{\nu}, h\rangle$ in semigroup S. Then the following statement holds.*

(1) $\mathcal{A}\underline{\sqcup}(\mathcal{B}\overline{\sqcap}C) = (\mathcal{A}\underline{\sqcup}\mathcal{B})\overline{\sqcap}(\mathcal{A}\underline{\sqcup}\mathcal{C})$,
(2) $\mathcal{A}\overline{\sqcap}(\mathcal{B}\underline{\sqcup}C) = (\mathcal{A}\overline{\sqcap}\mathcal{B})\underline{\sqcup}(\mathcal{A}\overline{\sqcap}\mathcal{C})$,
(3) $\mathcal{A} \odot (\mathcal{B}\underline{\sqcup}C) = (\mathcal{A} \odot \mathcal{B})(\mathcal{A} \odot \mathcal{C})$,
(4) $\mathcal{A} \odot (\mathcal{B}\overline{\sqcap}C) = (\mathcal{A} \odot \mathcal{B})\overline{\sqcap}(\mathcal{A} \odot \mathcal{C})$.

Definition 2.15. [4] A cubic set $\mathcal{A} = \langle \overline{\mu}, f\rangle$ in a semigroup S is called a *cubic subsemigroup* of S if it satisfies:

(1) $\overline{\mu}(uv) \succeq \overline{\mu}(u) \curlywedge \overline{\mu}(v)$,
(2) $f(uvy) \leq f(u) \vee f(v)$ for all $u, v \in S$.

Definition 2.16. [4] A cubic set $\mathcal{A} = \langle \overline{\mu}, f\rangle$ in a semigroup S is called a *cubic left*(resp. right)ideal of S if it satisfies:

(1) $\overline{\mu}(uv) \succeq \overline{\mu}(v)$ $(\overline{\mu}(uv) \succeq \overline{\mu}(u))$,
(2) $f(uv) \leq f(v)$, $(f(uv) \leq f(u))$ for all $u, v \in S$.

A non-empty cubic set $\mathcal{A} = \langle \overline{\mu}, f\rangle$ of S is called a *cubic ideal* of S if it is a cubic left ideal and a cubic right ideal of S.

Definition 2.17. [15] A cubic semigroup $\mathcal{A} = \langle \overline{\mu}, f\rangle$ in a semigroup S is called a *cubic generalized bi-ideal* of S if it satisfies:

(1) $\overline{\mu}(uvw) \succeq \overline{\mu}(u) \curlywedge \overline{\mu}(w)$,
(2) $f(uvw) \leq f(u) \vee f(w)$ for all $u, v, w \in S$.

Definition 2.18. [15] A cubic subsemigroup $\mathcal{A} = \langle \overline{\mu}, f\rangle$ on a semigroup S is called a *cubic bi-ideal* of S if it satisfies:

(1) $\overline{\mu}(uvw) \succeq \overline{\mu}(u) \curlywedge \overline{\mu}(w)$,
(2) $f(uvw) \leq f(u) \vee f(w)$ for all $u, v, w \in S$.

Theorem 2.19. [4] *Let $\mathcal{A} = \langle \overline{\mu}, f\rangle$ and $\mathcal{B} = \langle \overline{\lambda}, g\rangle$ be a cubic subsemigroup of S. Then $A \overline{\sqcap} B = \langle \overline{\mu} \sqcap \overline{\lambda}, f \vee g\rangle$ is a cubic subsemigroup of S.*

Theorem 2.20. [4] *Let A be non-empty subset of a semigroup S. Then A is a subsemigroup of S if and only if the characteristic cubic set $\chi_A = \langle \overline{\mu}_{\chi_A}, f_{\chi_A}\rangle$ is a cubic subsemigroup of S.*

Definition 2.21. [4] Let $\mathcal{A} = \langle \overline{\mu}, f\rangle$ is cubic set in X. For any $k \in [0, 1]$ and $[s, t] \in C[0, 1]$, we define $U(A, [s, t], k)$ as follows:

$$U(\mathcal{A}, [s, t], k) = \{x \in X \mid \overline{\mu}(x) \succeq [s, t], f(x) \leq k\},$$

and we say it is a cubic level set of $\mathcal{A} = \langle \overline{\mu}, f\rangle$.

Theorem 2.22. [4] *Let S be a semigroup. Then $\mathcal{A} = \langle \overline{\mu}, f\rangle$ is a cubic subsemigroup of S if and only if the level set $U(\mathcal{A}, [s, t], k)$ is subsemigroup of S.*

3. Cubic (1,2)-ideals of semigroups

In this section, we define cubic (1,2)-ideal and discuss some of its properties.

Definition 3.1. A cubic subsemigroup $\mathcal{A} = \langle \overline{\mu}, f\rangle$ of a semigroup S is called a *cubic (1,2)-ideal* of S if it satisfies:
(1) $\overline{\mu}(ua(vw)) \succeq \overline{\mu}(u) \curlywedge \overline{\mu}(v) \curlywedge \overline{\mu}(w)$,
(2) $f(ua(vw)) \leq f(u) \vee f(v) \vee f(w)$ for all $a, u, v, w \in S$.

The following example is a cubic $(1, 2)$-ideal of a semigroup.

Example 3.2. Consider a semigroup $S = \{a, b, c\}$ defined by the following table:

$\cdot$	a	b	c	d
a	a	a	a	a
b	a	b	c	a
c	a	b	c	b
d	a	b	d	d

Define cubic set $\mathcal{A} = \langle \overline{\mu}, f\rangle$ in S as follows:

$S:$	$\overline{\mu}(x)$	$f(x)$
a	$[0.9, 1]$	0.2
b	$[0.6, 0.8]$	0.5
c	$[0.4, 0.6]$	0.2
d	$[0.2, 0.3]$	0.2

Then, by routine calculation one can easily verify that $\mathcal{A} = \langle \overline{\mu}, f\rangle$ of S is a cubic $(1, 2)$-ideal of S.

The following theorem we shown the intersection of any family of cubic $(1, 2)$ ideal of a semigroup.

Theorem 3.3. *The intersection of any family of cubic $(1, 2)$-ideals of semigroup S is a cubic $(1, 2)$-ideal of a semigroup S.*

Proof. Let $\{\mathcal{A}_i\}_{i\in I}$ be a family of a cubic $(1,2)$-ideals of semigroup S and let $u, v \in S$. Then

$$\underset{i\in I}{\sqcap}\overline{\mu}_i(uv) \succeq \underset{i\in I}{\sqcap}(\overline{\mu}_i(u) \curlywedge \overline{\mu}_i(v)) = \underset{i\in I}{\sqcap}\overline{\mu}_i(u) \curlywedge \underset{i\in I}{\sqcap}\overline{\mu}_i(v).$$

And

$$\bigvee_{i\in I} f_i(uv) \leq \bigvee_{i\in I}(f_i(u) \vee f_i(v)) = \bigvee_{i\in I} f_i(u) \vee \bigvee_{i\in I} f_i(v).$$

Hence $\overline{\underset{i\in I}{\sqcap}}\mathcal{A}_i = \langle \underset{i\in I}{\sqcap}\overline{\mu}_i, \bigvee_{i\in I} f_i\rangle$ is a cubic subsemigroup of S. In a similar way,
let $a, u, v, w \in S$ we get that

$$\begin{aligned}\underset{i\in I}{\sqcap}\{\overline{\mu}_i(ua(vw))\} &\succeq \underset{i\in I}{\sqcap}(\overline{\mu}_i(u) \curlywedge \overline{\mu}_i(v) \curlywedge \overline{\mu}_i(w)) \\ &= \underset{i\in I}{\sqcap}\overline{\mu}_i(u) \curlywedge \underset{i\in I}{\sqcap}\overline{\mu}_i(v) \curlywedge \underset{i\in I}{\sqcap}\overline{\mu}_i(w).\end{aligned}$$

Thus $\underset{i\in I}{\sqcap}\{\overline{\mu}_i(ua(vw))\} \succeq \underset{i\in I}{\sqcap}\overline{\mu}_i(u) \curlywedge \underset{i\in I}{\sqcap}\overline{\mu}_i(v) \curlywedge \underset{i\in I}{\sqcap}\overline{\mu}_i(w)$. And

$$\begin{aligned}\bigvee_{i\in I}\{f_i(ua(vw))\}\} &\leq \bigvee_{i\in I}\bigvee_{i\in I}(f_i(u) \vee f_i(v) \vee f_i(w)) \\ &= \bigvee_{i\in I} f_i(u) \vee \bigvee_{i\in I} f_i(v) \vee \bigvee_{i\in I} f_i(w).\end{aligned}$$

Thus $\bigvee_{i\in I}\{f_i(ua(vw))\} \leq \bigvee_{i\in I} f_i(u) \vee \bigvee_{i\in I} f_i(v) \vee \bigvee_{i\in I} f_i(w)$.
Hence $\bigcap_{i\in I}\mathcal{A}_i = \langle \bigcap \overline{\mu}_i, \bigvee_{i\in I} f_i\rangle$ is a cubic $(1,2)$-ideal of S. □

In the following theorems, we give a relationship between $(1,2)$-ideal of a semigroup and the characteristic cubic set.

Theorem 3.4. *Let S be a semigroup and let A be non-empty subset of S. Then A is a $(1,2)$-ideal of S if and only if the characteristic cubic set $\chi_A = \langle \overline{\mu}_{\chi_A}, f_{\chi_A}\rangle$ is a cubic $(1,2)$-ideals of S.*

Proof. Suppose that A is a $(1,2)$-ideal of S. Then A is a subsemigroup of S. By Theorem 2.20 we have $\chi_A = \langle \overline{\mu}_{\chi_A}, f_{\chi_A}\rangle$ is a cubic subsemigroup of S.

Let $a, u, v, w \in S$. Then the following cases:

If $u, v, w \in A$ then, $ua(vw) \in A$. Thus, $\overline{\mu}_{\chi_A}(u) = \overline{\mu}_{\chi_A}(v) = \overline{\mu}_{\chi_A}(w) = \overline{\mu}_{\chi_A}(ua(vw)) = \overline{1}$ and $f_{\chi_A}(u) = f_{\chi_A}(v) = f_{\chi_A}(w) = f_{\chi_A}(ua(vw)) = 0$.
Hence

$$\overline{\mu}_{\chi_A}(ua(vw)) \succeq \overline{\mu}_{\chi_A}(u) \curlywedge \overline{\mu}_{\chi_A}(v) \curlywedge \overline{\mu}_{\chi_A}(w)$$

and

$$f_{\chi_A}(ua(vw)) \leq f_{\chi_A}(u) \vee f_{\chi_A}(v) \vee f_{\chi_A}(w).$$

Assume that $u \notin A$ or $v \notin A$ or $w \notin A$. Then,

$$\overline{\mu}_{\chi_A}(ua(vw)) \succeq \overline{\mu}_{\chi_A}(u) \curlywedge \overline{\mu}_{\chi_A}(v) \curlywedge \overline{\mu}_{\chi_A}(w)$$

and

$$f_{\chi_A}(ua(vw)) \leq f_{\chi_A}(u) \vee f_{\chi_A}(v) \vee f_{\chi_A}(w).$$

Therefore $\chi_A = \langle \overline{\mu}_{\chi_A}, f_{\chi_A}\rangle$ is a cubic $(1,2)$-ideal of S.

Conversely, suppose that $\chi_A = \langle \overline{\mu}_{\chi_A}, f_{\chi_A}\rangle$ is a cubic $(1,2)$-ideal of S and let $u, v \in A$. Then by Theorem 2.20 we have A is a subsemigroup of S.

Assume that $(ua(vw)) \notin A$. Then there exist $x, a, y, z \in S$ and $u, v, w \in A$. Thus, $\overline{\mu}_{\chi_A}(ua(vw)) = \overline{0}$, $\overline{\mu}_{\chi_A}(u) = \overline{\mu}_{\chi_A}(v) = \overline{\mu}_{\chi_A}(w) = \overline{1}$ and $f_{\chi_A}(ua(vw)) = 1$, $f_{\chi_A}(u) = f_{\chi_A}(v) = f_{\chi_A}(w) = 0$.

Since $\chi_A = \langle \overline{\mu}_{\chi_A}, f_{\chi_A} \rangle$ is a cubic (1, 2)-ideal of S. we have

$$\overline{\mu}_{\chi_A}(ua(vw)) \succeq \overline{\mu}_{\chi_A}(u) \curlywedge \overline{\mu}_{\chi_A}(v) \curlywedge \overline{\mu}_{\chi_A}(w)$$

and

$$f_{\chi_A}(xa(yz)) \leq f_{\chi_A}(u) \vee f_{\chi_A}(v) \vee f_{\chi_A}(w).$$

Thus, $\overline{\mu}_{\chi_A}(ua(vw)) = \overline{1}$ and $f_{\chi_A}(ua(vw)) = 0$. It is a contradiction.
Hence $(ua(vw)) \in A$ for all $u, v, w \in A$ and $a, u, v, w \in S$.
Therefore A is a (1, 2)-ideal of S. □

In the following theorems, we give a relationship between cubic (1, 2)-ideal of a semigroup and the cubic level set.

Theorem 3.5. *Let S be a semigroup then $\mathcal{A} = \langle \overline{\mu}, f \rangle$ is a cubic (1, 2)-ideal of S if and only if every nonempty cubic level set of $U(\mathcal{A}, [s, t], k)$ is a (1, 2)-ideal of S.*

Proof. Suppose that $\mathcal{A} = \langle \overline{\mu}, f \rangle$ is a cubic (1, 2)-ideal of S. Let $u, v \in U(\mathcal{A}, [s, t], k)$.
By Theorem 2.22 we have $U(\mathcal{A}, [s, t], k)$ is a subsemigroup of S.

Assume that $(ua(vw)) \notin U(\mathcal{A}, [s, t], k)$. Then there exist $u, v, w \in U(\mathcal{A}, [s, t], k)$, $a, u, v, w \in S$ and $[s, t] \in C[0, 1], k \in [0, 1]$. Thus, $\overline{\mu}(u) \succeq [s, t], \overline{\mu}(v) \succeq [s, t]$, $\overline{\mu}(w) \succeq [s, t], \overline{\mu}(ua(vw)) \prec [s, t]$ and $f(u) \leq k, f(v) \leq k, f_A(w) \leq k, f_A(xy) > k$.

Since $\mathcal{A} = \langle \overline{\mu}, f \rangle$ is a cubic (1, 2)-ideal of S we have

$$\overline{\mu}(ua(vw)) \succeq \overline{\mu}(u) \curlywedge \overline{\mu}(v) \curlywedge \overline{\mu}(w) \quad \text{and} \quad f(ua(vw)) \leq f(u) \vee f(v) \vee f(w).$$

Thus $\overline{\mu}(ua(vw)) \succeq [s, t]$ and $f(ua(vw)) \leq k$. It is a contradiction.
Hence $(ua(vw)) \in U(\mathcal{A}, [s, t], k)$ for all $u, v, w, \in U(\mathcal{A}, [s, t], k), a, u, v, w \in S$ and $[s, t] \in C[0, 1], k \in [0, 1]$. Therefore $U(\mathcal{A}, [s, t], k)$ is a cubic level set is a (1, 2)-ideal of S.

Conversely, suppose that $U(\mathcal{A}, [s, t], k)$ is a cubic level set is a (1, 2)-ideal of S.
Let $u, v \in S$. By Theorem 2.22 we have $\mathcal{A} = \langle \overline{\mu}, f \rangle$ is a cubic subsemigroup of S.
Let $a, u, v, w \in S$. Then the following cases:

If $u, v, w \in A$, then $ua(vw) \in A$. Thus, $\overline{\mu}(u) \succeq [s, t], \overline{\mu}(v) \succeq [s, t], \overline{\mu}(w) \succeq [s, t]$, $\overline{\mu}(ua(vw)) \succeq [s, t]$ and $f(u) \leq k, f(v) \leq k, f(w) \leq k, f(ua(vw)) \leq k$. Hence,

$$\overline{\mu}(ua(vw)) \succeq \overline{\mu}(u) \curlywedge \overline{\mu}(v) \curlywedge \overline{\mu}(w) \quad \text{and} \quad f(ua(vw)) \leq f(u) \vee f(v) \vee f(w).$$

Suppose that $u \notin A$ or $v \notin A$ or $w \notin A$. Then

$$\overline{\mu}(ua(vw)) \succeq \overline{\mu}(u) \curlywedge \overline{\mu}(v) \curlywedge \overline{\mu}(w) \quad \text{and} \quad f(ua(vw)) \leq f(u) \vee f(v) \vee f(w).$$

Hence $\mathcal{A} = \langle \overline{\mu}, f \rangle$ is a cubic (1, 2)-ideal of S. □

The following theorem we present relationship of cubic ideal and cubic (1, 2)-ideals.

Theorem 3.6. *Every cubic ideal of a semigroup S is a cubic (1, 2)-ideal of S.*

Proof. Let $\mathcal{A} = \langle \overline{\mu}, f \rangle$ be a cubic ideal of S and let $u, v \in S$. Since $\mathcal{A} = \langle \overline{\mu}, f \rangle$ is a cubic ideal of S we have $\mathcal{A} = \langle \overline{\mu}, f \rangle$ is a cubic right ideal of S. Thus $\overline{\mu}(uv) \succeq \overline{\mu}(u)$ and $f(uv) \leq f(u)$ Hence, $\overline{\mu}(uv) \succeq \overline{\mu}(u) \succeq \overline{\mu}(u) \curlywedge \overline{\mu}(v)$ and $f(uv) \leq f(u) \leq f(u) \vee f(v)$. Therefore $\mathcal{A} = \langle \overline{\mu}, f \rangle$ is a cubic subsemigroup of S.

Let $a, u, v, w \in S$. Since $\mathcal{A} = \langle \overline{\mu}, f \rangle$ is a cubic ideal of S we have $\overline{f}$ is a cubic left ideal of S. Thus, $\overline{\mu}(ua(vw)) = \overline{\mu}((uav)w) \succeq \overline{\mu}(w)$ and $f(ua(vw)) = f((uav)w) \leq f(w)$ so $\overline{\mu}(ua(vw)) \succeq \overline{\mu}(w) \succeq \overline{\mu}(u) \curlywedge \overline{\mu}(v) \curlywedge \overline{\mu}(w)$ and $f(ua(vw)) \leq f(w) \leq f(u) \vee f(v) \vee f(w)$.
Hence $\mathcal{A} = \langle \overline{\mu}, f \rangle$ is a cubic (1, 2)-ideal of S. □

The following theorem we shown that cubic bi-ideal is a cubic (1, 2)-ideals on a semigroup S.

Theorem 3.7. *In semigroup S, cubic bi-ideal is a cubic $(1,2)$-ideal of S.*

Proof. Let $\mathcal{A} = \langle \overline{\mu}, f\rangle$ be a cubic bi-ideal of semigroup S and let $a, u, v, w \in S$. Then,

$$\overline{\mu}(ua(vw)) = \overline{\mu}((uav)w) \succeq \overline{\mu}(uav) \curlywedge \overline{\mu}(w) \succeq (\overline{\mu}(u) \curlywedge \overline{\mu}(v)) \curlywedge \overline{\mu}(w) = \overline{\mu}(u) \curlywedge \overline{\mu}(v) \curlywedge \overline{\mu}(w)$$

and

$$f(ua(vw)) = f((uav)w) \leq f(uav) \vee f(w) \leq (f(u) \vee f(v)) \vee f(w) = f(u) \vee f(v) \vee f(w).$$

Hence $\mathcal{A} = \langle \overline{\mu}, f\rangle$ is a cubic $(1,2)$-ideal of semigroup S. □

Remark 3.8. In example 3.2 we can show that the converse of Theorem 3.7 is not true. Consider $\overline{\mu}(bcb) = [0.4.0.6] \not\succeq [0.6, 0.8] = \overline{f}(b) \curlywedge \overline{f}(b)$ and $f(bcb) = 0.2 \leq 0.5 = f(b) \vee f(b)$. Then $\mathcal{A} = \langle \overline{\mu}, f\rangle$ is not a cubic bi-ideal of S.

The following theorem we present relationship of cubic $(1,2)$-ideal and cubic bi-ideals.

Theorem 3.9. *In regular semigroup S, every cubic $(1,2)$-ideal is a cubic bi-ideal and conversely.*

Proof. Assume that S is a regular semigroup and let $\mathcal{A} = \langle \overline{\mu}, f\rangle$ is a cubic $(1,2)$-ideal of semigroup S and let $x, y, z \in S$. Then there exist an element $a \in S$ such that $x = xax$. Thus, $xy = (xax)y = (xa(xy))$ so, $xy \in (xSx)S \subseteq xSx$ implies that, $xy = xsx$ for some $s \in S$. Consider

$$\begin{aligned} \overline{\mu}(xyz) &= \overline{\mu}((xsx)z) = \overline{\mu}(xs(xz)) \\ &\succeq \overline{\mu}(x) \curlywedge \overline{\mu}(x) \curlywedge \overline{\mu}(z) = \overline{\mu}(x) \curlywedge \overline{\mu}(z) \end{aligned}$$

and

$$\begin{aligned} f(xyz) &= f((xsx)z) = f(xs(xz)) \\ &\leq f(x) \vee f(x) \vee f(z) = f(x) \vee f(z). \end{aligned}$$

Hence $\mathcal{A} = \langle \overline{\mu}, f\rangle$ is a cubic bi-ideal of semigroup S. □

The following lemma will be used to prove in Theorem 3.11.

Lemma 3.10. [2] *In regular semigroup S, every cubic bi-ideal of a semigroup S is cubic generalized bi-ideals and conversely.*

The following theorem we present relationship of cubic generalized bi-ideals and cubic $(1,2)$-ideals.

Theorem 3.11. *In regular semigroup S, every cubic $(1,2)$-ideal is a cubic generalized bi-ideals and conversely.*

Proof. Suppose that $\mathcal{A} = \langle \overline{\mu}, f\rangle$ is a cubic $(1,2)$-ideal. Then by Theorem 3.9 and Lemma 3.10, $\mathcal{A} = \langle \overline{\mu}, f\rangle$ is a cubic generalized bi-ideal.

Conversely assume that $\mathcal{A} = \langle \overline{\mu}, f\rangle$ is a cubic generalized bi-ideal of S and let $u, v \in S$. Then there exists $k \in S$ such that $v = vkv$. Thus, $\overline{\mu}(uv) = \overline{\mu}(u(vkv)) = \overline{\mu}(u(vk)v) \succeq \overline{\mu}(u) \curlywedge \overline{\mu}(v)$ and $f(uv) = f(u(vkv)) = f(u(vk)v) \leq f(u) \vee f(v)$. Hence $\mathcal{A} = \langle \overline{\mu}, f\rangle$ is a cubic subsemigroup of S. Let $a, u, v, w \in S$. It follows form Theorem 3.7. Thus $\mathcal{A} = \langle \overline{\mu}, f\rangle$ is a cubic $(1,2)$-ideal of S. □

4. Homomorphic inverse image operation to get cubic set

In this section, we study some properties of homomorphic and inverse image of cubic set.

Definition 4.1. [4] Let $\mathcal{C}(X)$ be the family of cubic set in a set X. Let X and Y be sets. A mapping $h : X \to Y$ induces two mapping $\mathcal{C}_h : \mathcal{C}(X) \to \mathcal{C}(Y)$, $\mathcal{A} \mapsto \mathcal{C}_h(\mathcal{A})$ and $\mathcal{C}_h^{-1} : \mathcal{C}(Y) \to \mathcal{C}(X)$, $\mathcal{B} \mapsto \mathcal{C}_h^{-1}(\mathcal{B})$ where $\mathcal{C}_h(\mathcal{A})$ is given by

$$\mathcal{C}_h(\overline{\mu})(y) = \begin{cases} \bigsqcup\limits_{y=h(x)} \overline{\mu}(x), & \text{if} \quad h^{-1}(y) \neq 0, \\ \overline{0}, & \text{otherwise} \end{cases}$$

$$\mathcal{C}_h(f)(y) = \begin{cases} \bigwedge\limits_{y=h(x)} f(x), & \text{if} \quad h^{-1}(y) \neq 0, \\ 1, & \text{otherwise} \end{cases}$$

for all $y \in Y$. The *inverse image* $\mathcal{C}_h^{-1}(\mathcal{B})$ is defined by $\mathcal{C}_h^{-1}(\overline{\lambda})(x) = \overline{\lambda}(h(x))$ and $\mathcal{C}_h^{-1}(g)(x) = g(h(x))$ for all $x \in X$. Then the mapping $\mathcal{C}_h$ (resp. $\mathcal{C}_h^{-1}$) is called a cubic transformation (inverse cubic transformation) induced by h: A cubic set $\mathcal{A} = \langle \overline{\mu}, f \rangle$ in X has the *cubic property* if for any subset T of X there exists $x_0 \in T$ such that $\overline{\mu}(x_0) = \bigsqcup\limits_{x \in T} \overline{\mu}(x)$ and $f(x_0) = \bigwedge\limits_{x \in T} f(x)$.

Theorem 4.2. *Let $h : X \to Y$ be a homomorphism of semigroups and let $\mathcal{C}_h : \mathcal{C}(X) \to \mathcal{C}(Y)$ and be the cubic transformation cubic transformation induced by h. If $\mathcal{A} = \langle \overline{\mu}, f \rangle \in \mathcal{C}(X)$ is a cubic $(1,2)$-ideal of X which has the cubic property, then $\mathcal{C}_h(\mathcal{A})$ is a cubic $(1,2)$-ideal of Y.*

Proof. Assume that $\mathcal{A} = \langle \overline{\mu}, f \rangle \in \mathcal{C}(X)$ is a cubic $(1,2)$-ideal of X and let $h(x), h(y) \in h(X)$ $x_0 \in h^{-1}(h(x))$, $y_0 \in h^{-1}(h(y))$ be such that $\overline{\mu}(x_0) = \bigsqcup\limits_{z \in h^{-1}(h(x))} \overline{\mu}(a)$, $f(x_0) = \bigwedge\limits_{z \in h^{-1}(h(x))} f(a)$ and $\overline{\mu}(y_0) = \bigsqcup\limits_{z \in h^{-1}(h(y))} \overline{\mu}(a)$, $f(y_0) = \bigwedge\limits_{z \in h^{-1}} f(b)$ respectively. Then

$$\begin{aligned} \mathcal{C}_h(\overline{\mu})(h(x)h(y)) &= \bigsqcup_{z \in h^{-1}(h(x)h(y))} \overline{\mu}(z) \succeq \overline{\mu}(x_0 y_0) \succeq \overline{\mu}(x_0) \curlywedge \overline{\mu}(y_0) \\ &= \bigsqcup_{a \in h^{-1}(h(x))} \overline{\mu}(a) \curlywedge \bigsqcup_{a \in h^{-1}(h(y))} \overline{\mu}(b) = \mathcal{C}_h(\overline{\mu}(a))(h(x)) \curlywedge \mathcal{C}_h(\overline{\mu}(a))(h(y)). \end{aligned}$$

and

$$\begin{aligned} \mathcal{C}_h(f)(h(x)h(y)) &= \bigwedge_{z \in h^{-1}(h(x)h(y))} f(z) \succeq f(x_0 y_0) \leq f(x_0) \vee f(y_0) \\ &= \bigwedge_{a \in h^{-1}(h(x))} f(a) \vee \bigwedge_{a \in h^{-1}(h(y))} f(b) = \mathcal{C}_h(f(a))(h(x)) \curlywedge \mathcal{C}_h(f(a))(h(y)). \end{aligned}$$

Thus $\mathcal{C}_h(\overline{\mu})(h(x)h(y)) \succeq \mathcal{C}_h(\overline{\mu}(a))(h(x)) \curlywedge \mathcal{C}_h(\overline{\mu}(a))(h(y))$ and $\mathcal{C}_h(f)(h(x)h(y)) \leq \mathcal{C}_h(f(a))(h(x)) \curlywedge \mathcal{C}_h(f(a))(h(y))$.
Hence $\mathcal{C}_h(\mathcal{A})$ is a cubic subsemigroup of Y.

Similarly, let $h(x), h(y), h(w), h(z) \in h(x)$ and let $x_0 \in h^{-1}(h(x))$, $y_0 \in h^{-1}(h(y))$, $w_0 \in h^{-1}(h(w))$, $z_0 \in h^{-1}(h(z))$ be such that $\overline{\mu}(x_0) = \bigsqcup\limits_{a \in h^{-1}(h(x))} \overline{\mu}(a)$, $f(x_0) = \bigwedge\limits_{a \in h^{-1}(h(x))} f(a)$, $\overline{\mu}(y_0) = \bigsqcup\limits_{b \in h^{-1}(h(y))} \overline{\mu}(b)$, $f(y_0) = \bigwedge\limits_{b \in h^{-1}(h(y))} f(b)$, $\overline{\mu}(w_0) = \bigsqcup\limits_{c \in h^{-1}(h(z))} \overline{\mu}(c)$, $f(w_0) = \bigwedge\limits_{c \in h^{-1}(h(w))} f(c)$ and $\overline{\mu}(z_0) =$

$\bigsqcup\limits_{d\in h^{-1}h(z))}\overline{\mu}(d), f(z_0)=\bigwedge\limits_{d\in h^{-1}(h(x))} f(d)$ respectively. Then

$$\begin{array}{rcl}\mathcal{C}_h(\overline{\mu})(h(x)h(y)(h(w)h(z))) &=& \bigsqcup\limits_{k\in h^{-1}(h(x)h(y)(h(w)h(z)))}\overline{\mu}(k)\succeq\overline{\mu}(x_0y_0(w_0z_0))\\ &\succeq& \overline{\mu}(x_0)\curlywedge\overline{\mu}(w_0)\curlywedge\overline{\mu}(z_0)\\ &=& \bigsqcup\limits_{a\in h^{-1}(h(x))}\overline{\mu}(a)\curlywedge\bigsqcup\limits_{c\in h^{-1}(h(w))}\overline{\mu}(c)\curlywedge\bigsqcup\limits_{d\in h^{-1}(h(z))}\overline{\mu}(d)\\ &=& \mathcal{C}_h(\overline{\mu}(a))(h(x))\curlywedge\mathcal{C}_h(\overline{\mu}(a))(h(w))\curlywedge\mathcal{C}_h(\overline{\mu}(a))(h(z)).\end{array}$$

and

$$\begin{array}{rcl}\mathcal{C}_h(f)(h(x)h(y)(h(w)h(z))) &=& \bigwedge\limits_{k\in h^{-1}(h(x)h(y)(h(w)h(z)))}\overline{f}(k)\succeq f(x_0y_0(w_0z_0))\\ &\leq& \overline{\mu}(x_0)\vee\overline{\mu}(w_0)\vee\overline{\mu}(z_0)\\ &=& \bigwedge\limits_{a\in h^{-1}(h(x))}f(a)\vee\bigwedge\limits_{c\in h^{-1}(h(w))}f(c)\vee\bigwedge\limits_{d\in h^{-1}(h(z))}f(d)\\ &=& \mathcal{C}_h(\overline{\mu}(a))(h(x))\vee\mathcal{C}_h(\overline{\mu}(a))(h(w))\vee\mathcal{C}_h(\overline{\mu}(a))(h(z)).\end{array}$$

Thus $\mathcal{C}_h(\overline{\mu})(h(x)h(y)(h(w)h(z)))\succeq\mathcal{C}_h(\overline{\mu}(a))(h(x))\curlywedge\mathcal{C}_h(\overline{\mu}(a))(h(w))\curlywedge\mathcal{C}_h(\overline{\mu}(a))(h(z))$ and $\mathcal{C}_h(f)(h(x)h(y)(h(w)h(z)))\leq\mathcal{C}_h(\overline{\mu}(a))(h(x))\vee\mathcal{C}_h(\overline{\mu}(a))(h(w))\vee\mathcal{C}_h(\overline{\mu}(a))(h(z))$.
Hence $\mathcal{C}_h(\mathcal{A})$ is a cubic $(1,2)$-ideal of Y. □

Theorem 4.3. *Let $h:X\to Y$ be a homomorphism of semigroups and let $\mathcal{C}_h^{-1}:\mathcal{C}(Y)\to\mathcal{C}(X)$ be the cubic inverse cubic transformation, induced by h. If $\mathcal{A}=\langle\overline{\mu},f\rangle\in\mathcal{C}(Y)$ is a cubic $(1,2)$-ideal of Y then $\mathcal{C}_h^{-1}(\mathcal{A})$ is a cubic $(1,2)$-ideal of X.*

Proof. Suppose that $\mathcal{A}=\langle\overline{\mu},f\rangle\in\mathcal{C}(Y)$ is a cubic $(1,2)$-ideal of Y and let $x,y\in X$. Then

$$\begin{array}{rcl}\mathcal{C}_h^{-1}(\overline{\mu}(xy)) &=& \overline{\mu}(h(xy))=\overline{\mu}(h(x)h(y))\succeq\overline{\mu}(h(x))\curlywedge\overline{\mu}(h(y))\\ &=& \mathcal{C}_h^{-1}(\overline{\mu}(x))\curlywedge\mathcal{C}_h^{-1}(\overline{\mu}(y)).\end{array}$$

and

$$\begin{array}{rcl}\mathcal{C}_h^{-1}(f(xy)) &=& f(h(xy))=f(h(x)h(y))\leq f(h(x))\vee f(h(y))\\ &=& \mathcal{C}_h^{-1}(f(x))\vee\mathcal{C}_h^{-1}(f(y)).\end{array}$$

Thus $\mathcal{C}_h^{-1}(\overline{\mu}(xy))\succeq\mathcal{C}_h^{-1}(\overline{\mu}(x))\curlywedge\mathcal{C}_h^{-1}(\overline{\mu}(y))$ and $\mathcal{C}_h^{-1}(f(xy))\leq\mathcal{C}_h^{-1}(f(x))\vee\mathcal{C}_h^{-1}(f(y))$.
Hence $\mathcal{C}_h^{-1}(\mathcal{A})$ is a cubic subsemigroup of S.

Let $x,y,w,z\in X$. Then

$$\begin{array}{rcl}\mathcal{C}_h^{-1}(\overline{\mu}(xy(wz)) &=& \overline{\mu}(h(xy(wz))=\overline{\mu}(h(x)h(y)h(w)h(z))\\ &\succeq& \overline{\mu}(h(x))\curlywedge\overline{\mu}(h(w))\curlywedge\overline{\mu}(h(z))\\ &=& \mathcal{C}_h^{-1}(\overline{\mu}(x))\curlywedge\mathcal{C}_h^{-1}(\overline{\mu}(w))\curlywedge\mathcal{C}_h^{-1}(\overline{\mu}(z))\end{array}$$

and

$$\begin{array}{rcl}\mathcal{C}_h^{-1}(f(xy(wz))) &=& f(h(xy))=f(h(x)h(y)h(w)h(z))\\ &\leq& f(h(x))\vee f(h(w))\vee f(h(z))\\ &=& \mathcal{C}_h^{-1}(f(x))\vee\mathcal{C}_h^{-1}(f(w))\vee\mathcal{C}_h^{-1}(f(z)).\end{array}$$

Thus $\mathcal{C}_h^{-1}(\overline{\mu}(xy(wz))\succeq\mathcal{C}_h^{-1}(\overline{\mu}(x))\curlywedge\mathcal{C}_h^{-1}(\overline{\mu}(w))\curlywedge\mathcal{C}_h^{-1}(\overline{\mu}(z))$ and
$\mathcal{C}_h^{-1}(f(xy(wz))\leq\mathcal{C}_h^{-1}(f(x))\vee\mathcal{C}_h^{-1}(f(w))\vee\mathcal{C}_h^{-1}(f(z))$.
Therefore $\mathcal{C}_h^{-1}$ is a cubic $(1,2)$-ideal of X. □

5. Conclusion

In this paper, we give concept of cubic $(1, 2)$-ideals and establish basic properties of cubic $(1, 2)$-ideals on semigroups. Finally we discussed the images or inverse images of a cubic $(1, 2)$-ideal of a semigroup. In future, we will focus characterizations of some semigroups by the properties of cubic subsemigroups of a semigroup.

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