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SOFT SOMEWHAT CONTINUOUS AND SOFT SOMEWHAT OPEN FUNCTIONS

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Abstract. In this paper, we define a soft somewhat open set using the soft interior operator. We study main properties the class of soft somewhat open sets that is contained in the class soft somewhere dense sets. Then, we introduce the classes of soft somewhat continuous and soft somewhat open functions and soft somewhat homeomorphisms. Moreover, we study properties and characterizations of soft somewhat continuous and soft somewhat open functions. At last, we discuss topological invariants for soft somewhat homeomorphisms. Multiple examples are offered to clarify some invalid results.

Keywords: soft semicontinuity, soft β-continuity, soft somewhat continuity, soft somewhat open, soft somewhere dense continuity.

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1. Introduction

In 1999, Molodtsov [21] suggested a different approach for dealing with problems of incomplete information under the name of soft set theory. This notion has been utilized in many directions, like: smoothness of function, Riemann integration, theory of measurement, probability theory, game theory and so on. The core concept of the theory of soft set is the nature of sets of parameters that provides a general framework for modeling uncertain data. This essentially contributes to the development of soft set theory during a short period of time. Maji et al. [20] studied a (detailed) theoretical structure of soft set theory. In particular, they established some operators and operations between soft sets. Then, some mathematicians reformulated the operators and operations between soft sets given in Maji et al.'s work as well as proposed different types of them; to see the recent contributions concerning soft operators and operations, we refer the reader to [7].

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In 2011, the concept of soft (general) topology was defined by Shabir and Naz [24] and Çağman et al. [10] independently. In 2013, Nazmul and Samanta [22] defined soft continuity of functions. Then various generalizations of soft continuity and soft openness of functions appeared in the literature. For instance, soft α-continuous functions [1], soft semicontinuous functions [19], soft β-continuous functions [26], soft somewhere dense continuous [5], soft α-open functions [1], soft semi-open functions [19], soft β-open functions [26], soft somewhere dense open [5], and so on. Different kinds of belong and nonbelong relations were studied in [24, 13]. These relations led to the variety and abundance of the forms of the concepts and notions on soft topology.

After this brief introduction, we recollect some preliminaries concepts in Section 2. Then, we devote Section 3 to introduce the concept of soft somewhat open sets and study its relationships with some generalizations of soft open sets. The goals of Section 4 and Section 5 are to investigate soft somewhat continuous functions and soft somewhat open functions which are respectively weaker than soft semicontinuous and soft semi-open functions but stronger than soft somewhere dense continuous and soft somewhere dense open functions. In Section (6), we make a conclusion and propose some further works.

2. Preliminaries

This section presents some basic definitions and notations that will be used in the sequel. Henceforth, we mean by X an initial universe, E a set of parameters and $\mathcal{P}(X)$ the power set of X.

Definition 2.1. [21] *A pair $(F,E) = \{(e, F(e)) : e \in E\}$ is said to be a soft set over X, where $F : E \to \mathcal{P}(X)$ is a (crisp) map. We write F_E in place of the soft set (F,E).*

The class of all soft sets on X is symbolized by $SS_E(X)$ (or simply $SS(X)$). If $A \subseteq E$, then it will be symbolized by $SS_A(X)$.

Definition 2.2. [3, 22] *A soft set F_E over X is called:*

(i) a soft element if $F(e) = \{x\}$ for all $e \in E$, where $x \in X$. It is denoted by $\{x\}_E$ (or shortly x).

(ii) a soft point if there are $e \in E$ and $x \in X$ such that $F(e) = \{x\}$ and $F(e') = \emptyset$ for each $e' \neq e$. It is denoted by P^x_e. An expression $P^x_e \in F_E$ means that $x \in F(e)$.

Definition 2.3. [2] *The complement of F_E is a soft set $X_E \setminus F_E$ (or simply F^c_E), where $F^c : E \to \mathcal{P}(X)$ is given by $F^c(e) = X \setminus F(e)$ for all $e \in E$.*

Definition 2.4. [21] *A soft subset F_E over X is called*

(i) null if $F(e) = \emptyset$ for any $e \in E$.

(ii) absolute if $F(e) = X$ for any $e \in E$.

The null and absolute soft sets are respectively symbolized by Φ_E and X_E.
Clearly, $X^c_E = \Phi_E$ and $\Phi^c_E = X_E$.

Definition 2.5. [20] *Let $A, B \subseteq E$. It is said that G_A is a soft subset of H_B (written by $G_A \sqsubseteq H_B$) if $A \subseteq B$ and $F(e) \subseteq G(e)$ for any $e \in A$. We call G_A soft equals to H_B if $G_A \sqsubseteq H_B$ and $H_B \sqsubseteq G_A$.*

The definitions of soft union and soft intersection of two soft sets with respect to arbitrary subsets of E was given by Maji et al. [20]. But it turns out that these definitions are misleading and ambiguous as reported by Ali et al. [2]. Therefore, we follow the definitions given by Ali et al. [2] and M. Terepeta [25].

Definition 2.6. *Let $\{F_E^\alpha : \alpha \in \Lambda\}$ be a collection of soft sets over X, where Λ is any indexed set.*

(1) The intersection of F_E^α, for $\alpha \in \Lambda$, is a soft set G_E such that $G(e) = \bigcap_{\alpha\in\Lambda} F^\alpha(e)$ for each $e \in E$ and denoted by $G_E = \bigsqcap_{\alpha\in\Lambda} F_E^\alpha$.
(2) The union of F_E^α, for $\alpha \in \Lambda$, is a soft set G_E such that $G(e) = \bigcup_{\alpha\in\Lambda} F^\alpha(e)$ for each $e \in E$ and denoted by $G_E = \bigsqcup_{\alpha\in\Lambda} F_E^\alpha$.

Definition 2.7. [24] *A subfamily $\mathcal{T}$ of $SS_E(X)$ is called a soft topology on X if*

(c1) Φ_E and X_E belong to $\mathcal{T}$,
(c2) finite intersection of sets from $\mathcal{T}$ belongs to $\mathcal{T}$, and
(c3) any union of sets from $\mathcal{T}$ belongs to $\mathcal{T}$.

Terminologically, we call $(X, \mathcal{T}, E)$ a soft topological space on X. The elements of $\mathcal{T}$ are called soft open sets, and their complements are called soft closed sets.

Henceforward, $(X, \mathcal{T}, E)$ means a soft topological space.

Definition 2.8. [24] *Let Y_E be a non-null soft subset of $(X, \mathcal{T}, E)$. Then $\mathcal{T}_Y := \{G_E \bigsqcap Y_E : G_E \in \mathcal{T}\}$ is called a soft relative topology on Y and $(Y, \mathcal{T}_Y, E)$ is a soft subspace of $(X, \mathcal{T}, E)$.*

Definition 2.9. [24] *Let F_E be a soft subset of $(X, \mathcal{T}, E)$. The soft interior of F_E is the largest soft open set contained in F_E and denoted by $\mathrm{Int}_X(F_E)$ (or shortly $\mathrm{Int}(F_E)$). The soft closure of F_E is the smallest soft closed set which contains F_E and denoted by $\mathrm{Cl}_X(F_E)$ (or simply $\mathrm{Cl}(F_E)$).*

Lemma 2.1. [15] *For a soft subset G_E of $(X, \mathcal{T}, E)$, $\mathrm{Int}(G_E^c) = (\mathrm{Cl}(G_E))^c$ and $\mathrm{Cl}(G_E^c) = (\mathrm{Int}(G_E))^c$.*

Definition 2.10. *A soft subset G_E of $(X, \mathcal{T}, E)$ is called*

(i) soft dense if $\mathrm{Cl}(G_E) = X_E$,
(ii) soft co-dense if $\mathrm{Int}(G_E) = \Phi_E$
(iii) soft semiopen [11] *if $G_E \sqsubseteq \mathrm{Cl}(\mathrm{Int}(G_E))$,*
(iv) soft β-open [26] *if $G_E \sqsubseteq \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(G_E)))$,*
(v) soft somewhere dense [4] *if $\mathrm{Int}(\mathrm{Cl}(G_E)) \neq \Phi_E$ (For a better connection between these soft sets, we force Φ_E to be soft somewhere dense).*

We call F_E a countable soft set if $F(e)$ is countable for each $e \in E$.

Definition 2.11. *A soft topological space $(X, \mathcal{T}, E)$ is called*

(i) soft separable [23] *if it has a countable soft dense subset.*
(ii) soft hyperconnected [16] *if any pair of non-null soft open subsets intersect.*
(iii) soft connected [18] *if it cannot be written as a union of two disjoint soft open sets.*
(iv) soft compact [8] *if every cover of X by soft open sets has a finite subcover. It is soft locally compact if each soft point has a soft compact neighborhood.*
(v) soft metrizable [12] *if $\mathcal{T}$ is induced by soft metric space.*

Definition 2.12. [24, 9] *A soft topological space $(X, \mathcal{T}, E)$ is called*

(i) soft T_0 if for each $P_e^x, P_e^y \in X$ with $P_e^x \neq P_e^y$, there exist soft open sets G_E, H_E such that $P_e^x \in G_E$, $P_e^y \notin H_E$ or $P_e^y \in G_E$, $P_e^x \notin H_E$.
(ii) soft T_1 if for each $P_e^x, P_e^y \in X$ with $P_e^x \neq P_e^y$, there exist soft open sets G_E, H_E such that $P_e^x \in G_E$, $P_e^y \notin H_E$ and $P_e^y \in G_E$, $P_e^x \notin H_E$,
(iii) soft T_2 (soft Hausdorff) if for each $P_e^x, P_e^y \in X$ with $P_e^x \neq P_e^y$, there exist soft open sets G_E, H_E containing P_e^x, P_e^y respectively such that $G_E \bigsqcap H_E = \Phi_E$.

Definition 2.13. *Let $(X, \mathcal{T}, E)$ and $(Y, \mathcal{S}, E')$ be soft topological spaces. A soft function $f : (X, \mathcal{T}, E) \to (Y, \mathcal{S}, E')$ is called*

(i) soft continuous [22] *(resp., soft semicontinuous* [19]*, soft SD-continuous* [5]*, soft β-continuous* [26]*) if the inverse image of each soft open subset of $(Y, \mathcal{S}, E')$ is a soft open (resp., soft semiopen, soft somewhere dense, β-open) subset of $(X, \mathcal{T}, E)$.*

(ii) soft open [22] *(resp., soft semiopen* [19]*, soft SD-open* [5]*, soft β-open* [26]*) if the image of each soft open subset of $(X, \mathcal{T}, E)$ is a soft open (resp., soft semiopen, soft somewhere dense, β-open) subset of $(Y, \mathcal{S}, E')$.*

(iii) soft homeomorphism [22] *if it is one to one soft open and soft continuous from $(X, \mathcal{T}, E)$ onto $(Y, \mathcal{S}, E')$.*

For the definition of soft functions between collections of all soft sets, we refer the reader to [17]. Henceforward, by the word "function" we mean "soft function".

3. Soft Somewhat Open Sets

In this section, we introduce the concept of soft somewhat open sets and establish main properties. With the help of examples, we show the relationships between soft somewhat open sets and some generalizations of soft open sets such that soft semiopen and soft somewhere dense sets.

Definition 3.1. *A subset G_E of a soft topological space $(X, \mathcal{T}, E)$ is said to be soft somewhat open (briefly soft sw-open) if either G_E is null or $\text{Int}(G_E) \neq \Phi_E$.*

The complement of each soft sw-open set is called soft sw-closed. That is, a set F_E is soft sw-closed if $\text{Cl}(F_E) \neq X_E$ or $F_E = X_E$.

Remark 3.1. *Let $(X, \mathcal{T}, E)$ be a soft topological space.*

(a) A non-null set G_E over X is soft sw-open iff there is a soft open set U_E such that $\Phi_E \neq U_E \sqsubseteq G_E$.

(b) A proper set H_E over X is soft sw-closed iff there is a soft closed set F_E such that $H_E \sqsubseteq F_E \neq X_E$.

Proposition 3.1. *(a) Every superset of a soft sw-open set is soft sw-open.*

(b) Every subset of a soft sw-closed set is soft sw-closed.

Proof. Straightforward. □

Proposition 3.2. *A non-null soft set is soft sw-open iff it is a soft neighbourhood of a soft point.*

Proof. Let G_E be a non-null soft sw-open set. Then there is a soft open set U_E such that $\Phi_E \neq U_E \sqsubseteq G_E$. Therefore, G_E is a soft neighbourhood of all soft points in U_E. Conversely, let G_E be a soft neighbourhood of a soft point P_e^x. Then there is a soft open set U_E such that $P_e^x \in U_E \sqsubseteq G_E$. Hence, we obtain $\text{Int}(G_E) \neq \Phi_E$, as required. □

Proposition 3.3. *Any union of soft sw-open sets is soft sw-open.*

Proof. Let $\{G_E^\alpha : \alpha \in \Lambda\}$ be any collection of soft sw-open subsets of a soft topological space $(X, \mathcal{T}, E)$. Now

$$\text{Int}(\bigsqcup_{\alpha\in\Lambda} G_E^\alpha) \sqsupseteq \bigsqcup_{\alpha\in\Lambda} \text{Int}(G_E^\alpha)) \neq \Phi_E.$$

Thus $\bigsqcup_{\alpha\in\Lambda} G_E^\alpha$ is soft sw-open. □

Corollary 3.1. *Any intersection of soft sw-closed sets is soft sw-closed.*

The intersection of two soft sw-open sets need not be soft sw-open, as showing in the next example:

Example 3.1. *Let $\mathbb{R}$ be the set of real numbers and $E = \{e_1, e_2\}$ be a set of parameters. Let $\mathcal{T}$ be the soft topology on $\mathbb{R}$ generated by $\{(e_i, B(e_i)) : B(e_i) = (a_i, b_i); a_i, b_i \in \mathbb{R}; a_i \leq b_i; i = 1, 2\}$. Take soft sw-open sets $G_E = \{(e_1, [0, 1]), (e_2, [0, 1])\}$ and $H_E = \{(e_1, [1, 2]), (e_2, [1, 2])\}$ over $\mathbb{R}$, then $G_E \bigsqcap H_E \neq \Phi_E$ but $\text{Int}(G_E \bigsqcap H_E) = \Phi_E$.*

Remark 3.2. *The intersection of a soft sw-open set with another soft open, soft closed or soft dense set need not be a soft sw-open set, and counterexamples showing this are easy to find.*

The result below explains the conditions under which the intersection of soft sw-open and soft open sets is a soft sw-open set.

Proposition 3.4. *The intersection of two soft sw-open sets in a soft hyperconnected space $(X, \mathcal{T}, E)$ is a soft sw-open set.*

Proof. If one of the two soft sw-open sets is null, the proof is trivial. Suppose G_E and H_E are two soft sw-open sets. Then $\text{Int}(G_E) = U_E \neq \Phi_E$ and $\text{Int}(H_E) = V_E \neq \Phi_E$. Now, $\text{Int}(G_E \bigsqcap H_E) = \text{Int}(G_E) \bigsqcap \text{Int}(H_E) = U_E \bigsqcap V_E$. Since $(X, \mathcal{T}, E)$ is soft hyperconnected, $U_E \bigsqcap V_E \neq \Phi_E$. Thus $\text{Int}(G_E \bigsqcap H_E) \neq \Phi_E$; hence, we obtain the desired result. □

Corollary 3.2. *The intersection of soft sw-open and soft open sets in a soft hyperconnected space $(X, \mathcal{T}, E)$ is a soft sw-open set.*

Corollary 3.3. *The family of soft sw-open subsets of a soft hyperconnected space $(X, \mathcal{T}, E)$ forms a soft topology.*

Lemma 3.1. *Let G_E, D_E be subsets of $(X, \mathcal{T}, E)$. If G_E is sw-open and D_E is soft dense over X, then $G_E \bigsqcap D_E$ is soft sw-open over D.*

Proof. Since $\text{Int}_D(G_E \bigsqcap D_E) = \text{Int}_D(G_E) \bigsqcap D_E \sqsupseteq \text{Int}(G_E) \bigsqcap D_E \neq \Phi_E$ (as D_E is soft dense), so $G_E \bigsqcap D_E$ is soft sw-open over D. □

Lemma 3.2. *Let $(Y, \mathcal{T}_Y, E)$ be a soft open subspace of $(X, \mathcal{T}, E)$ and let $G_E \sqsubseteq Y_E$. Then G_E is soft sw-open over Y iff it is soft sw-open over X.*

Proof. Assume G_E is soft sw-open over Y. There exists a soft open set U_E over Y such that $\Phi_E \neq U_E \sqsubseteq G_E$. Since Y_E is soft open over X, so U_E is soft open over X. Hence G_E is soft sw-open over X.

Conversely, assume G_E is soft sw-open over X. That is $\text{Int}_X(G_E) \neq \Phi_E$. By Theorem 2 in [24], $\text{Int}_X(G_E) \sqsubseteq \text{Int}_Y(G_E)$, therefore G_E is soft sw-open over Y. □

The following example shows that the above result is not true if Y_E is soft dense in $\widetilde{X}$.

Example 3.2. *Let $X = \{w, x, y, z\}$ and $E = \{e_1, e_2\}$. Set $\mathcal{T} = \{\Phi_E, F_E, G_E, H_E, X_E\}$, where*

$$\begin{aligned}
F_E &= \{(e_1, \{x, z\}), (e_2, \{w, x\})\} \\
G_E &= \{(e_1, X), (e_2, \{y, z\})\} \\
H_E &= \{(e_1, \{x, z\}), (e_2, \emptyset)\}.
\end{aligned}$$

Take $Y = \{x, y\}$, so $\mathcal{T}_Y = \{\Phi_E, I_E, J_E, K_E, Y_E\}$, where

$$\begin{aligned}
I_E &= \{(e_1, \{x\}), (e_2, \{x\})\} \\
J_E &= \{(e_1, Y), (e_2, \{y\})\} \\
K_E &= \{(e_1, \{x\}), (e_2, \emptyset)\} \\
Y_E &= \{(e_1, \{x, y\}), (e_2, \{x, y\})\}.
\end{aligned}$$

The set I_E is soft sw-open over the soft dense set Y but not soft sw-open over X.

Lemma 3.3. *Let G_E be a subset of $(X, \mathcal{T}, E)$. Then G_E is soft semiopen iff* $\mathrm{Cl}(G_E) = \mathrm{Cl}(\mathrm{Int}(G_E))$.

Proof. If G_E is soft semiopen, then $G_E \sqsubseteq \mathrm{Cl}(\mathrm{Int}(G_E))$ and so $\mathrm{Cl}(G_E) \sqsubseteq \mathrm{Cl}(\mathrm{Int}(G_E))$. For other side of inclusion, we always have $\mathrm{Int}(G_E) \sqsubseteq G_E$. Therefore $\mathrm{Cl}(\mathrm{Int}(G_E)) \sqsubseteq \mathrm{Cl}(G_E)$. Thus $\mathrm{Cl}(G_E) = \mathrm{Cl}(\mathrm{Int}(G_E))$.

Conversely, assume that $\mathrm{Cl}(G_E) = \mathrm{Cl}(\mathrm{Int}(G_E))$, but $G_E \sqsubseteq \mathrm{Cl}(G_E)$ always, so $G_E \sqsubseteq \mathrm{Cl}(\mathrm{Int}(G_E))$. Hence G_E is soft semiopen. □

Lemma 3.4. *Let G_E be a non-null subset of $(X, \mathcal{T}, E)$. If G_E is soft semiopen, then* $\mathrm{Int}(G_E) \neq \Phi_E$.

Proof. Suppose otherwise that if G_E is a non-null soft semiopen set such that $\mathrm{Int}(G_E) = \Phi_E$, by Lemma 3.3, $\mathrm{Cl}(G_E) = \Phi_E$ which implies that $G_E = \Phi_E$. Contradiction! □

Remark 3.3. *Since* $\mathrm{Int}(G_E) \sqsubseteq \mathrm{Int}(\mathrm{Cl}(G_E))$ *for each soft set G_E in a soft topological space $(X, \mathcal{T}, E)$, so each soft sw-open set is soft somewhere dense.*

Next, we put Remark 3.3, Lemma 3.4 and Proposition 2.8 in [4] into the following diagram:

$$\begin{array}{ccc} \text{soft semiopen set} & \longrightarrow & \text{soft } \beta\text{-open set} \\ \downarrow & & \downarrow \\ \text{soft } sw\text{-open set} & \longrightarrow & \text{soft somewhere dense set} \end{array}$$

Diagram I: Relationship between some generalizations of soft open sets

In general, none of these implications can be replaced by equivalence as shown below:

Example 3.3. *Consider the soft topology defined in Example 3.1. The soft set of rational numbers $\mathbb{Q}_E$ over $\mathbb{R}$ is soft β-open (consequently, is soft somewhere dense) but not soft sw-open (consequently, is not soft semi-open). On the other hand, the set* $\{(e_1, (0, 1)), (e_2, \{2\})\}$ *is clearly soft sw-open but not soft semiopen. The soft set F_E given in Example 2.9 in* [4] *is soft somewhere dense but not soft β-open.*

Lemma 3.5. [4, Lemma 2.24] *Let G_E be a subset of $(X, \mathcal{T}, E)$. Then* $\mathrm{Cl}(G_E) \sqcap U_E \sqsubseteq \mathrm{Cl}(G_E \sqcap U_E)$ *for each soft open set U_E over X.*

Lemma 3.6. *Let G_E, H_E be subsets of $(X, \mathcal{T}, E)$. If G_E is soft open and H_E is soft semiopen, then $G_E \sqcap H_E$ is soft semiopen over X.*

Proof. Assume H_E is soft semiopen and G_E is soft open. By Theorem 3.1 in [11], there exists a soft open set U_E over X such that $U_E \sqsubseteq H_E \sqsubseteq \mathrm{Cl}(U_E)$. Now $U_E \sqcap G_E \sqsubseteq H_E \sqcap G_E \sqsubseteq \mathrm{Cl}(U_E) \sqcap G_E$. By Lemma 3.5, $U_E \sqcap G_E \sqsubseteq H_E \sqcap G_E \sqsubseteq \mathrm{Cl}(U_E \sqcap G_E)$ and since $U_E \sqcap G_E$ is soft open, therefore by Theorem 3.1 in [11], $H_E \sqcap G_E$ is soft semiopen over X. □

Lemma 3.7. *Let G_E, H_E be subsets of $(X, \mathcal{T}, E)$. If G_E is soft open and H_E is soft semiopen, then $G_E \sqcap H_E$ is soft semiopen over G.*

Proof. Apply the same steps in the proof of above lemma and use the statement that $\mathrm{Cl}(U_E) \sqcap G_E = \mathrm{Cl}_{G_E}(U_E)$. □

Lemma 3.8. *A subset G_E of $(X, \mathcal{T}, E)$ is soft semiopen iff $G_E \sqcap U_E$ is soft sw-open for each soft open set U_E over X.*

Proof. Since each soft semiopen set is soft sw-open and by Lemma 3.6, the intersection of a soft semiopen set with a soft open set is semiopen, so the first part follows.

Conversely, let $P_e^x \in G_E$ and assume that $G_E \sqcap U_E$ is soft sw-open for each soft open set U_E over X. That is $\mathrm{Int}(G_E \sqcap U_E) \neq \Phi_E$. But $\Phi_E \neq \mathrm{Int}(G_E \sqcap U_E) = \mathrm{Int}(G_E) \sqcap \mathrm{Int}(U_E) = \mathrm{Int}(G_E) \sqcap U_E$, which implies that $P_e^x \in \mathrm{Cl}(\mathrm{Int}(G_E))$ and so $G_E \sqsubseteq \mathrm{Cl}(\mathrm{Int}(G_E))$. This proves that G_E is soft semiopen. □

Lemma 3.9. *Let F_E be a subset of $(X, \mathcal{T}, E)$. If F_E is soft semiclosed and soft somewhere dense, it is soft sw-open.*

Proof. Directly follows from Lemma 3.3 which implies that F_E is semiclosed iff $\mathrm{Int}(\mathrm{Cl}(F_E)) = \mathrm{Int}(F_E)$. □

4. Soft Somewhat Continuous Functions

We devote this section to presenting the concepts of soft somewhat continuous functions (briefly soft sw-continuous) and giving several characterizations of it. In addition, we illustrate its relationships with some types of soft continuity. Finally, we derive some results related to soft separable and hyperconnected spaces.

Definition 4.1. *Let $(X, \mathcal{T}, E)$ and $(Y, \mathcal{S}, E')$ be soft topological spaces. A function $f : (X, \mathcal{T}, E) \to (Y, \mathcal{S}, E')$ is said to be soft sw-continuous if the inverse image of each soft open set over Y is soft sw-open over X.*

The above definition can be stated as:

Remark 4.1. *A function $f : (X, \mathcal{T}, E) \to (Y, \mathcal{S}, E')$ is soft sw-continuous if for each $P_e^x \in X$ and each soft open set $V_{E'}$ over Y containing $f(P_e^x)$, there exists a soft sw-open set U_E over X containing P_e^x such that $f(U_E) \sqsubseteq V_{E'}$.*

From Diagram I, we conclude that

$$\begin{array}{ccc}
\text{soft semicontinuous} & \longrightarrow & \text{soft } \beta\text{-continuous} \\
\downarrow & & \downarrow \\
\text{soft } sw\text{-continuous} & \longrightarrow & \text{soft } SD\text{-continuous}
\end{array}$$

Diagram II: Relationship between some generalizations of soft continuity

None of the implications in the above diagram is reversible.

Example 4.1. *Let $X = \{x, y, z\}$ and $E = \{e_1, e_2\}$. Put $\mathcal{T} = \{\Phi_E, F_E, G_E, X_E\}$, where $F_E = \{(e_1, \{y\}), (e_2, \{y\})\}$, $G_E = \{(e_1, \{x, z\}), (e_2, \{x, z\})\}\}$ and $\mathcal{S} = \{\Phi_E, H_E, X_E\}$, where $H_E = \{(e_1, X), (e_2, \{x, y\})\}$. Let $f : (X, \mathcal{T}, E) \to (X, \mathcal{S}, E)$ be the soft identity function. Then f is soft sw-continuous but not soft semicontinuous.*

Example 4.2. *Let $X = \mathbb{R}$ be the set of real numbers and $E = \{e\}$ be a set of parameters. Let $\mathcal{T}$ be the soft topology on $\mathbb{R}$ generated by $\{(e, B(e)) : B(e) = (a, b); a, b \in \mathbb{R}; a < b\}$. Define a soft function $f : (X, \mathcal{T}, E) \to (X, \mathcal{T}, E)$ by*

$$f(x) = \begin{cases} x, & \text{if } x \notin \{0,1\}_E; \\ 0, & \text{if } x = 1; \\ 1, & \text{if } x = 0. \end{cases}$$

One can easily show f is soft sw-continuous (consequently, soft SD-continuous) because the inverse image of any soft basic open set always contains some soft basic open, so its soft interior cannot be null. On the other hand f is not soft β-continuous. Take the soft open set $G_E = \{(e, (-\varepsilon, \varepsilon))\}$, where $\varepsilon < 1$. Therefore

$$f^{-1}(G_E) = \{(e, (-\varepsilon, 0))\} \bigsqcup \{(e, (0, \varepsilon))\} \bigsqcup \{(e, \{1\})\}.$$

But $\mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(f^{-1}(G_E)))) = \{(e, [-\varepsilon, \varepsilon])\}$ and so $f^{-1}(G_E) \not\sqsubseteq \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(f^{-1}(G_E))))$. In conclusion, f cannot be soft β-continuous (consequently, is not soft semicontinuous).

Example 4.3. *Let $(X, \mathcal{T}, E)$ be the soft topological space given in Example 4.2 and let $f : (X, \mathcal{T}, E) \to (X, \mathcal{T}, E)$ be defined by*

$$f(x) = \begin{cases} 0, & x \notin \mathbb{Q}_E; \\ 1, & x \in \mathbb{Q}_E. \end{cases}$$

Then f is soft SD-continuous but not soft sw-continuous. The inverse image of any soft open set containing only 1 is $\mathbb{Q}_E$ which is not soft sw-open over X.

Definition 4.2. *For a subset G_E of a soft topological space $(X, \mathcal{T}, E)$, we introduce the following:*

(i) $\mathrm{Cl}_{sw}(G_E) = \bigcap \{F_E : F_E$ *is soft sw-closed over X and $G_E \sqsubseteq F_E\}$.*

(ii) $\mathrm{Int}_{sw}(G_E) = \bigcup \{O_E : O_E$ *is soft sw-open over X and $O_E \sqsubseteq G_E\}$.*

Proposition 4.1. *Let $(X, \mathcal{T}, E)$ and $(Y, \mathcal{S}, E')$ be soft topological spaces. For a function $f : (X, \mathcal{T}, E) \to (Y, \mathcal{S}, E')$, the following are equivalent:*

(1) f is soft sw-continuous,

(2) $f^{-1}(F_{E'})$ is soft sw-closed set over X, for each soft closed set $F_{E'}$ over Y,

(3) $f(\mathrm{Cl}_{sw}(G_E)) \sqsubseteq \mathrm{Cl}(f(G_E))$, for each set G_E over X,

(4) $\mathrm{Cl}_{sw}(f^{-1}(H_{E'})) \sqsubseteq f^{-1}(Cl(H_{E'}))$, for each set $H_{E'}$ over Y,

(5) $f^{-1}(\mathrm{Int}(H_{E'})) \sqsubseteq \mathrm{Int}_{sw}(f^{-1}(H_{E'}))$, for each set $H_{E'}$ over Y,

Proof. Follows from the definition of soft sw-continuity. □

Definition 4.3. [5, Definition 3.10] *Let (X, E) and (Y, E') be soft sets and let $A_E \in (X, E)$. The restriction of $f : (X, E) \to (Y, E')$ is the soft function $f_{A_E} : (X, E) \to (Y, E')$ defined by $f_{A_E}(P^x_e) = f(P^x_e)$ for all $P^x_e \in A_E$. An extension of a soft function f is a soft function g such that f is a restriction of g*

Theorem 4.1. *Let $(X, \mathcal{T}, E)$ and $(Y, \mathcal{S}, E')$ be soft topological spaces and let D_E be a soft dense subspace over X. If $f : (X, \mathcal{T}, E) \to (Y, \mathcal{S}, E')$ is soft sw-continuous over X, then $f|_{D_E}$ is soft sw-continuous over D.*

Proof. Standard (by using Lemma 3.1). □

Theorem 4.2. *Let $(X, \mathcal{T}, E)$ and $(Y, \mathcal{S}, E')$ be soft topological spaces. Let $f : (X, \mathcal{T}, E) \to (Y, \mathcal{S}, E')$ be a function and $\{G^\alpha_E : \alpha \in \Lambda\}$ be a soft open cover of X. Then f is soft sw-continuous, if $f|_{G^\alpha_E}$ is soft sw-continuous for each $\alpha \in \Lambda$.*

Proof. Let $V_{E'}$ be a soft open set over Y. By assumption, $\left(f|_{G_E^\alpha}\right)^{-1}(V_{E'})$ is soft sw-open over G_E^α. By Lemma 3.2, $\left(f|_{G_E^\alpha}\right)^{-1}(V_{E'})$ is soft sw-open over X for each $\alpha \in \Lambda$. But

$$f^{-1}(V_{E'}) = \bigsqcup_{\alpha\in\Lambda}\left[\left(f|_{G_E^\alpha}\right)^{-1}(V_{E'})\right],$$

which a union of soft sw-open sets and by Lemma 3.3, $f^{-1}(V_{E'})$ is soft sw-open over X. Hence f is soft sw-continuous. □

Theorem 4.3. *Let $(X,\mathcal{T},E)$ and $(Y,\mathcal{S},E')$ be soft topological spaces and let W_E be a soft open set over X. If $f:(W,\mathcal{T}_W,E)\to(Y,\mathcal{S},E')$ is a soft sw-continuous function such that $f(W_E)$ is soft dense over Y, then each extension function of f over X is soft sw-continuous.*

Proof. Let g be an extension of f and let $V_{E'}$ be a (non-null) soft open set over Y. If $g^{-1}(V_{E'})=\Phi_E$, then g is trivially soft sw-continuous. Suppose $g^{-1}(V_{E'})\neq\Phi_E$. By density of $f(W_E)$, $f(W_E)\sqcap V_{E'}\neq\Phi_{E'}$ which implies that $W_E\sqcap f^{-1}(V_{E'})\neq\Phi_E$. Therefore $f^{-1}(V_{E'})\neq\Phi_E$. By assumption, there exists a non-null soft open set U_E over W such that

$$U_E = U_E\bigsqcap W_E \sqsubseteq f^{-1}(V_{E'})\bigsqcap W_E = g^{-1}(V_{E'})\bigsqcap W_E\sqsubseteq g^{-1}(V_{E'}).$$

By Lemma 3.2, U_E is a soft open set over X and so $\Phi_E\neq U_E\sqsubseteq g^{-1}(V_{E'})$. Thus g is soft sw-continuous over X. □

Theorem 4.4. *Let $(X,\mathcal{T},E)$ and $(Y,\mathcal{S},E')$ be soft topological spaces. A function $f:(X,\mathcal{T},E)\to(Y,\mathcal{S},E')$ is soft semicontinuous iff $f|_{U_E}$ is sw-continuous for each soft open set U_E over X.*

Proof. Assume that f is soft semicontinuous and U_E is any soft open over X. Let $G_{E'}$ be a soft open set over Y. Then $f^{-1}(G_{E'})$ is soft semiopen and so, by Lemma 3.7, $(f|_{U_E})^{-1}(G_{E'})=f^{-1}(G_{E'})\sqcap U_E$ is soft semiopen over U. Thus $f|_{U_E}$ is soft semicontinuous and hence soft sw-continuous.

Conversely, suppose that $f|_{U_E}$ is soft sw-continuous for each soft open set U_E over X. Let $H_{E'}$ be soft open over Y. Then $(f|_{U_E})^{-1}(H_{E'})=f^{-1}(H_{E'})\sqcap U_E$ is soft sw-open over U. Since U_E is a soft open set over X, by Lemma 3.2, $f^{-1}(H_{E'})\sqcap U_E$ is soft sw-open over X and so, by Lemma 3.8, $f^{-1}(H_{E'})$ is soft semiopen over X. Thus f is soft semicontinuous. □

Theorem 4.5. *Let $(X,\mathcal{T},E)$ and $(Y,\mathcal{S},E')$ be soft topological spaces. For a function $f:(X,\mathcal{T},E)\to(Y,\mathcal{S},E')$, the following are equivalent:*

(1) f is soft sw-continuous,
(2) for each soft open set $V_{E'}$ over Y with $f^{-1}(V_{E'})\neq\Phi_E$, there exists a non-null soft open set U_E over X such that $U_E\sqsubseteq f^{-1}(V_{E'})$,
(3) for each soft closed set $F_{E'}$ over Y with $f^{-1}(F_{E'})\neq X_E$, there exists a proper soft closed K_E over X such that $f^{-1}(F_{E'})\sqsubseteq K_E$,
(4) for each soft dense set D_E over X, then $f(D_E)$ is soft dense over $f(X)$.

Proof. (1) $\Longrightarrow$ (2) Remark 3.1 and the definition of sw-continuity.

(2) $\Longrightarrow$ (3) Let $F_{E'}$ be a soft closed set over Y such that $f^{-1}(F_{E'})\neq X_E$. Then $Y_{E'}\setminus F_{E'}$ is soft open over Y with $f^{-1}(Y_{E'}\setminus F_{E'})\neq\Phi_E$. By (2), there exists a soft open set U_E over X such that $\Phi_E\neq U_E\sqsubseteq f^{-1}(Y_{E'}\setminus F_{E'})=X_E\setminus f^{-1}(F_{E'})$. This implies that $f^{-1}(F_{E'})\sqsubseteq X_E\setminus U_E\neq X_E$. If $K_E=X_E\setminus U_E$, then K_E is a proper soft closed set that satisfies the required property.

(3) $\Longrightarrow$ (4) Let D_E be soft dense over X. We need to prove that $f(D_E)$ is soft dense over $f(X)$. Suppose that $f(D_E)$ is not soft dense over $f(X)$. There exists a proper soft closed set $F_{E'}$ such that $f(D_E) \sqsubseteq F_{E'} \sqsubset f(X_E)$. Therefore $D_E \sqsubseteq f^{-1}(F_{E'})$. By (3), there exists a soft closed set K_E over X such that $D_E \subseteq f^{-1}(F_{E'}) \sqsubseteq K_E \neq X_E$. This contradicts that D_E is soft dense over X. Thus (4) holds.

(4) $\Longrightarrow$ (1) With out loss of generality, let $H_{E'}$ be a soft open set over Y with $f^{-1}(H_{E'}) \neq \Phi_E$, because if $f^{-1}(H_{E'}) = \Phi_E$, then it is trivially soft sw-open. Suppose that $f^{-1}(H_{E'})$ is not soft sw-open. That is $\text{Int}(f^{-1}(H_{E'})) = \Phi_E$. Therefore $\text{Cl}(X_E \setminus f^{-1}(H_{E'}) = X_E$. This implies that $X_E \setminus f^{-1}(H_{E'})$ is soft dense over X. By (4), $f(X_E \setminus f^{-1}(H_{E'}))$ is soft dense over $f(X)$, i.e., $\text{Cl}(f(X_E \setminus f^{-1}(H_{E'}))) = f(X_E)$. This yields that $\text{Cl}(f(X_E) \setminus H_{E'}) = f(X_E) \setminus H_{E'} = f(X_E)$ and so $H_{E'} = \Phi_{E'}$. Contradiction to the choice of $H_{E'}$. It follows that $\text{Int}(f^{-1}(H))$ must not be null. Thus $f^{-1}(H_{E'})$ is soft sw-open over X. □

Corollary 4.1. *Let $(X, \mathcal{T}, E)$ and $(Y, \mathcal{S}, E')$ be soft topological spaces. For a one to one function f from a space $(X, \mathcal{T}, E)$ onto a space $(Y, \mathcal{S}, E')$, the following are equivalent:*

(1) f is soft sw-continuous,

(2) for each soft co-dense set N_E over X, $f(N_E)$ is soft co-dense over Y.

We complete this section by discussing two results related to soft separable and hyperconnected spaces.

Theorem 4.6. *Let $(X, \mathcal{T}, E)$ and $(Y, \mathcal{S}, E')$ be soft topological spaces, and let f be a function from $(X, \mathcal{T}, E)$ onto $(Y, \mathcal{S}, E')$. If f is soft sw-continuous and $(X, \mathcal{T}, E)$ is soft separable, then $(Y, \mathcal{S}, E')$ is soft separable.*

Proof. Let D_E be a countable soft dense set over X. Clearly $f(D_E)$ is countable. By Theorem 4.5 (4), $f(D_E)$ is soft dense over $f(X) = Y$. Therefore $(Y, \mathcal{S}, E')$ is soft separable. □

Theorem 4.7. *Let $(X, \mathcal{T}, E)$ and $(Y, \mathcal{S}, E')$ be soft topological spaces. If f is a soft sw-continuous from $(X, \mathcal{T}, E)$ onto $(Y, \mathcal{S}, E')$ and $(X, \mathcal{T}, E)$ is soft hyperconnected, then $(Y, \mathcal{S}, E')$ is soft hyperconnected.*

Proof. Let $G_{E'}, H_{E'}$ be any two soft open sets over Y with $G_{E'} \neq \Phi_{E'} \neq H_{E'}$. Since f is soft sw-continuous, then $\text{Int}(f^{-1}(G_{E'})) \neq \Phi_E \neq \text{Int}(f^{-1}(H_{E'}))$. But $(X, \mathcal{T}, E)$ is soft hyperconnected, so

$$\text{Int}(f^{-1}(G_{E'})) \bigsqcap \text{Int}(f^{-1}(H_{E'})) \neq \Phi_E.$$

If

$$x \in \text{Int}(f^{-1}(G_{E'})) \bigsqcap \text{Int}(f^{-1}(H_{E'})) \sqsubseteq f^{-1}(G_{E'}) \bigsqcap f^{-1}(H_{E'}),$$

then $f(x) \in G_{E'} \bigsqcap H_{E'}$. Thus $(Y, \mathcal{S}, E')$ is soft hyperconnected. □

5. Soft Somewhat Open Functions

In this section, we formulate the concepts of soft somewhat open functions (briefly soft sw-open) and study its main properties. We characterized it using soft closed and soft dense sets.

Definition 5.1. *Let $(X, \mathcal{T}, E)$ and $(Y, \mathcal{S}, E')$ be soft topological spaces. A function $f : (X, \mathcal{T}, E) \to (Y, \mathcal{S}, E')$ is soft sw-open if for each soft open set U_E over X, $f(U_E)$ is soft sw-open over Y.*

Remark 5.1. *Let $(X, \mathcal{T}, E)$ and $(Y, \mathcal{S}, E')$ be soft topological spaces. A function $f : (X, \mathcal{T}, E) \to (Y, \mathcal{S}, E')$ is soft sw-open iff for each non-null soft open set U_E over X, there exits a non-null soft sw-open set $V_{E'}$ over Y such that $V_{E'} \sqsubseteq f(U_E)$.*

For a single soft point, we have

Proposition 5.1. *Let $(X, \mathcal{T}, E)$ and $(Y, \mathcal{S}, E')$ be soft topological spaces. A function $f : (X, \mathcal{T}, E) \to (Y, \mathcal{S}, E')$ is soft sw-open at $P_e^x \in X_E$ if for each soft open set U_E over X containing P_e^x, there exits a soft sw-open set $V_{E'}$ over Y such that $f(P_e^x) \in V_{E'} \sqsubseteq f(U_E)$.*

From [5, Proposition 4.7], Lemma 3.4 and Remark 3.3, one can obtain the following for functions:

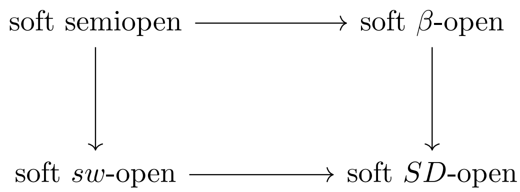

Diagram III: Relationship between some generalizations of soft openness

None of the implications in the above diagram is reversible and counterexamples are not difficult to obtain.

Proposition 5.2. *Let $(X, \mathcal{T}, E)$ and $(Y, \mathcal{S}, E')$ be soft topological spaces. For a function $f : (X, \mathcal{T}, E) \to (Y, \mathcal{S}, E')$, the following are equivalent:*

(1) f is soft sw-open,
(2) $f(\mathrm{Int}(G_E)) \sqsubseteq \mathrm{Int}_{sw}(f(G_E))$, for each set G_E over X,
(3) $f^{-1}(\mathrm{Cl}_{sw}(H_{E'})) \sqsubseteq \mathrm{Cl}(f^{-1}(H_{E'}))$, for each set $H_{E'}$ over Y.

Proof. Standard. □

Theorem 5.1. *Let $(X, \mathcal{T}, E)$ and $(Y, \mathcal{S}, E')$ be soft topological spaces and let G_E be a soft open subspace over X. If $f : (X, \mathcal{T}, E) \to (Y, \mathcal{S}, E')$ is soft sw-open over X, then $f|_{G_E}$ is sw-open over G.*

Proof. If U_E is any soft open over G_E, then U_E is also soft open over X because G_E is soft open. By assumption, $f(U_E)$ is soft sw-open and hence the result. □

Theorem 5.2. *Let $(X, \mathcal{T}, E)$ and $(Y, \mathcal{S}, E')$ be soft topological spaces and let D_E be a soft dense subspace over X. If $f : (D, \mathcal{T}_D, E) \to (Y, \mathcal{S}, E')$ is a soft sw-open function, then each extension of f is soft sw-open over X.*

Proof. Let g be any extension of f and let U_E be a soft open set over X. Since D_E is soft dense over X, so $U_E \sqcap D_E$ is a non-null soft open set over D_E. By assumption, there exists a non-null soft sw-open set $V_{E'}$ over Y such that $V_{E'} \sqsubseteq f(U_E \sqcap D_E) = g(U_E \sqcap D_E) \sqsubseteq g(U_E)$. Thus g is soft sw-open over X. □

Theorem 5.3. *Let $(X, \mathcal{T}, E)$ and $(Y, \mathcal{S}, E')$ be soft topological spaces. Let $f : (X, \mathcal{T}, E) \to (Y, \mathcal{S}, E')$ be a function and $\{G_E^\alpha : \alpha \in \Lambda\}$ be any soft cover over X. Then f is soft sw-open, if $f|_{G_E^\alpha}$ is soft sw-open for each $\alpha \in \Lambda$.*

Proof. Let U_E be a (non-null) soft open set over X. Then $U_E \sqcap G_E^\alpha$ is a non-null soft open set in G_E^α for each α. By assumption, $f(U_E \sqcap G_E^\alpha)$ is a soft sw-open set over Y. But

$$f(U_E) = \bigsqcup f\left(U_E \sqcap G_E^\alpha\right),$$

which a union of soft sw-open sets and by Lemma 3.3, $f(U_E)$ is a soft sw-open set over Y. Hence f is soft sw-open. □

Theorem 5.4. *Let $(X, \mathcal{T}, E)$ and $(Y, \mathcal{S}, E')$ be soft topological spaces. For a one to one function f from $(X, \mathcal{T}, E)$ onto $(Y, \mathcal{S}, E')$, the following are equivalent:*

(1) f is soft sw-open,

(2) for each soft closed set F_E over X with $f(F_E) \neq Y_{E'}$, there exists a proper soft closed $K_{E'}$ over Y such that $f(F_E) \sqsubseteq K_{E'}$.

Proof. (1) $\Longrightarrow$ (2) Let F_E be a soft closed over X with $f(F_E) \neq Y_{E'}$. This implies $X_E \setminus F_E$ is a non-null soft open set over X. By (1), there exists a soft open set $H_{E'}$ over Y such that $\Phi_{E'} \neq H_{E'} \sqsubseteq f(X_E \setminus F_E)$. Therefore $f(F_E) = Y_{E'} \setminus (f(X_E \setminus F_E)) \sqsubseteq Y_{E'} \setminus H_{E'}$. Set $K_{E'} = Y_{E'} \setminus H_{E'}$. So $K_{E'}$ is a soft closed set over Y that satisfies the required property.

(2) $\Longrightarrow$ (1) Reverse the above steps. □

Theorem 5.5. *Let $(X, \mathcal{T}, E)$ and $(Y, \mathcal{S}, E')$ be soft topological spaces. For a function f from $(X, \mathcal{T}, E)$ onto $(Y, \mathcal{S}, E')$, the following are equivalent:*

(1) f is soft sw-open,

(2) for each soft dense set $D_{E'}$ over Y, then $f^{-1}(D_{E'})$ is soft dense over X.

Proof. (1) $\Longrightarrow$ (2) Let $D_{E'}$ be a soft dense set over Y. Suppose otherwise that $f^{-1}(D_{E'})$ is not soft dense over X. Then there is a soft closed K_E over X such that $f^{-1}(D_{E'}) \sqsubset K_E \neq X_E$. But $X_E \setminus K_E$ is soft open over X so, by (1), there exists a soft open set $V_{E'}$ over Y such that $\Phi_{E'} \neq V_{E'} \sqsubseteq f(X_E \setminus K_E)$. Therefore $V_{E'} \sqsubseteq f(X_E \setminus K_E) \sqsubseteq f(f^{-1}(Y_{E'} \setminus D_{E'})) \sqsubseteq Y_{E'} \setminus D_{E'}$. Thus $D_{E'} \sqsubseteq Y_{E'} \setminus V_{E'} \neq \Phi_{E'}$. But $Y_{E'} \setminus V_{E'}$ is soft closed over Y which violates the soft density of $D_{E'}$ over Y. Hence $f^{-1}(D_{E'})$ must be soft dense over X.

(2) $\Longrightarrow$ (1) W.l.o.g, let U_E be a non-null soft open set over X. We need to prove that $\mathrm{Int}_Y(f(U_E)) \neq \Phi_{E'}$. Assume $\mathrm{Int}_Y(f(U_E)) = \Phi_{E'}$. Then $\mathrm{Cl}_Y(Y_{E'} \setminus f(U_E)) = Y_{E'}$. By (2), $\mathrm{Cl}_X\left(f^{-1}\left(Y_{E'} \setminus f(U_E)\right)\right) = X_E$. But $f^{-1}\left(Y_{E'} \setminus f(U_E)\right) \sqsubseteq X_E \setminus U_E$ and $X_E \setminus U_E$ is soft closed over X. Therefore $X_E = \mathrm{Cl}_X(f^{-1}(Y \setminus f(U_E))) \sqsubseteq X_E \setminus U_E$. This means that $U_E = \Phi_E$, which is contradiction. Thus $\mathrm{Int}_Y(f(U_E)) \neq \Phi_{E'}$ and hence f is soft sw-open. □

In the rest of this section, we define an sw-homeomorphism and show some soft topological properties which do not keep by soft sw-homeomorphisms

A soft one to one function f from $(X, \mathcal{T}, E)$ onto $(Y, \mathcal{S}, E')$ is called sw-homeomorphism if it is soft sw-continuous and soft sw-open. One can easily conclude that each homeomorphism is sw-homeomorphism but not the converse. Evidently, if f is soft sw-homeomorphism from $(X, \mathcal{T}, E)$ onto $(Y, \mathcal{S}, E')$, f^{-1} is sw-open.

It is worth stating that soft sw-homeomorphism does not preserve interesting soft topological properties, as showing in the following examples.

Example 5.1. *Let $X = Y = \mathbb{R}$ be the set of real numbers and let $E = \{e\}$ be a set of parameters. If $\mathcal{T}$ is the soft topology on X generated by $\{(e, B(e)) : B(e) = (a, b); a, b \in \mathbb{R}; a < b\}$ and $\mathcal{S}$ is the soft topology on Y generated by $\{(e, B(e)) : B(e) = [a, b); a, b \in \mathbb{R}; a < b\}$ (called soft Sorgenfrey line), then the identity function $i : (X, \mathcal{T}, E) \to (Y, \mathcal{S}, E)$ is soft sw-homeomorphism and $(X, \mathcal{T}, E)$ is soft metrizable, soft locally compact and soft connected, while $(Y, \mathcal{S}, E)$ does not have any of these properties.*

If we take $A = [0,1]$, then $i|_{A_E}$ is soft sw-homeomorphism and $(A, \mathcal{T}_A, E)$ is soft compact, but $(A, \mathcal{S}_A, E)$ is not.

Example 5.2. *Consider X, E and $\mathcal{T}$ given in Example 5.1. Let $\sigma = \{\Phi_E, X_E, \mathcal{T} \setminus \{G_E : G_E \in \mathcal{T}, (e,0) \text{ or } (e,1) \in G_E\}\}$ be another soft topology over X. The identity function $i : (X, \mathcal{T}, E) \to (X, \sigma, E)$ is soft sw-homeomorphism and $(X, \mathcal{T}, E)$ is soft Hausdorff but (X, σ, E) is not soft T_0 (consequently, not soft T_1).*

6. Conclusion and future works

Uncertain phenomena exist in many aspects of our daily life. One of the theories proposed to handle uncertainty is the soft set theory. Typologists applied soft sets to initiate a new mathematical structure called soft topology which is the framework of this study.

In this article, we have introduced the concept of soft somewhat open sets as a new generalization of soft open sets. We have shown that the family of soft somewhat open sets lies between the families of soft semiopen sets and soft somewhere dense sets on one hand. On the other hand, the families of soft somewhat open sets and soft β-open sets are independent of each other. These relationships have been illustrated and main properties have been established with the aid of examples. Then, we have employed soft somewhat open sets to define soft somewhat continuous, and soft somewhat open functions. We have characterized these two functions and investigated the main features. Some nice connections under certain soft topological space are studied in [6]. The reason for defining these concepts was to discuss the differences between soft homeomorphism and soft somewhat homeomorphism regarding the preservation of certain soft topological properties.

In the upcoming work, we plan to study some topological concepts using soft somewhat open sets such as soft compactness, soft Lindelöfness, and soft connectedness. The investigation of some applications soft somewhat homeomorphisms is also planned

Furthermore, we explore soft somewhat open sets in the content of supra soft topology.

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References

[1] Akdag M. and Ozkan A., (2014), Soft α-open sets and soft α-continuous functions, Abstr. Appl. Anal., 2014 , pp. 1–7.

[2] Ali M., Feng F., Liu X., Min W. K. and Shabir M., (2009), On some new operations in soft set theory, Comput Math Appl, 57 , pp. 1547–1553.

[3] Allam A., Ismail T. and Muhammed R., (2017), A new approach to soft belonging, Ann. Fuzzy Math. Inform, 13, pp. 145–152.

[4] Al-shami T. M., (2018), Soft somewhere dense sets on soft topological spaces, Commun. Korean Math. Soc, 33(4), pp. 1341–1356.

[5] Al-shami T. M., Alshammari I. and Asaad B. A., (2020), Soft maps via soft somewhere dense sets, Filomat, 34(10), pp. 3429–3440.

[6] Al-shami T. M., Ameen Z. A. and Asaad B. A., Soft bi-continuity and related soft functions, to appear.

[7] Al-shami T. M. and El-Shafei M. E., (2020), T-soft equality relation, Turkish Journal of Mathematics, 44(94), pp. 1427-1441.

[8] Aygünoğlu A. and Aygün H., (2012), Some notes on soft topological spaces, Neural Comput & Applic, 21, pp. 113–119.

[9] Bayramov S. and Gunduz C., (2018), A new approach to separability and compactness in soft topological spaces, TWMS Journal of Pure and Applied Mathematics, 9(21), pp. 82–93.

[10] Çağman N., Karataş S. and Enginoglu S., (2011), Soft topology, Comput Math Appl, 62, pp. 351–358.
[11] Chen B., (2013), Soft semi-open sets and related properties in soft topological spaces, Appl. Math. Inf. Sci., 7, pp. 287-294.
[12] Das S. and Samanta S., (2013), Soft metric, Annals of Fuzzy Mathematics and Information, 6(1), pp. 77-94.
[13] El-ShafeiM. E., Abo-Elhamayel M. and Al-shami T. M., (2018), Partial soft separation axioms and soft compact spaces, Filomat, 32(13,), pp. 4755-4771.
[14] El-Shafei M. E. and Al-shami T. M., (2021), Some operators of a soft set and soft connected spaces using soft somewhere dense sets, Journal of Interdisciplinary Mathematics, , Accepted.
[15] Hussain S. and Ahmad B., (2011), Some properties of soft topological spaces, Comput Math Appl, 62, pp. 4058-4067.
[16] Kandil A., Tantawy O., El-Sheikh S. and Abd El-latif A., (2014), Soft connectedness via soft ideals, J. New Results in Science, 4, pp. 90-108.
[17] Kharal A. and Ahmad B., (2011), Mappings of soft classes, New Math. Nat. Comput., 7(3), pp. 471-481.
[18] Lin F., (2013), Soft connected spaces and soft paracompact spaces, Int. J. Eng. Math., 7(2), pp. 1-7.
[19] Mahanta J. and Das P. K., (2014), On soft topological space via semiopen and semiclosed soft sets, Kyungpook Math J., 4, pp. 221-23.
[20] Maji P. K., Biswas R. and Roy A. R., (2003), Soft set theory, Computers and Mathematics with Applications, 45, pp. 555–562.
[21] Molodtsov D., (1999), Soft set theory first results, Comput Math Appl, 37, pp. 19-31.
[22] Nazmul S. K. and Samanta S. K., (2013), Neighbourhood properties of soft topological spaces, Ann. Fuzzy Math. Inform., 6, pp. 1-15.
[23] Rong W., (2012), The countabilities of soft topological spaces, International Journal of Computational and Mathematical Sciences, 6, pp. 159-162.
[24] Shabir M. and Naz M., (2011), On soft topological spaces, Comput Math Appl, 61, pp. 1786–1799.
[25] Terepeta M., (2019), On separating axioms and similarity of soft topological spaces, Soft Comput, 23, pp. 1049–1057.
[26] Yumak Y. and Kaymakci A. K., (2015), Soft β-open sets and their applications, J. New Theory, 4, pp. 80-89.

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