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INTEGRAL EQUATIONS FOR THE PROBLEM OF WAVE DIFFRACTION ON A FLAT STRIP: ALTERNATIVE REPRESENTATION

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Abstract. This study investigates an effective method for solving one class of integral equations. It addresses various two dimensional problems related to do diffraction theory by metal screens, which are reduced to these integral equations. A novel approach for solving this class of integral equations is proposed. The study foceses on investigating diffraction on a strip using a combination of analytical and numerical methods and the results are obtained through simulation.

Keywords: Fractional boundary condition, electromagnetic diffraction, fractional strip, integral equation, fractional calculus.

AMS Subject Classification: 35J05, 35A35, 45L05, 45E99

1. Introduction

The development of numerical techniques for solving scattering problems has always paralleled the advancement in computer technology. Although numerical methods may be considered less complicated compared to analytical methods due to the matrix inversion procedure for analysis computer capacity restricts the size of the problem that can be handled. In general, numerical methods can provide accurate solutions for obstacles with a maximum dimension of a few wavelengths. However, while integral equations are usually solved using numerical methods, they can also be converted into a set of algebraic

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equations using certain analytical techniques. Subsequently, the this matrix equation can be solved using standard matrix inversion algorithms. The time required to solve this matrix equation is proportional to the size of the resultant matrix. Therefore, for large bodies, particularly in RCS estimation, the computation time can be excessively long. To address this issue, the size of the matrix must be minimized. Although there are several powerful analytical techniques available, the main advantage of numerical techniques is their applicability to scatterers of arbitrary shape, limited primarily by the size of the scatterer. However, this limitation poses a practical problem. Theoretically, a set of linear equations that describe the scattering problem can be generated, but the resulting set may be too large to be solved. Fortunately, the development in computer technology has made it possible to solve many electromagnetic problems with the desired degree of accuracy. The difficulty of the problem increases when dealing with complex-shaped bodies. These difficulties can be overcome by utilizing a combination of analytical and numerical methods. An alternative method was developed by Veliev et al. [7], where the solution offers any desired level of accuracy. The scattered field was represented using the Fourier transform of the corresponding surface current density, which offers several advantages for constructing the solution to the problem. The scattering of electromagnetic waves from geometric and physical discontinuities is one of the most important areas in electromagnetic wave theory. Due to its simple geometry, the scattering problem related to a strip has been extensively studied by many scientists. Various techniques have been suggested for resolving the issues associated with a strip [1]. With the development of numerical techniques for solving scattering problems and simultaneous advancements in computer technology [6], it has become possible to solve many electromagnetic problems with the required degree of accuracy. In this study, diffraction on a strip was investigated using analytical and numerical methods, and the results were obtained through simulation.

2. MATERIAL AND METHODS

This study aims to address the two-dimensional thin strip diffraction problem with a new method. The research objective is to develop and generalize an alternative and new approach to the problem, as stated in the summary. The proposed method is expected to offer simpler calculations, faster processing, and wider applicability to various materials when compared to the existing methods in the literature. The problem considered in this study is the diffraction of a plane H-polarized wave, as shown in Figure 1. To describe the scattering properties of surfaces in various geometries, we utilize the integral boundary condition, which corresponds to a boundary condition between the Dirichlet and Neumann boundary conditions. The integral boundary condition is a generalization of the Dirichlet and Neumann boundary conditions. In our study, we employ the hybrid method as presented in previous works. The hybrid method combines the advantages of both analytical and numerical approaches to develop methods. While some implicit expressions can only be obtained for the high-frequency regime, hybrid methods can calculate field expressions in wider frequency regimes, allowing the investigation of resonances for thin strip problems with hybrid methods. To solve the problem presented in this study, we use the method of orthogonal polynomials to tackle the diffraction problems. Firstly, in Figure 1, we define the scattered area as an integral and employ Green and Fourier analysis to obtain this integral. The problem is then solved using an analytical approach. For the general solution, the integral equation is expressed as the sum of the special orthogonal functions, taking into account the current density on the strips, geometry, and other relevant conditions.

3. FORMULATION OF THE PROBLEMS

We will consider the problem of diffraction of a plane H-polarized wave in Figure 1.

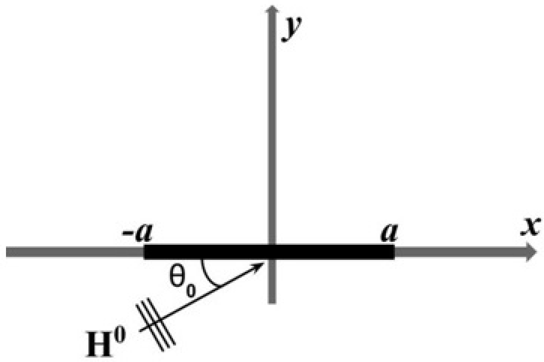

FIGURE 1. The geometry of the problem

$$H_z^0 = e^{ik\left(\alpha_0 x + \sqrt{1-\alpha_0^2 y}\right)} \tag{1}$$

On a flat strip (see Fig 1) Here, $\alpha_0 = \cos\theta_0$, and θ_0 is the angle of incidence. We assume that the wavelength of the incident wave is commensurate with the size of the strip, and the angle of incidence is not very small. Otherwise, the complex ray method developed in [8], [9] can be used. Total magnetic field can be expressed as

$$H_z = H_z^0 + H^s \tag{2}$$

Where the function H^s describes the scattered wave, which can be represented as [2],[3],[5].

$$H_z\left(\eta', \xi'\right) = \frac{i}{2}\int_{-1}^{1} \mu\left(\eta\right)\frac{\partial y}{\partial \xi'} H_0^{(1)}\left(\epsilon_j\sqrt{\left(\eta' - \eta^0\right)^2 + \xi'^2}\right) d\eta^0 \tag{3}$$

Here dimensionless coordinates $\eta = \frac{x}{a}, \xi = \frac{y}{a}$ are entered, and $H_0^{(1)}$ is the Hankel function of the first kind and zero order. In the representation Eq.3, the function describes the surface current density function, which at the ends of the interval (-1,1) obeys the condition

$$\mu_E^{(j)}\left(\eta\right) \sim \left(1-\eta^2\right)\frac{1}{2}, \eta \to \pm 1 \tag{4}$$

For the scattered field, we can write down the representation in Fourier domain of the current density function, which will have the form

$$H_z^s\left(\eta', \xi'\right) = -\frac{\epsilon}{2\pi}\frac{\left|\xi'\right|}{\xi'}\int_{-\infty}^{\infty} h\left(\alpha\right)e^{i\epsilon\left[\alpha\eta' + \sqrt{1-\alpha^2}\left|\xi'\right|\right]}d\alpha \tag{5}$$

$$\mu\left(\eta_j\right) = \frac{\epsilon}{2\pi}\int_{-\infty}^{\infty} h\left(\alpha\right)e^{i\epsilon\alpha\eta}d\alpha \tag{6}$$

For the current density function, we write the representation in the form of a series in Gegenbauer orthogonal polynomials $\{C_n^\nu\}_{n=0}^{\infty}$ [2].

$$\mu\left(\eta\right) = \left(1-\eta^2\right)^\nu \sum_{n=0}^{\infty} x_n C_n^{\nu+\frac{1}{2}}\left(\eta\right) \tag{7}$$

where, x_n is unknown coefficient. When, $\nu = \frac{1}{2}$,$C_n^{\nu+\frac{1}{2}}\left(\eta\right)\Big|_{\nu=\frac{1}{2}} = U_n\left(\eta\right)$ [5]. Note that $U_n\left(\eta\right)$ is the Chebyshev polynomials of the second kind. Representation Eq.7 for the current density function allows us to obtain the following representation for the Fourier images

$$h\left(\alpha\right) = \frac{\epsilon_j}{2\pi}\int_{-1}^{1} \mu\left(\eta\right)e^{-i\epsilon\alpha\eta}d\eta$$

$$h\left(\alpha\right) = \frac{2\pi}{\Gamma\left(\nu+\frac{1}{2}\right)}\sum_{n=0}^{\infty}\left(-i\right)^n x_n \beta_n^{\left(\nu+\frac{1}{2}\right)}\frac{J_{n+\nu+\frac{1}{2}(\epsilon\alpha)}}{\left(2\epsilon\alpha\right)^{\nu+\frac{1}{2}}}\Big|_{\nu=\frac{1}{2}}\Big)$$

$$= 2\pi\sum_{n=0}^{\infty}\left(-i\right)^n x_n\left(n+1\right)\frac{J_{n+1}\left(\epsilon\alpha\right)}{\left(2\epsilon\alpha\right)}$$

$$\beta_n^{\left(\nu+\frac{1}{2}\right)} = \frac{\Gamma\left(n+2\nu+1\right)}{\Gamma\left(n+1\right)}\Big|_{\nu=\frac{1}{2}} = n+1 \tag{8}$$

Now using the Neumann boundary conditions on the surface of the strip

$$\left[\frac{\partial}{\partial\overrightarrow{n}}\left(H_z^0 + H_z^p\right)\right]_L = 0 \tag{9}$$

To determine the unknown coefficients x_{2k}, we obtain a system of infinite algebraic equations of the form

$$\begin{aligned}(-1)^k_{x_{2k}} - \sum_{n=0}^{\infty}\left(-1\right)^n\left(2n+1\right)x_{2n}d_{2k2n} &= -2i\frac{\sqrt{1-\alpha_0^2}}{\alpha_0^2}J_{2k+1}\left(\epsilon_1\alpha_0\right)\\ (-1)^k_{x_{2k+1}} - \sum_{n=0}^{\infty}\left(-1\right)^n\left(2n+2\right)x_{2n+1}d_{2k+12n+1} &= 2\frac{\sqrt{1-\alpha_0^2}}{\alpha_0^2}J_{2k+2}\left(\epsilon_1\alpha_0\right)\end{aligned} \tag{10}$$

Where matrix elements d_{kn} are of the form

$$d_{kn}^{(1)} = 2\int_0^{\infty}\gamma\left(\alpha\right)J_{k+1}\left(\epsilon\alpha\right)J_{n+1}\left(\epsilon\alpha\right)\frac{d\alpha}{\alpha} \tag{11}$$

where, $\gamma(\alpha) = 1 + \frac{i}{|\alpha|}\sqrt{1-\alpha^2}$
Now the values of the unknowns from Eq.10 substituting into the expressions for the Fourier images, we obtain the following integral [4], equations for the Fourier images of the form

$$h_1(\pm\alpha) = \frac{1}{2}\left[h_1^+(\alpha) \pm h_1^-(\alpha)\right]$$
$$h_1^+(\alpha) = 2\pi\sum_{n=0}^{\infty}(-1)^n(2n+1)\,x_{2n}^{(1)}\frac{J_{2n+1}(\epsilon_1\alpha)}{\epsilon_1\alpha} \tag{12}$$
$$h_1^-(\alpha) = 2\pi\sum_{n=0}^{\infty}(-1)^n(2n+2)\,x_{2n+1}^{(1)}\frac{J_{2n+2}(\epsilon_1\alpha)}{\epsilon_1\alpha}$$

After that, we obtain integral equation from the Fourier domain.

$$h_1^+(\alpha) = -i\frac{4\pi}{\epsilon_1}\sqrt{1-\alpha_0^2}K^+(\alpha,\alpha_0) + 2\int_0^{\infty}\beta\gamma(\beta)K^+(\alpha,\beta)\,h_1^+(\beta)\,d\beta$$

$$h_1^-(\alpha) = -\frac{4\pi}{\epsilon_1}\sqrt{1-\alpha_0^2}K^-(\alpha,\alpha_0) + 2\int_0^{\infty}\beta\gamma(\beta)K^-(\alpha,\beta)\,h_1^-(\beta)\,d\beta \tag{13}$$

The kernels of these equations have the form of a bilinear expansion through Bessel functions $J_n(x)$

$$K^+(\alpha,\beta) = \frac{1}{\alpha\beta}\sum_{K=0}^{\infty}(2K+1)J_{2K+1}(\epsilon\alpha)\,J_{2K+1}(\epsilon\beta)$$
$$K^-(\alpha,\beta) = \frac{1}{\alpha\beta}\sum_{K=0}^{\infty}(2K+2)J_{2K+2}(\epsilon\alpha)\,J_{2K+2}(\epsilon\beta) \tag{14}$$

These bilinear expansions in the form of infinite series in Bessel functions can be summed up (see) [5]. In this case, sufficiently simple expressions for these kernels are obtained, which have the form.

$$K^+(\alpha,\beta) = \frac{\epsilon}{2(\alpha^2-\beta^2)}\left[\alpha J_1(\epsilon\alpha)\,J_0(\epsilon\beta) - \beta J_0(\epsilon\alpha)\,J_1(\epsilon\beta)\right]$$
$$K^+(\alpha,\alpha) = \frac{\epsilon^2}{4}\left[J_0^2(\epsilon\alpha) + J_1^2(\epsilon\alpha)\right] \tag{15}$$
$$K^-(\alpha,\beta) = \frac{\epsilon}{2(\alpha^2-\beta^2)}\left[\beta J_1(\epsilon\alpha)\,J_0(\epsilon\beta) - \alpha J_0(\epsilon\alpha)\,J_1(\epsilon\beta)\right]$$
$$K^-(\alpha,\alpha) = -\frac{\epsilon}{2\alpha}J_0(\epsilon\alpha)\,J_1(\epsilon\alpha) + K^+(\alpha,\alpha) \tag{16}$$

Equations Eq.12 are the new integral equations with simple kernel. At the end, we present expressions for the radiation pattern and cross-section of total scattering, which are expressed through unknown coefficients as follows, respectively:

$$\Phi(\Psi) = -\frac{\pi\sin(\Psi)}{2\epsilon\cos(\Psi)}\sum_{n=0}^{\infty}(-i)^n(n+1)\,x_n^{(1)}J_{n+1}(\epsilon\cos(\Psi)) \tag{17}$$

$$\frac{\sigma_s^H}{4\alpha} = \frac{\pi}{\epsilon^2}\frac{\sin\theta_0}{\cos\theta_0}\sum_{n=0}^{\infty}(n+1)J_{n+1}\left(\epsilon\cos\theta_0\right)\mathrm{Re}\left\{(-i)^n x_n^{(1)}\right\} \tag{18}$$

4. Results of Numerical Simulation

In this section the numerical results are presented based on above given algorithm. Fig.2-7 show the near field distribution, the far field radiation pattern and the current distribution, the far field radiation pattern and the current distribution at the given parameters.

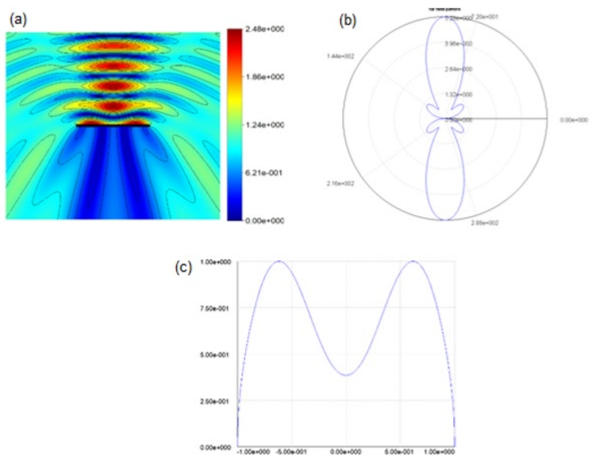

Figure 2. a)Near Field Distrubition $\varepsilon = ka = 5, \theta_0 = -\frac{\pi}{2}$, b)Far Field Pattern $\varepsilon = ka = 5, \theta_0 = -\frac{\pi}{2}$, c)Current Density $\varepsilon = ka = 5, \theta_0 = -\frac{\pi}{2}$

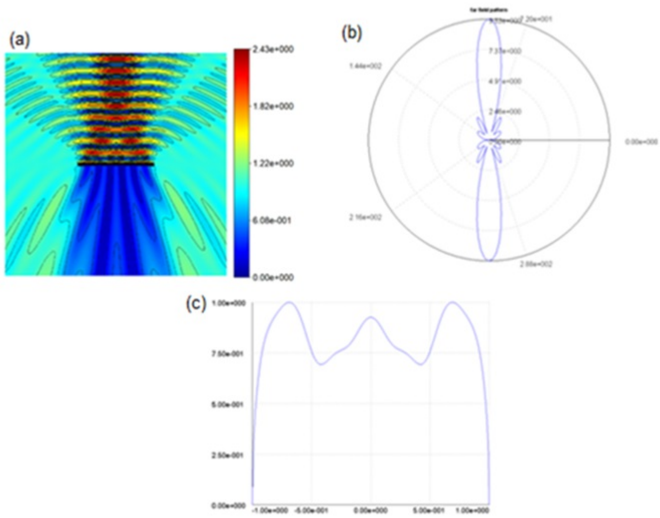

Figure 3. a)Near Field Distrubition $\varepsilon = ka = 10, \theta_0 = -\frac{\pi}{2}$, b)Far Field Pattern $\varepsilon = ka = 10, \theta_0 = -\frac{\pi}{2}$, c)Current Density $\varepsilon = ka = 10, \theta_0 = -\frac{\pi}{2}$

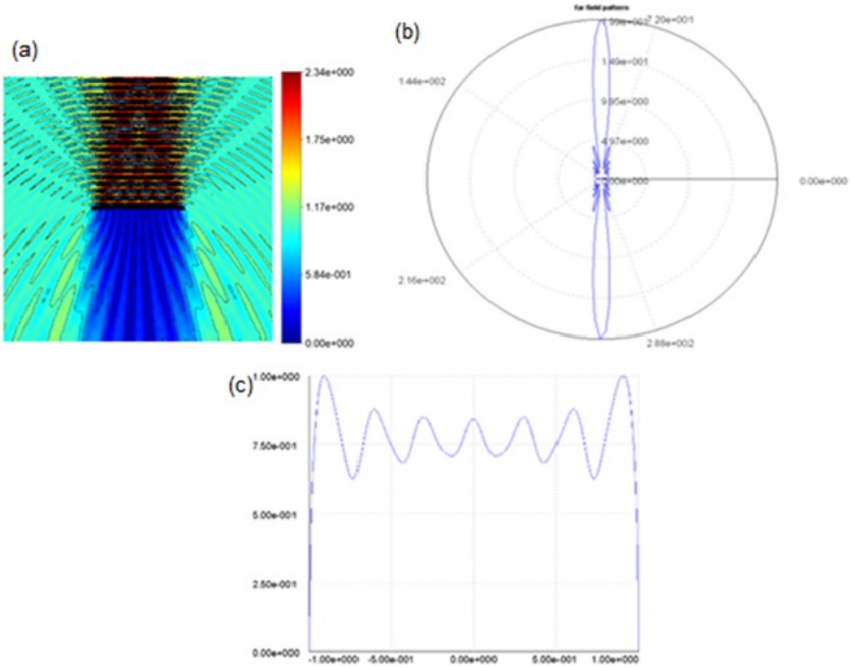

FIGURE 4. a)Near Field Distrubition $\varepsilon = ka = 20, \theta_0 = -\frac{\pi}{2}$, b)Far Field Pattern $\varepsilon = ka = 20, \theta_0 = -\frac{\pi}{2}$, c)Current Density $\varepsilon = ka = 20, \theta_0 = -\frac{\pi}{2}$

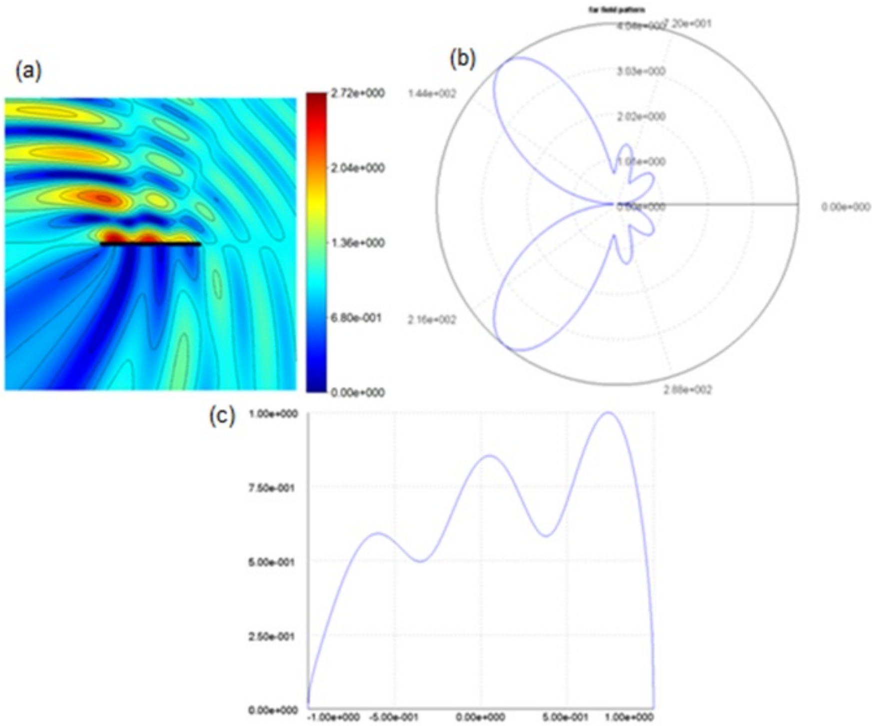

FIGURE 5. a)Near Field Distrubition $\varepsilon = ka = 5, \theta_0 = -\frac{3\pi}{4}$, b)Far Field Pattern $\varepsilon = ka = 5, \theta_0 = -\frac{3\pi}{4}$, c)Current Density $\varepsilon = ka = 5, \theta_0 = -\frac{3\pi}{4}$

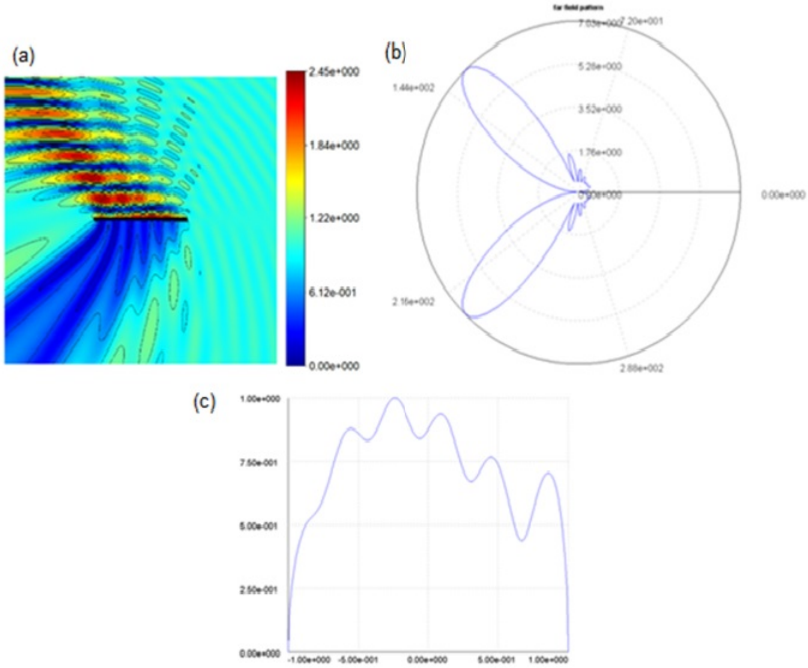

FIGURE 6. a)Near Field Distrubition $\varepsilon = ka = 10, \theta_0 = -\frac{3\pi}{4}$, b)Far Field Pattern $\varepsilon = ka = 10, \theta_0 = -\frac{3\pi}{4}$, c)Current Density $\varepsilon = ka = 10, \theta_0 = -\frac{3\pi}{4}$

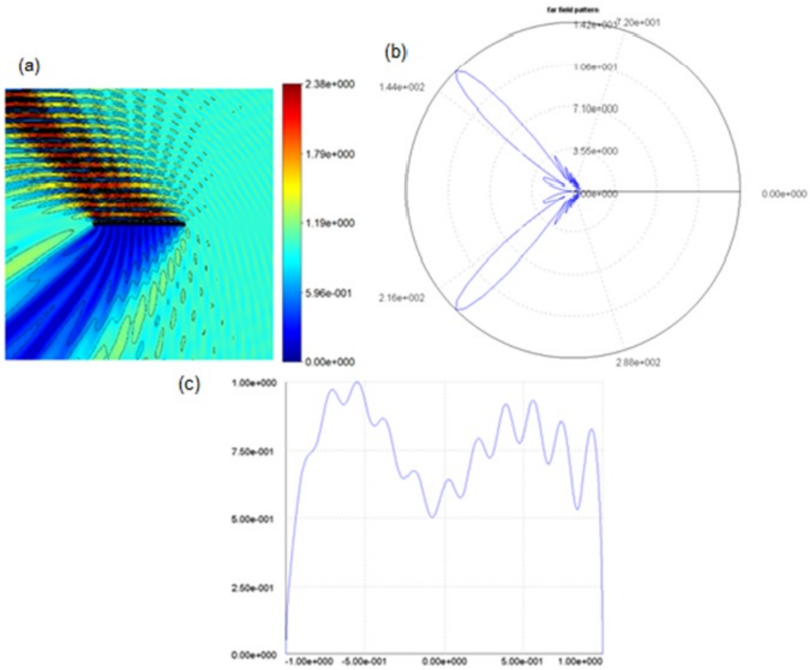

FIGURE 7. a)Near Field Distrubition $\varepsilon = ka = 20, \theta_0 = -\frac{3\pi}{4}$, b)Far Field Pattern $\varepsilon = ka = 20, \theta_0 = -\frac{3\pi}{4}$, c)Current Density $\varepsilon = ka = 20, \theta_0 = -\frac{3\pi}{4}$

5. Conclusion

In this article, the original theoretical background is given for diffraction problem solution by the strip. The electromagnetic scattering problem on a flat strip is analyzed using the hybrid method as a comparison of the moments method and other methods. Here, investigations are mainly done by boundary conditions and diffraction of strips at different varying angles. Also, Dirichlet and Neumann boundary conditions are used to explain the scattering in the integral boundary condition solution. The problems are two-dimensional and are expressed in terms of the scattered wavelength integral, starting with the solution of the total magnetic field. Here, the integral is the Fourier transform of the current density on the strip. The boundary condition is used to obtain the fractional integral equation of the Hankel function with the Fourier transform, then the Fourier transform of the current density on the strip needs to be determined. This is necessary to solve the integral equation for which the current density is expressed in the sum of orthogonal polynomials. Finally, the fourier transform of the current density is obtained. The unknown expressions were obtained from the coefficients for the radiation model and cross-section of the total scattering, and the analytical numerical results of the problem were analyzed based on the algorithm given above. From Figure 2 to Figure 7, the near-field distribution, the far-field radiation model and the current distribution in the given parameters were obtained numerically. Based on the theory the numerical calculations are done and results are presented.

References

[1] Butler, C. M., (1985), General Solution of the Narrow Strip Integral Equations. IEEE Trans. on Ant. and Proc., 33(10), pp. 1085-1090.

[2] Hönl, H., Maue, A. W., Westpfahl, K., (1987), Theorie der beugung In Kristalloptik, Beugung/Crystal Optics, Diffraction, pp. 218-573.

[3] Ivakhnychenko, M. V., Veliev, E. I., and Ahmedov, T. M., (2008), Scattering properties of the strip with fractional boundary conditions and comparison with the impedance strip,PIERC (Progress in Electromagnetics Research C), 02, pp. 189-205.

[4] Prudnikov, A. P., Brychkov, I. A., and Marichev, O. I.,(2002), Special functions,Taylor and rancis, pp. 750.

[5] Otsuki, T., (1976), Reexamination of diffraction problem of a slit by a method of Fourier-orthogonal functions transformation, J. Phys. Soc. Jpn.,, 41(6), pp. 2046-2051.

[6] Uchida, K., Noda, T., Matsunaga,T.,(1989), Electromagnetic wave scattering by a conducting strip-Spectral domain analysis, Electronics and Communications in Jpn.,72(11), pp. 717-722.

[7] Veliev, E.I., Ahmedov T.M., Ivakhnychenko M.V.,(2011), Fractional operators approach and fractional boundary conditions.In: Zhurbenko V (editor). Electromagnetic Waves. Rijeka, Croatia: InTech Open.

[8] Hasanov (Hasanoglu), E., (2004), A new theory of complex rays, IMA Journal of Applied Mathematics, V. 69, pp.521-537.

[9] Hasanov (Hasanoglu), E., Polat, B., (2000), On the Realization of Optical Mappings and Transformation of Amplitudes by Means of an Aspherical "Thick" Lens. Int.J. Electron. Commun.(AEU) V.54, pp. 109-113.

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