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ON S-I-QUOTIENT MAPPINGS, S-I-cs-NETWORKS, S-I-cs'-NETWORKS AND S-wcs'-NETWORKS

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Abstract. In this paper, we define and introduce the notions of S-I-quotient mappings, S-I-cs-networks, S-I-cs'-networks and S-wcs'-networks, and study some characterizations of S-I-quotient mappings and S-I-cs'networks, especially S-J-quotient mappings and S-J-cs-networks on an ideal J of $\mathbb{N}$. With these notions, we get that if X is a S-J-FU space with a point-countable S-J-cs'-network, then X is a meta-Lindelof space.

Keywords: Ideal convergence, ideal spaces, S-I-sequential neighbourhood, S-I-quotient mappings, S-I-cs-networks, S-I-FU spaces.

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1. Introduction

The notion of statistical convergence was originally introduced by H. Fast [7] and H. Steinhaus [13], which is a generalization of the usual notion of convergence. It is doubtless that the study of statistical convergence and its various generalizations has become an active research area in the last few years (see [6, 9, 14]). In particular, P. Kostyrko, T. Salat and W. Wilczynski [8] introduced two interesting generalizations of statistical convergence by using the notion of ideals of subsets of positive integers, which were named as I and I^*-convergence, and prove some properties of I and I^*-convergence in metric spaces. Moreover, B.K. Lahiri and P. Das [10] discussed I and I^*-convergence in topological spaces. Some further results connected with I and I^*-convergence can be found in [3, 4, 5]. It is well-known that mappings and networks are important tools of investigating topological spaces. Continuous mappings, quotient mappings , pseudo-open mappings, cs-networks, sn-networks, k-networks and so on are the most important tools for studying convergence, sequential spaces, Frechet-Urysohn spaces [12] and generalized metric spaces. Recently, I-quotient mappings and I-cs'-networks was defined in the topological spaces [17] taking into account notions defined in [15] and [16].

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Taking into account mentioned above, this paper draws into S-I-quotient mappings and S-I-cs'-networks for an ideal I on $\mathbb{N}$ by using the notion of S-I-convergence defined by Guevara, Sanabria and Rosas [1]. Besides, we discus about some of their properties.

Throughout this paper, the letter X always denotes a topological space. The cardinality of a set C is denoted by $|C|$. The set of all positive integers, the first infinite ordinal, and the first uncountable ordinal are denoted by $\mathbb{N}$, ω and ω_1, respectively. Throughout this paper, we write S-I instead of semi-I. Besides, a semi-I-open set is equal to a S-I-open set and semi-I-continuous is equal to S-I-continuous.

2. Preliminaries

The notion of I-convergence of sequences in a topological space is a generalization of statistical convergence which is based on the ideal of subsets of the set $\mathbb{N}$ of all positive integers. Let $A = 2^{\mathbb{N}}$ be the family of all subsets of $\mathbb{N}$. An ideal $I \subset A$ is a hereditary family of subsets of $\mathbb{N}$ which is stable under finite unions [8]. An ideal I is said to be non-trivial if $I \neq \emptyset$ and $\mathbb{N} \notin I$. A non-trivial ideal $I \subset A$ is called admissible if $\{\{n\} : n \in \mathbb{N}\} \subset I$. It is well-known that every non-trivial ideal defines a dual filter $F_I = \{A \subset \mathbb{N} : \mathbb{N} - A \in I\}$ on $\mathbb{N}$.

Example 2.1. *The following are admissible ideals on* $\mathbb{N}$.

(1) *Let I_f be the family of all finite subsets of $\mathbb{N}$. Then, I_f is an admissible ideal on $\mathbb{N}$.*
(2) *Let I_δ be the family of subsets $A \subset \mathbb{N}$ with $\delta(A) = 0$. Then, I_δ is an admissible ideal on $\mathbb{N}$ and the dual filer $F_{I_\delta} = \{A \subset \mathbb{N} : \delta(A) = 1\}$. Here, $\delta(A)$ denotes the asymptotic density of A which is denoted by $\delta(A) = \lim\limits_{n\to\infty} \dfrac{1}{n}|\{w \in A : w \leq n\}|$, if the limits exists.*

Definition 2.1. [1] *A sequence $\{x_n\}_{n\in\mathbb{N}}$ in a topological space X is said to be S-I-convergent to a point $x \in X$ provided for any semi-open set U containing x, we have $\{n \in \mathbb{N} : x_n \notin U\} \in I$, which is denoted by S-I-$\lim\limits_{n\to\infty} x_n = x$ or $x_n \xrightarrow{SI} x$, and the point x is called the S-I-limit of the sequence $\{x_n\}_{n\in\mathbb{N}}$.*

Definition 2.2. [2] *Let I be an ideal on $\mathbb{N}$ and X be a topological space. Then,*

(1) *A subset $F \subset X$ is said to be S-I-closed if for each sequence $\{x_n\}_{n\in\mathbb{N}} \subset F$ with $x_n \xrightarrow{SI} x \in X$, we have $x \in F$.*
(2) *A subset $U \subset X$ is said to be S-I-open if $X - U$ is S-I-closed.*
(3) *X is called an S-I-sequential space if each S-I-closed subset of X is closed.*

Remark 2.1. *Every open set is semi-open.*

Remark 2.2. *Each sequential space is a S-I-sequential space* [2].

Definition 2.3. [2] *Let I be an ideal on $\mathbb{N}$, X and Y be topological spaces and $f : X \to Y$ be a mapping.*

(1) *f is called preserving S-I-convergence provided for each sequence $\{x_n\}_{n\in\mathbb{N}}$ in X with $x_n \xrightarrow{SI} x$, the sequence $\{f(x_n)\}_{n\in\mathbb{N}}$ S-I-converges to $f(x)$.*
(2) *f is called S-I-continuous provided U is S-I-open in Y, then $f^{-1}(U)$ is S-I-open in X.*

Remark 2.3. *A mapping $f : X \to Y$ is S-I-continuous if and only if whenever F is S-I-closed in Y, then $f^{-1}(F)$ is S-I-closed in X* [2].

Lemma 2.1. [2] *Let I be an ideal on $\mathbb{N}$ and X be a topological space. If a sequence $\{x_n\}_{n\in\mathbb{N}}$ S-I-converges to a point $x \in X$ and $\{y_n\}_{n\in\mathbb{N}}$ is a sequence in X with $\{n \in \mathbb{N} : x_n \neq y_n\} \in I$, then the sequence $\{y_n\}_{n\in\mathbb{N}}$ S-I-converges to $x \in X$.*

Lemma 2.2. [2] *Let I be and ideal on $\mathbb{N}$. The following statements are equivalent for a topological space X and a subset $A \subset X$.*

(1) *A is S-I-open.*
(2) *$\{n \in \mathbb{N} : x_n \in A\} \notin I$ for each sequence $\{x_n\}_{n\in\mathbb{N}}$ in X with $x_n \xrightarrow{SI} x \in A$.*
(3) *$|\{n \in \mathbb{N} : x_n \in A\}| = \omega$ for each sequence $\{x_n\}_{n\in\mathbb{N}}$ in X with $x_n \xrightarrow{SI} x \in A$.*

Lemma 2.3. [2] *Let X and Y be topological space and $f : X \to Y$ be a mapping, Then, the following statements hold:*

(1) *If f is continuous, then f preserves S-I-convergence.*
(2) *If f preserves S-I-convergence, then f is S-I-continuous.*

Definition 2.4. [2] *Let $A \subset X$ and $\{x_n\}_{n\in\mathbb{N}}$ be a sequence in X. If I is an ideal on $\mathbb{N}$, then $\{x_n\}_{n\in\mathbb{N}}$ is S-I-eventually in A if there is $E \in I$ such that for all $n \in \mathbb{N} - E$, $x_n \in A$.*

Remark 2.4. *If A is a subset of X with the property that every sequence S-I-converging to a point in A is S-I-eventually in A, then A is S-I-open. When we assume J to be a maximal ideal, the following proposition shows that such sets must coincide with J-open sets.*

Proposition 2.1. [2] *If J is a maximal ideal of $\mathbb{N}$, then $A \subset X$ is S-J-open if and only if for each S-J-converging sequence $\{x_n\}_{n\in\mathbb{N}}$ with $x_n \xrightarrow{SJ} x \in A$, then $\{x_n\}_{n\in\mathbb{N}}$ is S-J-eventually in A.*

Remark 2.5. *By Definition 2.2, the union of a family of S-I-open sets in a topological space is S-I-open. Whenever J is a maximal ideal, the intersection of two S-J-open sets is a S-J-open set.*

Proposition 2.2. [2] *If J is a maximal ideal of $\mathbb{N}$ and U, V are two S-J-open subsets of X, then $U \cap V$ is S-J-open in X.*

Remark 2.6. *It is well known that the sequential coreflection sX of a space X is the set X endowed with the topology consisting of sequentially open subsets of X. Let J be a maximal ideal of $\mathbb{N}$ and X be a topological space. By Definition 2.2 and Proposition 2.2, the family of all S-J-open subsets of X forms a topology of the set X. The S-J-sequential coreflection of a space X is the set X endowed with the topology consisting of S-J-open subsets of X, which is denoted by S-J-sX. The spaces X and S-J-sX have the same S-J-convergent sequences. Besides, S-J-sX is a S-J-sequential space, further a space X is a S-J-sequential space if and only if S-J-sX.*

Throughout this paper, if no otherwise specified, we consider ideal I is always an admissible ideal on $\mathbb{N}$, all mappings are continuous and surjection; and all spaces are Hausdorff.

3. S-I-quotient mapping and its properties

In this section, we introduce the notion of S-I-quotient mappings, and obtain some characterizations of S-I-quotient mappings, especially S-J-quotient mappings under a maximal ideal of $\mathbb{N}$.

Definition 3.1. *Let I be an ideal on $\mathbb{N}$ and $f : X \to Y$ be a mapping. Then,*

(1) *f is said to be S-I-quotient mapping provided $f^{-1}(U)$ is S-I-open in X, then U is S-I-open in Y.*
(2) *f is said to be S-I-covering mapping if, whenever $\{y_n\}_{n\in\mathbb{N}}$ is a sequence in Y S-I-converging to y in Y, there exist a sequence $\{x_n\}_{n\in\mathbb{N}}$ of points $x_n \in f^{-1}(y_n)$ for all $n \in \mathbb{N}$ and $x \in f^{-1}(y)$ such that $x_n \xrightarrow{SI} x$.*

Theorem 3.1. *Every S-I-covering mapping is S-I-quotient.*

Proof. Let X and Y be arbitrary topological spaces, and $f : X \to Y$ be a S-I-covering mapping. Suppose that H is a non-S-I-closed in Y. Then, there exists a sequence $\{y_n\}_{n\in\mathbb{N}} \subset H$ such that $y_n \xrightarrow{SI} y \notin H$. Since f is S-I-covering, there exists a sequence $\{x_n\}_{n\in\mathbb{N}}$ of points $x_n \in f^{-1}(y_n)$ for all $n \in \mathbb{N}$ and $x \in f^{-1}(y)$ such that $x_n \xrightarrow{SI} x$. We can see that $\{x_n\}_{n\in\mathbb{N}} \subset f^{-1}(H)$ and that $x \notin f^{-1}(H)$. Therefore, $f^{-1}(H)$ is non-S-I-closed. Hence, f is S-I-quotient. □

Lemma 3.1. *The S-J-sequential coreflection S-J-sX is S-J-sequence space and the identity $id_X : S - J - sX \to X$ is a semi-continuous and S-J-covering mapping .*

Proof. Suppose that U is a semi-open subset in X. Since U is S-J-open in X, it is open in S-J-sX, so $id_X : S - J - sX \to X$ is semi-continuous. Now, suppose that $\{x_n\}_{n\in\mathbb{N}}$ is a sequence in X which S-J-converges to a point $x \in X$. If V is a semi-open subset in S-J-sX with $x \in V$, the set V is S-J-open in X. By Proposition 2.1, there is $W \in J$ such that for all $n \in \mathbb{N} - W$, $x_n \in V$, i.e., $\{n \in \mathbb{N} : x_n \notin V\} \subset W$ and thus $\{n \in \mathbb{N} : x_n \notin V\} \in J$. Hence, the sequence $x_n \xrightarrow{SJ} x$ in S-J-sX. This means that the spaces X and S-J-sX have the same S-J-convergent sequences. Therefore, $id_X : S - J - sX \to X$ is a S-J-covering mapping. For each $A \subset X$, by Proposition 2.1, A is S-J-open in S-J-sX if and only if A is S-I-open in X if and only if A is semi-open in S-J-sX. Therefore, S-J-sX is a S-J-sequence space. □

Definition 3.2. *Let I be an ideal on $\mathbb{N}$, X be a topological space and $P \subset X$. P is called a S-I-sequential neighbourhood of x, if for each sequence $\{x_n\}_{n\in\mathbb{N}}$ S-I-converges to a point $x \in P$, then $\{x_n\}_{n\in\mathbb{N}}$ is S-I-eventually in P, i.e., there is $\tilde{I} \in I$ such that $\{n \in \mathbb{N} : x_n \notin P\} = \tilde{I}$.*

Remark 3.1. *Let J be a maximal ideal of $\mathbb{N}$ and $A \subset X$. By Proposition 2.1, A is S-J-open in X if and only if A is S-J-sequential neighbourhood of x for each $x \in A$.*

Proposition 3.1. *Let J be a maximal ideal of $\mathbb{N}$ and $A \subset X$. If A is not a S-J-sequential neighbourhood of x, then there is a sequence $\{x_n\}_{n\in\mathbb{N}}$ in $X - A$ such that $x_n \xrightarrow{SJ} x$.*

Proof. If A is not a S-J-sequential neighbourhood of x, then there is a sequence $\{y_n\}_{n\in\mathbb{N}}$ in X such that $y_n \xrightarrow{SJ} y$, but $\{n \in \mathbb{N} : y_n \notin A\} \notin J$. Since J is a maximal ideal of $\mathbb{N}$, this means that $\{n \in \mathbb{N} : y_n \in A\} \in J$. Now, let $\{n \in \mathbb{N} : y_n \in A\} = \tilde{J} \in J$. And since J is a non-trivial ideal, it follows that $A \neq X$. Taking a point $a \in X - A$. Define a sequence $\{x_n\}_{n\in\mathbb{N}}$ by $x_n = a$ if $n \in \tilde{J}$; $x_n = y_n$ if $n \in \mathbb{N} - \tilde{J}$. Then, the sequence $\{x_n\}_{n\in\mathbb{N}}$ in $X - A$ and $x_n \xrightarrow{SJ} x$ from Lemma 2.2. □

Theorem 3.2. *Let I be an ideal on $\mathbb{N}$. If $f : X \to Y$ is a S-I-quotient mapping, then for each S-I-convergent sequence $\{y_n\}_{n\in\mathbb{N}}$ in Y with $y_n \xrightarrow{SI} x$, there is a sequence $\{x_i\}_{i\in\mathbb{N}}$ in X such that $\{x_i\}_{i\in\mathbb{N}} \subset f^{-1}(\{y_n\}_{n\in\mathbb{N}})$ and $x_i \xrightarrow{SJ} x \notin f^{-1}(\{y_n\}_{n\in\mathbb{N}})$.*

Proof. Suppose that $f : X \to Y$ is a S-I-quotient mapping and $\{y_n\}_{n\in\mathbb{N}}$ is sequence in Y with $y_n \xrightarrow{SJ} y$. Without loss of generality, we can assume that $y_n \neq y$ for each $n \in \mathbb{N}$. Now, let $U = Y - \{y_n\}_{n\in\mathbb{N}}$. Then, U is not S-I-open in Y. Since f is a S-I-quotient mapping, $f^{-1}(U) = f^{-1}(Y - \{y_n\}_{n\in\mathbb{N}}) = X - f^{-1}(\{y_n\}_{n\in\mathbb{N}})$ is not S-I-open in X. Therefore, there is a sequence $\{x_i\}_{i\in\mathbb{N}}$ in $X - f^{-1}(U) = f^{-1}(\{y_n\}_{n\in\mathbb{N}})$ such that $x_i \xrightarrow{SJ} x \notin f^{-1}(\{y_n\}_{n\in\mathbb{N}\}})$. □

In the following theorem we discus about some properties by using quotient mappings and S-I-quotient mappings.

Theorem 3.3. *Let $f : X \to Y$ be a mapping. Then, the following statements hold:*

(1) *If X is a S-I-sequential space and f is quotient, then Y is a S-I-sequential space and f is S-I-quotient.*
(2) *If Y is a S-I-sequential space and f is S-I-quotient, then f is quotient.*
(3) *X is a S-I-sequential space if and only if for an arbitrary topological space Y, if f is quotient, then f is S-I-quotient.*

Proof. (1) Let X be a S-I-sequential space and f be quotient. First, we will show that the space Y is a S-I-sequential space, i.e., each S-I-open set in Y is open. Now, assume that U is S-I-open in Y. Since f is a quotient mapping and X is a S-I-sequential space, it suffices to show that $f^{-1}(U)$ is S-I-open in X. Taking an arbitrary sequence $\{x_n\}_{n\in\mathbb{N}} \subset X$ such that $x_n \xrightarrow{SI} x \in f^{-1}(U)$ in X. Since f is a quotient mapping from X onto Y, it follows that $f(x_n) \xrightarrow{SI} f(x) \in U$ is S-I-open in Y, it follows Lemma 2.2 that $|\{n \in \mathbb{N} : f(x_n) \in U\}| = \omega$, i.e., $|\{n \in \mathbb{N} : x_n \in f^{-1}(U)|\} = \omega$. Therefore, $f^{-1}(U)$ is S-I-open in X. Next, suppose that $U \subset Y$ such that $f^{-1}(U)$ is S-I-open in X. Then, $f^{-1}(U)$ is open in X since X is a S-I-sequential space, due to f is quotient, U is open in Y, furthermore U is S-I-open in Y. Therefore, f is S-I-quotient.

(2) Let Y be a S-I-sequential space and f be S-I-quotient. If $f^{-1}(U)$ is open in X, then $f^{-1}(U)$ is S-I-open in X. Since f is S-I-quotient, U is S-I-open in Y. We can see that Y is a S-I-sequential space, hence U is open in Y. Therefore, f is quotient.

(3) By part (1) and (2) of this theorem, the necessity is obvious.

Sufficiency: If X is not a S-I-sequential space, then there exists a S-I-closed subset H in X such that H is not closed in X. Now, Let $Y = \{0, 1\}$, define a mapping $f : X \to Y$ by $f(x) = 0$ if $x \in H$; $f(x) = 1$, if $x \in X - H$. The topology on Y is endowed the continuous quotient topology induced by f . Then, f is quotient. We can check that $f^{-1}(\{1\})$ is not open in X, therefore $\{1\}$ is not open in Y. It follows that the constant sequence $0, 0, ..., 0, ...$ S-I-converges to 1 in Y, and hence $\{0\}$ is not semi-I-closed in Y. But $f^{-1}(\{0\}) = H$ is S-I-closed in X. Therefore, f is not S-I-quotient.

□

The following example shows that there exists a continuous and S-I-covering mapping which is not quotient.

Example 3.1. *Let the space $X = [0, \omega_1]$ be endowed the following topology: the only non-isolated point ω_1 has the semi-neighbourhoods of the usual ordered topology. Then X is not discrete, and there is not any non-trivial convergent sequence in X. The set $Z = [0, \omega_1]$ is endowed the discrete topology. Taking $h : Z \to X$ is the identity mapping. Obviously, g is continuous, but g is not quotient. Now, suppose that a sequence $\{x_n\}_{n\in\mathbb{N}}$*

S-I-converges to x in X. Now, we will show that $\{n \in \mathbb{N} : x_n \neq x\} \in I$, *i.e.,* $\{n \in \mathbb{N} : x_n \notin \{x\}\} \in I$. *Let* $\{x_n\}_{n\in\mathbb{N}}$ *be a sequence in X and* $x \in X$, *there exits a semi-neighbourhood* U_x *of x in X with* $x_n \notin U_x$ *for each* $x_n \neq x$. *If the sequence* $x_n \xrightarrow{SI} x \in X$, *then* $\{n \in \mathbb{N} : x_n \neq x\} = \{n \in \mathbb{N} : x_n \notin U_x\} \in I$. *If the sequence* $\{x_n\}_{n\in\mathbb{N}}$ *satisfies* $\{n \in \mathbb{N} : x_n \neq x\} \in I$ *and U is a semi-neighbourhood of x in X, we can assume* $U_x \subset U$, *then* $\{n \in \mathbb{N} : x_n \notin U\} \subset \{n \in \mathbb{N} : x_n \notin U_x\} = \{n \in \mathbb{N} : x_n \neq x\} \in I$. *Therefore,* $x_n \xrightarrow{SI} x$ *in Z. Hence, h is S-I-covering.*

Theorem 3.4. *Let J be a maximal ideal of* $\mathbb{N}$ *and X be a topological space. Then, X is a S-J-sequential space if and only if each S-J-quotient mapping onto X is quotient.*

Proof. The necessity is obtained by Theorem 3.3(2). On the other hand, suppose that every S-J-quotient mapping onto X is quotient. By Lemma 3.1 and Theorem 3.1 , the identity $id_X : S - J - sX \to X$ is a continuous and S-J-quotient mapping, thus it is a quotient mapping. Since S-J-sX is a S-J-sequence space, it follows from Theorem 3.3(1) that X is a S-J-sequential space. □

The following example shows that there exists a quotient mapping which is not S-I-quotient.

Example 3.2. *The space* $X = [0, \omega_1]$ *is defined by Example 3.1. The set* $Y = \{0, 1\}$ *is endowed the topology* $\{\emptyset, \{0\}, Y\}$. *Define a mapping* $f : X \to Y$ *by* $f([0, \omega_1]) = \{0\}$, *and* $f(\omega_1) = 1$. *Then, f is a quotient mapping. On the other hand, by Example 3.1, the set* $f^{-1}(\{0\}) = [0, \omega_1)$ *is S-I-closed in X. Since the constant sequence* $0, 0, ..., 0, ...$ *S-I-converges to* 1 *in Y, the set* $\{0\}$ *is not S-I-closed in Y. Therefore, f is not S-I-quotient.*

Definition 3.3. *Let J be a maximal ideal of* $\mathbb{N}$ *and* $A \subset X$. *Denote,*

1. $[A]_{J_S} = \{x \in X :$ *there is a sequence* $\{x_n\}_{n\in\mathbb{N}}$ *in A such that* $x_n \xrightarrow{SJ} x\}$.
2. $(A)_{J_S} = \{x \in X :$ *A is a S-J-sequential neighbourhood of* $x\}$.

Definition 3.4. *Let J be a maximal ideal of* $\mathbb{N}$ *and* $A \subset X$. *A subset* $U \subset X$ *is said to be S-J-sequential neighbourhood of A if* $A \subset (U)_{J_S}$.

Proposition 3.2. *Let J be a maximal ideal of* $\mathbb{N}$ *and* $A \subset X$. *Then,* $[A]_{J_S} = X - (X - A)_{J_S}$.

Proof. Suppose that $x \in [A]_{J_S}$, then there is a sequence $\{x_n\}_{n\in\mathbb{N}}$ in A such that $x_n \xrightarrow{SJ} x\}$. Thus, $X - A$ is not a S-J-sequential neighbourhood of x in X. In indeed, if $X - A$ is a S-J-sequential neighbourhood of x in X, then $\{x_n\}_{n\in\mathbb{N}}$ is S-J-eventually in $X - A$, i.e., there is $E \in J$ such that for all $n \in \mathbb{N} - E$, $x_n \in X - A$. Since J is an admissible ideal, this contradicts to $\{x_n\}_{n\in\mathbb{N}}$ in A. Therefore, $x \notin (X - A)_{J_S}$ and further $x \in X - (X - A)_{J_S}$.

On the other hand, assume that $x \in X - (X - A)_{J_S}$, then $x \notin (X - A)_{J_S}$, and hence $X - A$ is not a S-J-sequential neighbourhood of $x \in X$. By Proposition 3.1, there is a sequence $\{x_n\}_{n\in\mathbb{N}}$ in A such that $x_n \xrightarrow{SJ} x$. Therefore, $x \in [A]_{J_S}$. □

By Definition 2.2, Definition 3.3 and Proposition 3.2, the following proposition holds.

Proposition 3.3. *Let J be a maximal ideal of* $\mathbb{N}$ *and* $A, B \subset X$. *Then,*

1. $[\emptyset]_{J_S} = \emptyset$, $Int(A) \subset (A)_{J_S} \subset A \subset [A]_{J_S} \subset Cl(A)$, *where* $Int(A)$ *and* $Cl(A)$ *denote interior and closure of A, respectively.*
2. *A is S-J-open in X if and only if* $A = (A)_{J_S}$.
3. *A is S-J-closed in X if and only if* $A = [A]_{J_S}$.
4. *If* $B \subset A$, *then* $(B)_{J_S} \subset (A)_{J_S}$ *and* $[B]_{J_S} \subset [A]_{J_S}$.

(5) $(A \cap B)_{J_S} = (A)_{J_S} \cap (B)_{J_S}$ *and* $[A \cup B]_{J_S} = [A]_{J_S} \cup [B]_{J_S}$.

Proof. (1)-(4) are followed directly by the definitions, for that reason we only proof (5).

Since $A \cap B \subset A$ and $A \cap B \subset B$, it follows that $(A \cap B)_{J_S} \subset (A)_{J_S} \cap (B)_{J_S}$. On the other hand, assume that $x \in (A)_{J_S} \cap (B)_{J_S}$. Then, for each sequence $\{x_n\}_{n\in\mathbb{N}}$ in X with $x_n \xrightarrow{SJ} x$, there is $E, F \in J$, such that for each $n \in \mathbb{N} - E$, $x_n \in A$ and for each $n \in \mathbb{N} - F$, $x_n \in B$. Since $E \cup F \in J$ and for each $n \in \mathbb{N} - (E \cup F)$, $x_n \in A \cap B$. This means that $A \cap B$ is a S-I-sequential neighbourhood of x in X. Therefore, $x \in (A \cap B)_{J_S}$.

Now, if we replace $X - A$ with A and $X - B$ with B, it follows that $((X-A)\cap(X-B))_{J_S} = (X-A)_{J_S} \cap (X-B)_{J_S}$. Hence, $[A \cup B]_{J_S} = X - (X - (A \cup B))_{J_S} = X - ((X-A) \cap (X-B))_{J_S} = X - ((X-A)\cap(X-B))_{J_S}) = (X-(X-A)_{J_S}) \cup (X-(X-B)_{J_S}) = [A]_{J_S} \cup [B]_{J_S}$. □

Theorem 3.5. *Let J be a maximal ideal of $\mathbb{N}$ and $f : X \to Y$ be a mapping. Then, the following conditions are equivalent.*

1. *For each S-J-convergent sequence $\{y_n\}_{n\in\mathbb{N}}$ in Y with $y_n \xrightarrow{SJ} y$, there is a sequence $\{x_i\}_{i\in\mathbb{N}}$ in X with $x_i \xrightarrow{SJ} x \in f^{-1}(y)$ and $\{x_i\}_{i\in\mathbb{N}} \subset f^{-1}(\{y_n\}_{n\in\mathbb{N}})$.*
2. *For each $A \subset Y$, it has $f([f^{-1}(A)]_{J_S}) = [A]_{J_S}$.*
3. *If $y \in [A]_{J_S} \subset Y$, then $f^{-1}(y) \cap [f^{-1}(A)]_{J_S} \neq \emptyset$.*
4. *If $y \in [A]_{J_S} \subset Y$, then there is a point $x \in f^{-1}(y)$ such that whenever V is a S-J-sequential neighbourhood of x, $y \in [f(V) \cap A]_{J_S}$.*
5. *If $y \in [A]_{J_S} \subset Y$, then there is a point $x \in f^{-1}(y)$ such that whenever V is a S-J-sequential neighbourhood of x, $f(V) \cap A \neq \emptyset$.*
6. *For each $y \in Y$, if U is a S-J-sequential neighbourhood of $f^{-1}(y)$, then $f(U)$ is a S-J-sequential neighbourhood of y.*

Proof. (1) ⇒ (2) Suppose that $x \in [f^{-1}(A)]_{J_S}$. Then, there is a sequence $\{x_n\}_{n\in\mathbb{N}}$ in $f^{-1}(A)$ such that $x_n \xrightarrow{SJ} x$. Therefore, $\{f(x_n)\}_{n\in\mathbb{N}} \subset A$ and $f(x_n) \xrightarrow{SJ} f(x)$. This means that $f(x) \in [A]_{J_S}$. Hence, $f([f^{-1}(A)]_{J_S}) \subset [A]_{J_S}$.

On the other hand, assume that $y \in [A]_{J_S}$. Then, there is a sequence $\{y_n\}_{n\in\mathbb{N}}$ in A such that $y_n \xrightarrow{SJ} y$. By hypothesis (1), there is a sequence $\{x_i\}_{i\in\mathbb{N}}$ in X with $\{x_i\}_{i\in\mathbb{N}} \subset f^{-1}(\{y_n\}_{n\in\mathbb{N}}) \subset f^{-1}(A)$ and $x_i \xrightarrow{SJ} x \in f^{-1}(y)$. Thus, $x \in [f^{-1}(A)]_{J_S}$, therefore $y = f(x) \in f([f^{-1}(A)]_{J_S})$, moreover $[A]_{J_S} \subset f([f^{-1}(A)]_{J_S})$.

(2) ⇒ (3) Let $y \in [A]_{J_S}$ for each $A \subset Y$. By hypothesis (2), it follows that $y \in f([f^{-1}(A)]_{J_S})$. Therefore, $f^{-1}(y) \cap [f^{-1}(A)]_{J_S} \neq \emptyset$.

(3) ⇒ (4) Let $y \in [A]_{J_S} \subset Y$. By hypothesis (3), assume that $x \in f^{-1}(y) \cap [f^{-1}(A)]_{J_S}$. Then, there is a sequence $\{x_n\}_{n\in\mathbb{N}}$ in $f^{-1}(A)$ such that $x_n \xrightarrow{SJ} x$. If V is a S-J-sequential neighbourhood of x, then there is $E \in J$ such that $x_n \in V$ for all $n \in \mathbb{N} - E$. Therefore, $f(x_n) \in f(V) \cap A$ for all $n \in \mathbb{N} - E$ and $f(x_n) \xrightarrow{SJ} f(x)$. Taking a point $a \in f(V) \cap A$. Define a sequence $\{y_n\}_{n\in\mathbb{N}}$ by $y_n = f(x_n)$ if $n \in \mathbb{N} - E$; $y_n = a$ if $n \in E$. Then, $\{y_n\}_{n\in\mathbb{N}} \subset f(V) \cap A$ and $y_n \xrightarrow{SJ} f(x) = y$ from Lemma 2.1. Hence, $y \in [f(V) \cap A]_{J_S}$.

(4) ⇒ (5) It is obvious.

(5) ⇒ (6) Let $y \in Y$ and U be a S-J-sequential neighbourhood of $f^{-1}(y)$. If $f(U)$ is not a S-J-sequential neighbourhood of y, then $y \in Y - (f(U))_{J_S} = [Y - f(U)]_{J_S}$. By hypothesis (5), it follows that $f(U) \cap (Y - f(U)) = \emptyset$, which is a contradiction.

(6) ⇒ (3) Let $y \in [A]_{J_S} \subset Y$. Now, suppose that $f^{-1}(y) \cap [f^{-1}(A)]_{J_S} = \emptyset$. Then, $f^{-1}(y) \subset X - [f^{-1}(A)]_{J_S} = (X - f^{-1}(A))_{J_S}$, This means that $X - f^{-1}(A)$ is a S-J-sequential neighbourhood of $f^{-1}(y)$. By hypothesis (6), $y \in (f(X - f^{-1}(A)))_{J_S} = (Y - A)_{J_S} = Y - [A]_{J_S}$, which is a contradiction.

(3) $\Rightarrow$ (1) Let $\{y_n\}_{n\in\mathbb{N}}$ be a S-J-convergent sequence in Y with $y_n \xrightarrow{SJ} y$. Now, put $A = \{y_n\}_{n\in\mathbb{N}}$, then $A \in [A]_{J_S}$. By hypothesis (3), there is $x \in f^{-1}(y) \cap [f^{-1}(A)]_{J_S}$. Therefore, there is a sequence $\{x_i\}_{i\in\mathbb{N}}$ in X with $\{x_i\}_{i\in\mathbb{N}} \subset f^{-1}(A) \subset f^{-1}(\{y_n\}_{n\in\mathbb{N}})$ and $x_i \xrightarrow{SJ} x \in f^{-1}(y)$. □

Remark 3.2. *One of the above six conditions can deduce that f is a S-J-quotient mapping. In indeed, let U be non-S-I-closed in Y. Then, there is a sequence $\{y_n\}_{n\in\mathbb{N}}$ in U S-J-converging to $y \in Y - U$. Thus $y \neq y_n$ for each $n \in \mathbb{N}$. By the assumption of the condition (1), there is a sequence $\{x_i\}_{i\in\mathbb{N}}$ in X such that $\{x_i\}_{i\in\mathbb{N}} \subset f^{-1}(\{y_n\}_{n\in\mathbb{N}}) \subset f^{-1}(U)$ and $x_i \xrightarrow{SJ} x \in f^{-1}(y) \notin f^{-1}(U)$. This implies that $f^{-1}(U)$ is non-S-J-closed in X. Therefore, f is a S-J-quotient mapping.*

Remark 3.3. *If the maximal ideal J is replaced by I_f in Theorem 3.5, then* $(1) \Leftrightarrow (2) \Leftrightarrow (3) \Leftrightarrow (4) \Leftrightarrow (5) \Leftrightarrow (6) \Leftrightarrow$ *f is a S-I_f-quotient mapping. But the following example shows that there exist a T_1 space X, an ideal I of $\mathbb{N}$ and a S-S-quotient mapping f such that f does not satisfy the condition* (6) *of Theorem 3.5.*

Example 3.3. *Let $I = \{A \subset \mathbb{N} : A$ contains at most only finite odd positive integers $\}$. Then, I is an admissible ideal of $\mathbb{N}$. Now, Let Y be the set ω which is endowed with the finite complement topology. Then, Y is a first-countable T_1-space. Now, putting $X_0 = Y - \{0\}$ and $X_1 = \{2k : k \in \omega\}$ as the subspaces of the space Y and $X = X_0 \bigoplus X_1$. A mapping $f : X \to Y$ is defined by the natural mapping. It can see that the mapping f is a continuous quotient mapping. Since X_0 and X_1 are first-countable space, X is a first-countable space. Hence, X is a S-I-sequential space. By Theorem 3.3, it follows that f is a S-I-quotient mapping.*

Next, note that the set X_1 is open in X and $f^{-1}(0) \subset X_1$ and hence X_1 is a S-I-sequential neighbourhood of $f^{-1}(0)$. For each open neighbourhood U of 0 *in Y , $\{n \in \mathbb{N} : n \notin U\}$ is a finite subset, therefore $\{n \in \mathbb{N} : n \notin U\} \in I$. This means that the sequence $\{n\}_{n\in\mathbb{N}}$ in Y satisfies $n \xrightarrow{SI} 0 \in f^{-1}(y)$. But, $\{n \in \mathbb{N} : n \notin f(X_1)\} = \{2k+1, k \in \omega\} \notin I$. Thus $f(X_1)$ is not a S-I-sequential neighbourhood of* 0 *in Y.*

Open Problem 3.1. *For some maximal ideal J of $\mathbb{N}$ and an S-J-quotient mapping f, does it satisfy the condition* (6) *of Theorem 3.5?*

Remark 3.4. *Theorem 3.2 is different from Theorem 3.5(1). In Theorem 3.5(1), $x_i \xrightarrow{SJ} x \in f^{-1}(y)$. But we do not know whether the S-I-limit point x in $f^{-1}(y)$ or not in Theorem 3.2.*

4. On spaces with S-I-cs-networks, S-I-cs'-networks and S-wcs'-networks

In this section, we introduce the notions of S-I-cs-networks, S-I-cs'-networks and S-wcs'-networks for a space X; and obtain that if X is an S-J-FU space with a point-countable S-J-cs'-network, then X is a meta-Lindelof space, for an ideal J of $\mathbb{N}$.

Definition 4.1. *Let I be an ideal on $\mathbb{N}$, X be a topological space and P be a cover of X.*

1. [11] *P is a network of X if whenever $x \in U$ with U open in X, then $x \in \tilde{P} \subset U$ for some $\tilde{P} \in P$.*
2. *P is a S-network of X if whenever $x \in U$ with U semi-open in X, then $x \in \tilde{P} \subset U$ for some $\tilde{P} \in P$.*
3. *P is called S-I-cs-network of X if whenever $\{x_n\}_{n\in\mathbb{N}}$ is a sequence in X S-I-covering to a point $x \in U$ with U semi-open in X, then $\{x_n\}_{n\in\mathbb{N}}$ is S-I-eventually in $\tilde{P}$ and $x \in \tilde{P} \subset U$ for some $\tilde{P} \in P$.*

(4) *P is called S-I-cs$'$-network of X if whenever $\{x_n\}_{n\in\mathbb{N}}$ is a sequence in X S-I-covering to a point $x \in U$ with U semi-open in X, then there is $\tilde{P} \in P$ and some $n_0 \in \mathbb{N}$ such that $\{x, x_{n_0}\} \subset \tilde{P} \subset U$.*

(5) *P is called S-I-wcs$'$-network of X if whenever $\{x_n\}_{n\in\mathbb{N}}$ is a sequence in X S-I-covering to a point $x \in U$ with U semi-open in X, then there is $\tilde{P} \in P$ and some $n_0 \in \mathbb{N}$ such that $\{x_{n_0}\} \subset \tilde{P} \subset U$.*

By Definition 4.1, we have the following diagram:

S-I-cs-*networks* $\Longrightarrow$ S-I-cs'-*networks* $\Longrightarrow$ S-I-wcs'-*networks* $\Longrightarrow$ S-*networks*.

Proposition 4.1. *Every S-network is a network.*

Proof. Let P be a S-network and U be an open subset of X containing x. Then, U is semi-open in X. Since P is a S-network for X, there is some $P \in \mathcal{P}$ such that $x \in P \subset U$. Thus $\mathcal{P}$ is a network for X. □

Definition 4.2. *Let J be a maximal ideal of $\mathbb{N}$ and X be a topological space. U is said to be S-J-sn-cover of X, if $\{(\tilde{U})_{J_S} : \tilde{U} \in U\}$ is a cover of X.*

Theorem 4.1. *Each S-I-cs-network is preserved by a S-I-covering mapping.*

Proof. Let $f : X \to Y$ be a S-I-covering mapping and P be a S-I-cs-network of X. Now, suppose that $\{y_n\}_{n\in\mathbb{N}}$ is a sequence S-I-converging to a point $y \in U$ with U semi-open in Y. Since f is a S-I-covering mapping, there exist a sequence of points $x_n \in f^{-1}(y_n)$ for all $n \in \mathbb{N}$ and $x \in f^{-1}(y)$ such that $x_n \xrightarrow{SI} x$. Since P is a S-I-cs-network of X, there is some $\tilde{P} \in P$ such that $\{x_n\}_{n\in\mathbb{N}}$ is S-I-eventually in $\tilde{P}$ and $x \in \tilde{P} \subset f^{-1}(U)$. Thus there is $E \in I$ such that $\{n \in \mathbb{N} : x_n \notin \tilde{P}\} \subset E$. Note that $\{n \in \mathbb{N} : y_n \notin f(\tilde{P})\} \subset \{n \in \mathbb{N} : x_n \notin \tilde{P}\} \subset E$, hence $y_n \in f(\tilde{P})$ for all $n \in \mathbb{N} - E$, i.e. $\{y_n\}_{n\in\mathbb{N}}$ is S-I-eventually in $f(\tilde{P})$ and $y \in f(\tilde{P}) \subset U$. This means that $f(P) = \{f(\tilde{P}) : \tilde{P} \in P\}$ is a S-I-cs-network of Y. □

Corollary 4.1. *Each S-I-cs$'$-network is preserved by a S-I-covering mapping.*

Proof. Proof follows from Theorem 4.1. □

Theorem 4.2. *Each S-I-wcs$'$-network is preserved by a S-I-quotient mapping.*

Proof. Let $f : X \to Y$ be a S-I-quotient mapping and P be a S-I-wcs'-network of X. Now, suppose that $\{y_n\}_{n\in\mathbb{N}}$ is a sequence S-I-converging to a point $y \in U$ with U semi-open in Y. Since f is a S-I-quotient mapping, there exist a sequence $\{x_i\}_{i\in\mathbb{N}}$ in X such that $\{x_i\}_{i\in\mathbb{N}} \subset f^{-1}(\{y_n\}_{n\in\mathbb{N}})$ and $x_i \xrightarrow{SI} x \in f^{-1}(\{y_n\}_{n\in\mathbb{N}})$. And because P is a S-I-wcs'-network of X, there is some $\tilde{P}_0 \in P$ and $i_0 \in \mathbb{N}$ such that $\{x_{i_0}\} \subset \tilde{P}_0 \subset f^{-1}(U)$. And hence $\{f(x_{i_0})\} \subset \{y_{n_0}\} \subset f(\tilde{P}_0) \subset U$ for some $n_0 \in \mathbb{N}$. This implies that $f(P) = \{f(\tilde{P}) : \tilde{P} \in P\}$ is a S-I-wcs'-network of Y. □

Lemma 4.1. *Let J be maximal ideal of $\mathbb{N}$ and P be a family of subsets of X. Then, P is a S-J-cs$'$-network of X if and only if, whenever U is a semi-open set containing x, $\bigcup\{\tilde{P} \in P : x \in \tilde{P} \subset U\}$ is a S-J-sequential neighbourhood of x.*

Proof. Necessity: Let U be a semi-open set containing x. If $\bigcup\{\tilde{P} \in P : x \in \tilde{P} \subset U\}$ is not a S-J-sequential neighbourhood of x, then there is a sequence $\{x_n\}_{n\in\mathbb{N}}$ such that $x_n \xrightarrow{SJ} x$ and $x_n \notin \bigcup\{\tilde{P} \in P : x \in \tilde{P} \subset U\}$. Since P is a S-J-cs'-network of X, there is $\tilde{P}_0 \in P$ and $n_0 \in \mathbb{N}$ such that $\{x, x_0\} \subset \tilde{P}_0 \subset U$, which is a contradiction.

Sufficiency: Suppose that$x_n \xrightarrow{SJ} x \in U \in \tau_X$ and $\bigcup\{\tilde{P} \in P : x \in \tilde{P} \subset U\}$ is a S-J-sequential neighbourhood of x. Then $\{x_n\}_{n\in\mathbb{N}}$ is S-J-eventually in $\bigcup\{\tilde{P} \in P : x \in \tilde{P} \subset U\}$

. Hence there exists $n_0 \in \mathbb{N}$ such that $x_{n_0} \in \bigcup\{\tilde{P} \in P : x \in \tilde{P} \subset U\}$. And hence there is $\tilde{P}_0 \in P$ such that $x_{n_0} \in \tilde{P}_0$ and $x \in \tilde{P}_0 \subset U$. Thus $\{x, x_0\} \subset \tilde{P}_0 \subset U$. This means that P is a S-J-cs'-network of X. □

Theorem 4.3. *Let J be a maximal ideal of $\mathbb{N}$ and a space X be of a point-countable S-J-cs'-network. Then each open cover of X has a point-countable S-J-sn refinement.*

Proof. Suppose that P is a point-countable S-J-cs'-network for a space X. Now, let $U = \{\tilde{U}_\alpha\}_{\alpha<\gamma}$ be a semi-open cover of X, where γ is an ordinal. For each $\alpha < \gamma$, put

$$V_\alpha = \bigcup\{\tilde{P} \in P : \tilde{P} \subset \tilde{U}_\alpha, \tilde{P} \nsubseteq \tilde{U}_\beta \ if \ \beta < \alpha\}.$$

It is clear that $V_\alpha \subset \tilde{U}_\alpha$. Next we shall show that the family $\mathcal{V} = \{V_\alpha\}_{\alpha<\gamma}$ is a point-countable S-J-sn-cover of X. For each $x \in X$, let $\alpha(x) = \min\{\alpha < \gamma : x \in \tilde{U}_\alpha\}$. Then $x \in \tilde{U}_{\alpha(x)}$ and

$$\bigcup\{\tilde{P} \in P : x \in \tilde{P} \subset \tilde{U}_{\alpha(x)}\} \subset \bigcup\{\tilde{P} \in P : \tilde{P} \subset \tilde{U}_\alpha, \tilde{P} \nsubseteq \tilde{U}_\beta \ if \ \beta < \alpha(x)\}.$$

Since P is an S-J-cs'-network for a space X, it follows from Lemma 4.1 that

$$\begin{aligned} x &\in (\bigcup\{\tilde{P} \in P : x \in \tilde{P} \subset \tilde{U}_{\alpha(x)}\})_{J_S} \\ &\subset (\bigcup\{\tilde{P} \in P : \tilde{P} \subset \tilde{U}_\alpha, \tilde{P} \nsubseteq \tilde{U}_\beta \ if \ \beta < \alpha(x)\})_{J_S} \\ &= (V_{\alpha(x)})_{J_S}. \end{aligned}$$

This means that $\mathcal{V} = \{V_\alpha\}_{\alpha<\gamma}$ is a S-J-sn-cover of X.

We claim that $\mathcal{V}$ is point-countable. Consider, to the contrary i.e., there exists a point $x \in X$ and an uncountable subset Ω of γ such that $x \in V_\alpha$ for each $\alpha \in \Omega$. Therefore, there is $P_\alpha \in \mathcal{P}$ such that $x \in P_\alpha \subset U_\alpha$ and $P_\alpha \nsubseteq U_\beta$ for $\beta < \alpha$. Since P is a point-countable family and Ω is an uncountable set, there are $\alpha, \beta \in \Omega$ with $\alpha \neq \beta$ such that $P_\alpha = P_\beta$. Suppose that $\beta < \alpha$, then $U_\beta \supseteq P_\beta = P_\alpha \nsubseteq U_\beta$, which is a contradiction. □

Definition 4.3. [2] *A space X is called S-I-Frechet-Urysohn (or shortly S-I-FU space) if for each $A \subset X$ and each $x \in Cl(A)$, there exists a sequence in A S-I-converging to the point x in X. Here, $Cl(A)$ denotes closure of A.*

Definition 4.4. [11] *A space X is called a meta-Lindelof space if each open cover of X has a point-countable open refinement.*

Corollary 4.2. *Let J be a maximal ideal of $\mathbb{N}$. If X is a S-J-FU space with a point-countable S-J-cs'-network, then X is a meta-Lindelof space.*

Proof. X is a S-J-FU if and only if $Cl(A) = [A]_{J_S}$, for each $A \subset X$ if and only if $Int(A) = (A)_{J_S}$ for each $A \subset X$. □

Remark 4.1. *In the last corollary, $Int(A)$ denotes interior of A.*

Theorem 4.4. *Let J be a maximal ideal of $\mathbb{N}$. The following are equivalent for a space X.*

1. *S-J-sX is a S-J-Frechet-Urysohn space.*
2. *$Cl_{S\text{-}J\text{-}sX}(A) = [A]_{J_S}$, for each $A \subset X$.*
3. *$[A]_{J_S}$ is S-J-closed in X, for each $A \subset X$.*
4. *$(A)_{J_S}$ is S-I-open in X, for each $A \subset X$.*

Proof. Since the spaces X and S-J-sX have the same S-J-convergent sequences, by the Definitions 4.3 and 4.4; and Proposition 3.2, it follows that (1) $\Leftrightarrow$ (2) and (3) $\Leftrightarrow$ (4). Hence, it suffices to show that (2) $\Leftrightarrow$ (3). If $Cl_{S\text{-}J\text{-}sX}(A) = [A]_{J_S}$, then $[A]_{J_S}$ is closed in S-J-sX, and hence $[A]_{J_S}$ is S-J-closed in X, for each $A \subset X$. On the other hand, if $[A]_{J_S}$ is S-J-closed in X, then $[A]_{J_S}$ is closed in S-J-sX, and further $Cl_{semi\text{-}J\text{-}sX}(A) = [A]_{J_S}$, for each $A \subset X$. □

5. Conclusion

In this paper, we have introduced the notion of S-I-quotient mappings, S-I-cs-networks and some of their applications. For future works, these properties could be studied by applying the concept of P-I-convergence and b-I-convergence, and see if these properties are satisfied, and even find relationships between them.

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