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MODELING INTERNET ACCESS AT HOME BY FRACTIONAL CALCULUS AND A CORRELATION ANALYSIS WITH HUMAN DEVELOPMENT INDEX

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Abstract. In this study, we modeled broadband internet access at home data and investigated the relationship between the human development index and internet access in countries. The broadband Internet access at home is modeled using two different Fractional methods and the conventional Polynomial approach for seven countries, including France, Germany, Italy, Mexico, Spain, Turkey, and the UK. The results showed the superiority of Fractional Model-3 over the Fractional Model-1 and Polynomial Method for modeling internet access at home. After analyzing the modeling performance of the above models, Pearson correlations between the human development index, an indicator of human development published by UNDP, and internet access are analyzed. Computed correlations showed that the internet access percentage and the human development index are positively correlated.

Keywords: digital divide, fractional calculus, HDI, internet access, mathematical modeling.

AMS Subject Classification: 83-02, 99A00.

1. Introduction

Developments in Information and communication technologies (ICTs) and increasing Internet usage are important structural factors in modern society. Internet access has a strong influence in the developing world. It has become an indispensable part of our

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daily lives because of its place in expressing opinions, accessing information, and other fundamental human rights. The issue of internet access has been included even in countries' human rights discussions, and the Internet's significance has been declared in United Nations (UN) report [1]. However, not all countries or individuals can access the Internet. Providing internet access to users is executed by Internet Service Providers (ISPs) using fiber optics, copper wire, satellite, and other forms. ISPs offer connectivity and transfer data with an extensive range via different network technologies. Accessing the Internet is significant for communication technologies and information access. Countries have varying levels of internet access percentage and usage. "Digital divide" is a term that represents inequality in accessing/using the Internet and technologies and can be observed in a country or between countries. The global digital divide is the digital divide between countries.

The Human Development Index (HDI) is a summary measuring key elements of human development such as long and healthy life, being knowledgeable, and having a decent standard of living [2]. It has been published annually by the United Nations Development Programme (UNDP) since 1990 and has been covering 177 countries since 2006. The intellectual history and development of HDI are reported in [2] where HDI is examined with the theory of human development and a chronicle of the past, and present measures of social welfare used in the field of economics and development, including national income, are provided. The HDI emphasizes the development of a country with economic growth, people, and capabilities. Therefore, the HDI improvement should be the ultimate goal of the corresponding countries [3].

The effect of improving ICT in developed and developing countries has been the object of interest. Therefore, the relationship between the HDI and internet penetration, access, and usage have been the subject of scientific research. In literature, the relationship between internet access and the level of development of several countries has been investigated [4, 5, 6, 7].

In [8], the relation of Internet penetration rate with human development level between 2000 and 2010 was investigated. The growth of the Internet penetration rate in developed countries was compared with developing countries. In [9], the influence of internet usage and innovation on human development in 15 Economic Community of West African States (ECOWAS) countries using the fixed and random effects panel data techniques were examined. The study finds that Internet usage, innovation, and interaction have a significant and positive relationship with human development. Then, the relationship between ICT, economic growth, and the human development index (HDI) considering urbanization, foreign direct investment (FDI), and trade for the period from 1990 to 2014 was debated [10]. In this study, autoregressive distributed lag (ARDL) and vector error correction model (VECM) approaches have been used to analyze the data. The empirical results found that ICT promotes the human development index.

Internet penetration is related to a country's technological advancements, which can indicate human development, and technological capabilities discriminate between developed and underdeveloped countries. The significance of the relationship between the human development index and internet access and the success of Fractional Calculus for modeling the dynamics of time-series data are the foundations of our motivation in this work.

This paper focuses on modeling the internet access percentage and investigating the correlational relationship between HDI and internet access. Precisely, three mathematical approaches using Fractional Calculus and Polynomial Models are utilized for modeling internet access percentages of France, Germany, Italy, Mexico, Spain, Turkey, and the UK.

The Internet access percentages are acquired from the ICT Access and Usage by Households and Individuals dataset of OECD. Results showed that both Fractional methods are superior to the polynomial approach for this dataset. After analyzing the performances of Fractional and Polynomial methods, an analysis of the relationship between internet access and the human development index is made by computing Pearson correlation for various years. For each calculated year, the relationship between human development and internet access of countries is found to be strongly positive.

The structure of this paper is as follows. Section 2 provides a theoretical background for Fractional Model-1 and Fractional Model-3. Experiments and analyses are carried out in Section 3. Lastly, Section 4 is devoted to the conclusion.

2. Approach

In this section, the mathematical formulation of the problem is presented. Two different approaches with fractional calculus are expressed in detail. The fractional models 1 and 3 are presented in Sections 2.1 and 2.2, respectively. The history of non-integer order integrals and derivatives is ancient as early as differential calculus. The fractional calculation is widely used in engineering and science [11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23]. In calculus, the n-th order derivative of a function requires derivation of the function n times. However, this is applicable only for positive integers. For the derivation of a function with positive integer order, the output only depends on the given input. This property is called the locality. In the fractional derivative D_x^α, there is a possibility to have a fractional-order α which can be a non-integer value. This brings non-locality and hereditary properties. The result of the equation does not only depend on the current input. This leads to memory property, which is essential on many financial, biological, and engineering systems to model, analyze the dataset in which the present values are expected to correlate with previous ones.

2.1. **Theory on Concept of Fractional Model.**

The present study employs Fractional Calculus to model ICT Access as proposed in [11, 12]. In this section, mathematical foundations of Fractional Model-1 are given. Before going into details of modeling, it should be highlighted that, throughout the present study, Caputo's definition of the fractional derivative is employed [18] which is used to determine fractional derivative. The formula of the fractional derivative is as follows:

$$\mathfrak{D}_x^\alpha f(x) = \frac{d^\alpha f(x)}{dx^\alpha} = \frac{1}{\Gamma(1-\alpha)} \int_0^x \frac{f^{(1)}(t)dt}{(x-t)^\alpha}. \tag{1}$$

Here, $f^{'}$ stands for the first derivative, and α can take values between 0 and 1. In (1), $\Gamma(1-\alpha)$ stands for the Gamma function. The reason why Caputo's type definition is used is that the Laplace transform for the fractional derivative is simpler [11, 12, 18].

In Equation 2, the fractional derivative of $f(x)$ in the order of α with respect to x is equated to the expression inspired by the Taylor expansion and the first derivative of the function.

$$\mathcal{D}_x^\alpha f(x) = \frac{d^\alpha f(x)}{dx^\alpha} = \sum_{n=1}^{\infty} a_n n x^{n-1}. \tag{2}$$

Note that,we aim to model a discrete dataset with a continuous curve with a minimum error. Therefore, $f(x)$ stands for households with broadband Internet access at home percentages data of the 7 countries concerning years between 2005 to 2018 in the present study. To solve Equation 2, the following steps should be tracked. First, the differential equation is reduced to an algebraic equation. Then, the unknown and required function $f(x)$ is kept alone. To follow such a procedure, the Laplace Transform of $\mathcal{L}$ is taken and the Laplace Transform table is used. F(s) represents the Laplace Transformed version of $f(x)$. As mentioned before, α is ranging in the interval of [0, 1] [12, 18].

$$\mathcal{L}\{\mathrm{f(x)}\} = \mathrm{F(s)} = \frac{f(0)}{s} + \sum_{n=1}^{\infty} \frac{a_n}{s^{\alpha+n}} \Gamma(\mathrm{n}+1). \tag{3}$$

After taking Laplace Transform, an algebraic Equation (4) is obtained. Then, the function can be separated from the other expression. After that, by Inverse Laplace Transform, f(x) is obtained. Note that, in Equation (4), $\mathcal{L}^{-1}$ indicates inverse Laplace Transform.

$$\mathcal{L}^{-1}\{\mathrm{F(s)}\} = f(\mathrm{x}) = f(0) + \sum_{n=1}^{\infty} \frac{a_n \Gamma(\mathrm{n}+1) x^{\alpha+n-1}}{\Gamma(\alpha+\mathrm{n})}. \tag{4}$$

As noticed in (4), a_n, $f(0)$ are unknown coefficients which need to be determined. To find these coefficients and to represent a discrete dataset as a continuous curve with a minimum error, the least-squares method is employed after this point. Before passing through the expression related to the least-squares method, truncation in (4) is needed for the numerical calculation of $f(x)$. Then, in (5), one can have the approximated version of $f(x)$.

$$f(x) \cong f(0) + \sum_{n=1}^{N} \frac{a_n \Gamma(n+1) x^{\alpha+n-1}}{\Gamma(\alpha+n)}. \tag{5}$$

It is time to express the dataset. P_K values represent the value of households with broadband Internet access at home percentages in the dataset for given years and p_0, $p_1 \ldots p_K$ are the specific values of each country at a specific time.

$$P_K = \begin{bmatrix} p_0 & p_1 & \ldots p_i \ldots & p_K \end{bmatrix},$$
$$x_K = \begin{bmatrix} x_0 & x_1 & \ldots x_i \ldots & x_K \end{bmatrix}.$$

Here, x_i corresponds time for those countries' data and. The error ϵ_i is calculated as follows,

$$(\epsilon_i)^2 = (p_i - f(x_i))^2. \tag{6}$$

Here, $i = 0, 1, \ldots K$.

As seen in Equation 6, the square of differences between values of p_i (actual value) and $f(x_i)$ (proposed value) gives the square of error. Our aim is to minimize the total error contributing to each data point. The total error is defined as Equation 7.

$$\epsilon_T^2 = \sum_{i=0}^{K} \left[\mathrm{p}_i - \left\{ f(0) + \sum_{n=1}^{N} \frac{a_n \Gamma(\mathrm{n}+1) x_i^{\alpha+n-1}}{\Gamma(\alpha+\mathrm{n})} \right\} \right]^2. \tag{7}$$

In the light of Equation 7, to find a minimized total error, the derivative of ϵ_T^2 with respect to unknown coefficients ($f(0)$ and a_n) is equated to 0 as in Equation 8.

$$\frac{\partial \epsilon_T^2}{\partial f(0)} = 0, \frac{\partial \epsilon_T^2}{\partial a_1} = 0, \frac{\partial \epsilon_T^2}{\partial a_2} = 0, \ldots \frac{\partial \epsilon_T^2}{\partial a_N} = 0. \tag{8}$$

Then, a system of Linear Algebraic Equations is obtained after implementing (8) as given below:

$$[A]_{N+1xN+1}[\Omega]_{N+1x1} = [B]_{N+1x1}. \tag{9}$$

Here,

$$A=\begin{bmatrix} k+1 & \frac{1}{\Gamma(\alpha+1)}\sum_{i=0}^{K}x_i^{\alpha} & \cdots & \frac{n!}{\Gamma(\alpha+n)}\sum_{i=0}^{K}x_i^{\alpha+n-1} \\ \sum_{i=0}^{K}x_i^{\alpha} & \frac{1}{\Gamma(\alpha+1)}\sum_{i=0}^{K}x_i^{2\alpha} & \cdots & \frac{n!}{\Gamma(\alpha+n)}\sum_{i=0}^{K}x_i^{2\alpha+n-1} \\ \vdots & \vdots & \cdots & \vdots \\ \sum_{i=0}^{K}x_i^{\alpha+n-1} & \frac{1}{\Gamma(\alpha+1)}\sum_{i=0}^{K}x_i^{2\alpha+n-1} & \cdots & \frac{n!}{\Gamma(\alpha+n)}\sum_{i=0}^{K}x_i^{2(\alpha+n-1)} \end{bmatrix}, \tag{10}$$

$$[\Omega]=\begin{bmatrix} f(0) & a_1 & a_2 & \ldots & a_N \end{bmatrix}^T, \tag{11}$$

and

$$[B]=\left[\sum_{i=0}^{K}\mathrm{P}_i\sum_{i=0}^{K}\mathrm{P}_i x_i^{\alpha}\sum_{i=0}^{K}\mathrm{P}_i x_i^{\alpha+1}\ldots\sum_{i=0}^{K}\mathrm{P}_i x_i^{\alpha+N-1}\right]^T. \tag{12}$$

Note that, the transpose of the matrix is shown as T After inversion of the system of algebraic equation, the unknown coefficients (represented by Ω vector) can be found.

$$[\Omega]_{N+1x1}=[\mathrm{A}]^{-1}_{N+1xN+1}[B]_{N+1x1}. \tag{13}$$

Here, the inverse matrix of A is indicated with $[A]^{-1}$.

2.2. **Theory on the Concept of Fractional Model-3.** In this section, the Fractional Model-3 is explained in detail [24]. Similar to Fractional Model-1, the fractional derivative of $f(x)$ function representing the modeled data is expressed as (14). The assumption is deduced from the modified version of the Taylor Expansion and the first derivative of the function.

$$\mathcal{D}_x^{\alpha}f(x)=\frac{d^{\alpha}f(x)}{dx^{\alpha}}=\sum_{n=1}^{\infty}a_n(n\alpha)x^{n\alpha-1}. \tag{14}$$

To solve the differential equation above, again, the Laplace Transform ($\mathcal{L}$) of the corresponding equation is taken. Then, the differential equation is converted into algebraic equation as given in (15) [11, 12, 24].

$$\mathcal{L}\{\mathrm{f(x)}\}=F(s)=\frac{f(0)}{s}+\sum_{n=1}^{\infty}\frac{a_n(n\alpha)}{s^{\alpha n}}\Gamma(n\alpha). \tag{15}$$

Then, by inversion of Laplace Transform, the function f(x) is obtained as a compact mathematical form. To proceed further in computation, the infinite summation is truncated to N.

$$\mathcal{L}^{-1}\{F(s)\}=f(x)\cong f(0)+\sum_{n=1}^{N}\frac{a_n\Gamma(n\alpha+1)x^{\alpha(n+1)-1}}{\Gamma(\alpha(n+1))}. \tag{16}$$

By (16), $f(x)$ function can be determined if one can obtain the unknown coefficients such as $f(0)$ and a_n. To acquire them, the least-squares method is employed as it has been done previously.

$$P_i=[p_0 \quad p_1\ldots p_K],$$
$$x_i=[x_0 \quad x_1\ldots x_K].$$

Again, P_i values represent the value of households with broadband Internet access at home percentages in the dataset for given years and p_0, p_1... p_K are the specific values of each country at a specific time. Besides, The x_i corresponds time for those countries' data for specific time values. The error ϵ_i is calculated as follows:

$$(\epsilon_i)^2=(p_i-f(x_i))^2. \tag{17}$$

Then, the square of the total error resulting from each data point is calculated as (18).

$$\epsilon_T^2 = \sum_{i=0}^{K} \left[p_i - \left\{ f(0) + \sum_{n=1}^{N} \frac{a_n \Gamma(n+1) x_i^{\alpha+n-1}}{\Gamma(\alpha+n)} \right\} \right]^2 . \tag{18}$$

In order to obtain an SLAE, the least-squares method is employed as given in (19).

$$\frac{\partial \epsilon_T^2}{\partial f(0)} = 0, \frac{\partial \epsilon_T^2}{\partial a_1} = 0, \frac{\partial \epsilon_T^2}{\partial a_2} = 0, \ldots \frac{\partial \epsilon_T^2}{\partial a_N} = 0. \tag{19}$$

Then, SLAE becomes as:

$$[A]_{N+1\times N+1} [\Omega]_{N+1\times 1} = [B]_{N+1\times 1}. \tag{20}$$

where,

$$A = \begin{bmatrix} k+1 & \sum_{i=1}^{k} c_1(x_i) & \sum_{i=1}^{k} c_2(x_i) & \ldots & \sum_{i=1}^{k} c_N(x_i) \\ \sum c_1(x_i) & \sum c_1(x_i) c_1(x_i) & \sum c_1(x_i) c_2(x_i) & \ldots & \sum c_1(x_i) c_N(x_i) \\ \sum c_2(x_i) & \sum c_2(x_i) c_1(x_i) & \sum c_2(x_i) c_2(x_i) & \ldots & \sum c_2(x_i) c_N(x_i) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \sum c_N(x_i) & \sum c_N(x_i) c_1(x_i) & \sum c_N(x_i) c_2(x_i) & \ldots & \sum_{i=1}^{k} c_N(x_i) c_N(x_i) \end{bmatrix}, \tag{21}$$

$$[\Omega] = \begin{bmatrix} f(0) & a_1 & a_2 & \ldots & a_n & \ldots & a_N \end{bmatrix}^T, \tag{22}$$

$$[\mathrm{B}] = \left[\sum_{i=0}^{K} P_i \sum_{i=0}^{K} P_i x_i^{\alpha} \sum_{i=0}^{K} P_i x_i^{\alpha+1} \quad \ldots \sum_{i=0}^{K} P_i x_i^{\alpha+N-1} \right]^T . \tag{23}$$

Here,

$$C_n(x, \alpha) = \frac{\Gamma(n\alpha+1)}{\Gamma(\alpha(n+1))} x^{\alpha(n+1)-1}.$$

After obtaining the unknown coefficients, $f(x)$ can be found by employing both models. In the following section, this mathematical background is utilized and the comparison with the actual dataset and the proposed modeling function $f(x)$ is explained in detail.

3. Experimental Results

This section provides modeling results and Pearson correlations between HDI scores and internet usage. For this purpose, OECD Dataset: ICT Access and Usage by Households and Individuals are used. For modeling the data, seven countries are chosen as Germany, France, Italy, Mexico, Spain, Turkey, and United Kingdom. The selected countries support internet access reliably, are demographically representative, and are present in G-20. According to the WESP[3], they are either high-income or upper-middle-income. The indicator is determined as households with broadband Internet access at home in percentage. The time interval for this data set covers the years between 2005 and 2018. There are several missing data for the following counties and years such as France (2005), Turkey (2006 and 2011), and the UK (2010). These values are explicitly found with modeling the dataset by Fractional Model-1 for N=15. After the modeling, the missing values are inserted into the dataset. In the following section, the results are reported with Mean Absolute Percentage Error (MAPE) which calculated as follows:

$$MAPE = \frac{1}{k}\sum_{i=1}^{k}\left|\frac{v(i)-\tilde{v}(i)}{v(i)}\right| \times 100 \tag{24}$$

Where $v(i)$ is the original value of data, and $\tilde{v}(i)$ indicates the modeled value of data.

By using MAPE values, the average value of the total MAPE (AMAPE) is evaluated regarding the formula given in Equation 12.

The missing values of countries in the OECD dataset are modeled by using Fractional Model-1 with exponent N=15. MAPE results of France, Germany, Italy, Spain, Turkey, and the UK are observed as 0.2526, 0.2443, 0.5348, 0.0821, 1.0393, and 0.1084, respectively. Since the error rates are close enough to 0, the results of this model are accepted as original data for missing data. Those missing values for corresponding years and countries are inserted into the dataset.

3.1. **Modeling the Internet Access at Home Percentage.** As aforementioned, one of the goals of this paper is to compare Polynomial Model and Fractional Models in the context of internet access modeling. For this purpose, experiments are carried out using two different exponent values.

Table 1 illustrates the modeling results of Fractional Model-1 (FM-1), Fractional Model-3 (FM-3), and Polynomial Model (Polynom.) for 2 different values of N. Modeling internet access at home percentage using Fractional Model-1, 0.9901 Fractional Model-3, and polynomial yields, 2.4364%, 2.2161%, and 3.0706%, respectively. The smallest errors are obtained for the UK and the largest errors are obtained for Mexico with both Fractional models and the Polynomial model. As expected, increasing the exponent value decreased the error rates. AMAPE of Fractional Model-1 is smaller than Polynomial Model by 0.531 %. When N is set to 10, the AMAPE of Fractional Model-1 and Fractional Model-3 are 1.111% and 0.9901%, respectively. For the same N value, AMAPE is 1.337% in Polynomial Model. Increasing N results in 1.4281 % average MAPE improvement for Fractional Model-1, 1.226% improvement for Fractional Model-3, and 1.7340% improvement for Polynomial model. Increasing N to 10 from 5 has a greater impact on Polynomial Model compared to the Fractional approaches. The table illustrates that both Fractional models outperform the polynomial method for both exponent values. The best-modeled country is Spain whilst the largest error rates are obtained for Mexico. Results show that both Fractional approaches are superior to the Polynomial Method and the best results are obtained by using Fractional Model-3 for both exponent values.

When fractional-order α is optimized as 1, the Polynomial and the Fractional Model are equivalent. In such cases, the same results are obtained for both Polynomial Model and Fractional Models. Fractional-order α values are determined by grid search for the minimum MAPE values. By applying Equation 12 for each exponent N, AMAPE results are compared. In Table 1, fractional-order α is found to be 1 for France when N=10 and Turkey when N=5. As seen from the table, when α is 1, polynomial and fractional models result in the same error rate.

Modeling curves of the households with broadband Internet access at home (%) of France, Germany, Italy, Mexico, Spain, Turkey, and the UK using Fractional Model-1 and Fractional Model-3 where N=5 and N=10 are illustrated in Figure 1 and 2, respectively. The MAPE results from Table 1 show that Fractional Model-3 provides better modeling performance for the original data. As seen from the figures, when exponent N is higher, the models fit better into the dataset. The modeling illustrated in Figure 1 does not

Table 1. Households with broadband Internet access at home (%).

N	Models	Results	France	Germany	Italy	Mexico	Spain	Turkey	UK
5	FM-1	MAPE	2.9789	1.8023	2.8079	5.1219	1.0212	3.1102	0.9324
		α value	0.01	0.01	0.01	0.01	0.01	1	0.36
	FM-3	MAPE	1.8666	1.5247	2.7307	5.0772	0.7782	2.5890	0.9464
		α value	0.02	0.25	0.50	0.63	0.02	0.11	0.85
	Polynom.	MAPE	3.1645	2.0324	3.1878	7.9775	1.0389	3.1102	0.9835
10	FM-1	MAPE	0.7813	0.6338	1.5849	2.1578	0.2583	1.8548	0.5061
		α-value	1	0.01	0.01	0.13	0.01	0.94	0.01
	FM-3	MAPE	0.7813	0.4157	1.4130	2.1641	0.2358	1.5641	0.3569
		α-value	1	0.58	0.58	0.86	0.64	0.49	0.22
	Polynom.	MAPE	0.7813	1.0221	2.2037	2.5329	0.3540	1.8548	0.6072

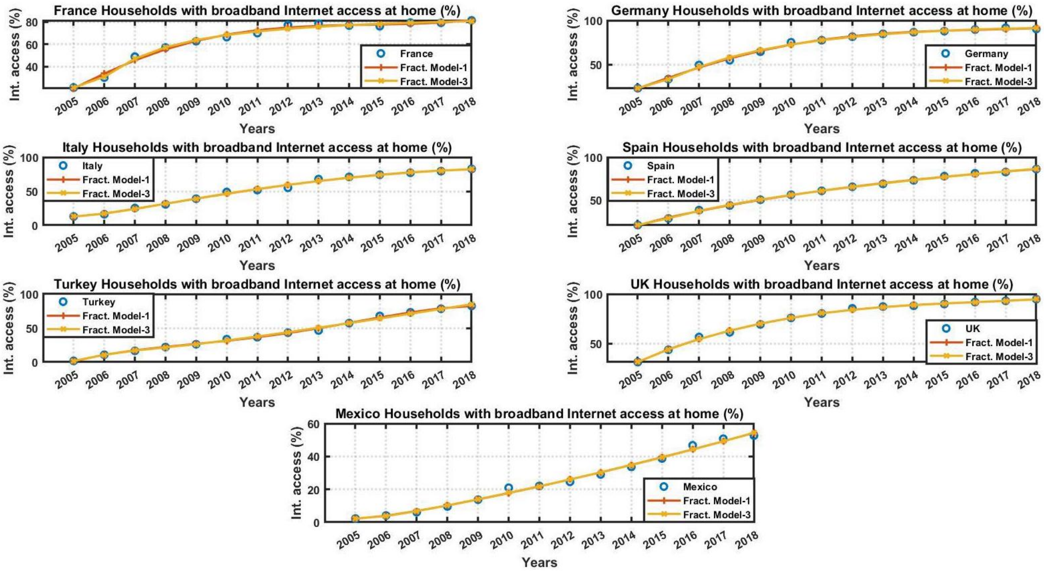

Figure 1. Modeling of the countries using Fractional model for N=5.

capture variations in the dataset as much as in Figure 1. Also, when the exponent N is 5, the difference between the modeling performance of Fractional Model-1 and Fractional Model-3 is more visible. Especially for France and Germany, the Fractional Model-3 is superior in constructing the data compared to Model-1. Among all the countries, Spain has the smoothest data curve, while France and Germany have the most fluctuated ones.

Figure 1 illustrates the modeling of the households with broadband Internet access at home (%) of France, Germany, Italy, Mexico, Spain, Turkey, and the UK with Fractional Model-1 and Fractional Model-2 where N=5. The largest households with broadband Internet access at home percentage value is observed for the UK in 2005. The second-largest Internet access at home percentage is observed in Germany. The percentages of Germany and Spain are very close, but Spain has smaller access for that year. By using Fractional Model-1 for exponent 15, the modeled value of France in 2005 is inserted into the dataset as 21.2941. The predicted households with broadband Internet access at home percentage value of France in 2005 is seen between Germany and Spain. The fifth-largest

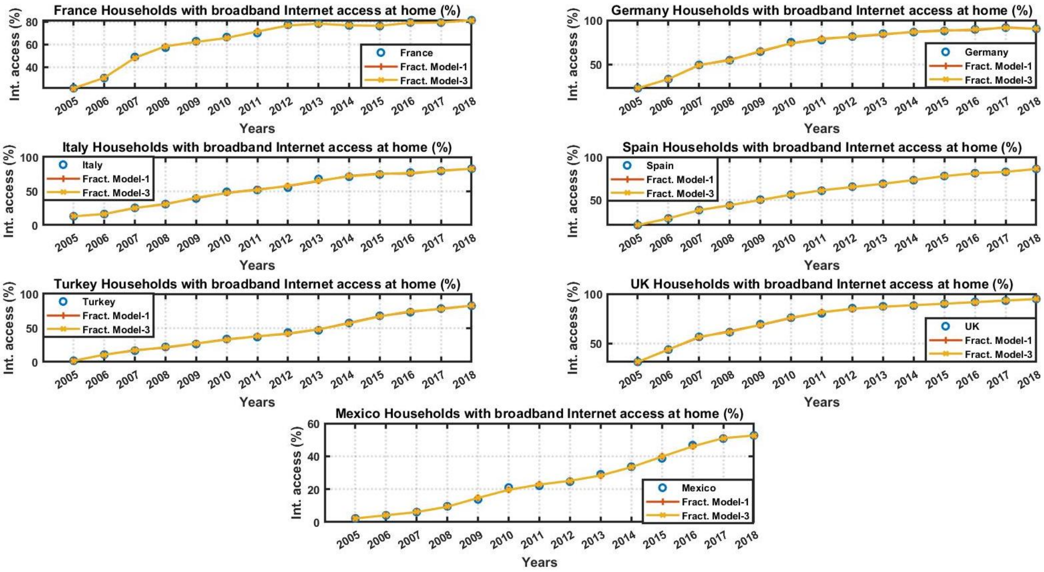

Figure 2. Modeling of the countries using Fractional model for N=10.

percentage is observed for Italy and the sixth-largest percentage is observed for Mexico. The last country is Turkey with a small amount of broadband Internet access at home percentage in 2005.

In 2006, although the percentage orders of the first 5 countries are the same as the previous year, the rankings of Mexico and Turkey are different. The increments in the percentage values are observed for each country. As mentioned earlier, the percentage value of Turkey in 2006 is not provided in the dataset. By using Fractional Model-1 with N=15, the expected percentage value of Turkey is modeled as nearly 10.7472 for 2006. While the internet access percentages of 7 countries increased year by year, the rankings in the unit of the percentage of countries in 2006 are preserved starting from that year until 2016. The missing value of the UK in 2010 is modeled as nearly 75.7964 with exponent 15, and it is inserted into the dataset. Similarly, the value of Turkey is also obtained by using the same method and it is around 37.7741. In 2016, the value of Spain is larger than France's because of the stableness in France's Internet access percentage between 2015-2017 and the continuity of increment for Spain. In 2017, France dropped to 5^th order in the percentage of Internet access. For the last year in this dataset, while other countries have the same order as the previous year, the order of France is 6, and Turkey is in the 5^{th} order. Mexico constantly ranked 7 except in the year 2005.

3.2. **Analysis of relationship between Human Development Index and Internet Access at Home Percentage.**

This section investigates the rankings and developments of France, Germany, Italy, Mexico, Spain, Turkey, and the UK and their link to Internet Access at home. Development measures of countries are obtained from United Nations [25], World Economic Situation and Prospects (WESP) [2], and World Bank [26]. Rankings in Human Development Index Reports [27, 28, 29, 30, 31, 32, 33] of six countries are shown in Table 2. As seen in Table 2, the UK was the highest-ranking country among 7 countries according to Human Development Report 2005 (HDR). In Figure 1, the Internet access percentage of the UK is the largest, which is consistent with UK's HDI rank. Similarly, in 2005, the second-highest-ranking country was France. The third and fourth countries are

Italy and Germany, respectively. These orders are not fully consistent with the rankings of internet access at home percentage. For instance, the largest internet access is always observed in the UK as seen in Figure 1. However, in 2007, 2011, 2013, 2015, and 2019 Human Development Reports, the UK was not in the highest rank among those seven countries. For the year 2007, France is the highest-ranking country in the HDR index, and for the interval 2007-2008 in 2, France's rank decreased. Also, for 2011, 2013, and 2015, the rank of France is 2, 2, and 3 in HDR among these pre-selected countries, respectively. However, as illustrated in both Figure 1 and Figure 2, France is at position three between 2008-2015 for internet access at home. For Spain, the ranking results of HDR were 5, 2, 3, 3, 4, and 3 for 2005, 2007, 2011, 2013, 2015, and 2019, respectively. In Figure 1, the orders of Spain are 4, 4, 4, 4, 4, and 3 for 2005, 2007, 2011, 2013, 2015, and 2018, respectively. For Turkey, the HDR rankings are 7, 7, 7, 7, 6, 6 in 2005, 2007, 2011, 2013, 2015, and 2019. In Figure 1 and 2, the country is also modeled in the 7^th position in 2005. After that year, the ranking of Turkey is 6 until 2018 in the same figures. In 2018, the ranking of Turkey is 5. For the last country, Mexico, the HDR rankings among seven countries are 6, 6, 6, 6, 7, 7. However, in internet access at home percentage figures, Mexico is ranked as the last country except for the first year. The HDR rankings of these seven countries are not always directly proportional to the ranking of internet access at home percentages. A mathematical analysis tool is needed to understand the relationship between human development and internet access. Therefore, the relationship between the human development index and internet access at home percentage is investigated using the Pearson correlation coefficient, denoted as R.

TABLE 2. Human Development Indexes of Six Countries.

Country/HDR Ranking	2005	2007	2011	2013	2015	2019
France	16	10	20	20	22	26
Germany	20	22	9	5	6	4
Italy	18	20	24	25	27	29
Mexico	53	52	57	61	74	76
Spain	21	13	23	23	26	25
Turkey	94	84	92	90	72	59
UK	15	16	28	26	14	15

The correlation coefficient R takes values between [-1, 1] where the sign of the R indicates the direction of a relationship. Positive R means that if the rank of a country increases, the internet usage at the home percentage of that country also increases. Negative R means that if the human development index of a country increases, the internet usage at home (%) of that country decreases. In this section, the correlation between the HDI rankings and internet usage at home (%) amongst years of a country is investigated. Table 3 illustrates the Pearson correlation coefficients between the internet access percentage and human development indices amongst France, Germany, Italy, Mexico, Spain, Turkey, and the UK for the years reported in the first row. For all years, the human development index and internet access percentage are found positively correlated. These results show a positive correlation between the human development index and internet access from home percentage. The largest positive correlation is found for 2015 as 0.8616 where the smallest R is calculated as 0.7385 for the year 2007. The absolute average correlation R between the human development index and internet access at home (%) for a single year is found as 0.79632. Table 3 illustrates that the human development index maintains a positive affair with internet access at home (%) for the computed years of the dataset. Countries with higher human development indices have larger internet access at home (%).

After stating the purpose of the study, the mathematical background, and the experimental results including comparisons, the conclusion is drawn in the following section.

TABLE 3. Pearson correlation coefficient between internet access percentage and human development index amongst France, Germany, Italy, Mexico, Spain, Turkey, and the UK for years 2005, 2007, 2011, 2013 and, 2015.

Year	Correlation
2005	0.8183
2007	0.7385
2011	0.7541
2013	0.8091
2015	0.8616

4. CONCLUSION

In this study, Internet access at home percentages of seven countries (France, Germany, Italy, Mexico, Spain, Turkey, and the UK) are modeled, Fractional Calculus and Polynomial Models are compared and the relationship between the development of countries and internet usage percentage is analyzed using the OECD dataset, between years 2005 and 2018. Here, internet access percentage is modeled using our previously proposed modeling methods, Fractional Model-1 and Fractional-Model 3, and the Polynomial method. Later, by using the development index and rankings of United Nations, Human Development Reports, World Economic Situation and Prospects (WESP), and World Bank, the development of countries and their link to internet access from home percentages are examined by computing Pearson correlation, R. Modeling experiments are carried out with two different exponent values. For N=5, Fractional Model-1, Fractional Model-3, and Polynomial model yield, 2.4364%, 2.2161%, and 3.0706%, respectively. When N is set to 10, the AMAPE of Fractional Model-1, Fractional Model-3, and Polynomial model are 1.111% and 0.9901%, 1.337%, respectively. Results showed that both fractional models outperform the polynomial approach. As expected, increasing the exponent N decreased the error rates for both models. Increasing N to 10 from 5 has a greater impact on Polynomial Model compared to the Fractional approaches. For N=5 and N=10, Fractional Model-3 outperforms the Polynomial model by 0.8545% and 0.3469% errors, respectively. Fractional Model-3 is superior to Fractional Model-1 and Polynomial Model for both N values. The smallest error rates are obtained with Fractional Model-3 where N is 10. Here, the best-modeled country is Spain with 0.2358 MAPE and the largest MAPE is found for Mexico as 2.1641%. Further, the UK is ranked 1 on Internet access at home percentages every year while the lowest ranking country is Mexico except in the year 2005. Results show that the human development index maintains a strong positive affair with the internet access from home percentage.

REFERENCES

[1] La Rue, F., 2011, Report of the Special Rapporteur on the promotion and protection of the right to freedom of opinion and expression, United Nations, https://www2.ohchr.org/english/bodies/hrcouncil/docs/17session/A.HRC.17.27_en.pdf.

[2] Stanton, E. A., 2007, The human development index: A history, PERI Working Papers, 2007(127).

[3] United Nations, 2014, Country classification, World Economic Situation and Prospects, WESP, pp. 143-150, New York, USA.

[4] Wallsten, S., 2015, Regulation and internet use in developing countries, Economic Development and Cultural Change, 53(2), pp.501-523.

[5] Gong, W., Li, Z. G., & Stump, R. L., 2007, Global internet use and access: cultural considerations, Asia Pacific Journal of Marketing and Logistics, 19(1), pp. 57-74.

[6] Sambuli, N., 2016, Challenges and opportunities for advancing Internet access in developing countries while upholding net neutrality, Journal of Cyber Policy, 1(1), pp. 61-74.

[7] Stork, C., Calandro, E., & Gillwald, A., 2013, Internet going mobile: internet access and use in 11 African countries, Info, 15(5), pp. 34-51.

[8] Pratama, A. R. I., & Al-Shaikh, M., 2012, Relation and growth of internet penetration rate with human development level from 2000 to 2010. Communications of the IBIMA, 2012, p. 1.

[9] Ejemeyovwi, J. O., Osabuohien, E. S., Johnson, O. D., & Bowale, E. I., 2019, Internet usage, innovation and human development nexus in Africa: the case of ECOWAS. Journal of Economic Structures, 8(1), p. 15.

[10] Khan, N. H., Ju, Y., & Hassan, S. T., 2019, Investigating the determinants of human development index in Pakistan: an empirical analysis, Environmental Science and Pollution Research, 26(19), pp. 19294-19304.

[11] Önal, N.Ö., Karaçuha, K., Erdinç, G.H., Karaçuha, B.B., Karaçuha, E.,,2019, A Mathematical Approach with Fractional Calculus for the Modelling of Children's Physical Development, Comput. Math. Methods Med., 3081264.

[12] Karacuha, K., Sağlamol, S. A., Ergün, E., Tuğrul, N. Ö. Ö., Şimsek, K., & Karaçuha, E., 2022, Mathematical Modeling of European Countries' Telecommunication Investments, El-Cezeri, 9(3), pp. 1028-1037.

[13] Karaçuha, E., Tabatadze, V., Karacuha, K., Önal, N. Ö., & Ergün, E., 2020, Deep assessment methodology using fractional calculus on mathematical modeling and prediction of gross domestic product per capita of countries, Mathematics, 8(4), p. 633.

[14] Karaçuha, E., Önal, N. Ö., Ergün, E., Tabatadze, V., Alkaş, H., Karaçuha, K., Tontuş H. Ö., & Nu, N. V. N., 2020, Modeling and prediction of the Covid-19 cases with deep assessment methodology and fractional calculus, IEEE Access, 8, pp. 164012-164034.

[15] Önal, N. Ö., Karacuha, K., & Karacuha, E., 2020, A comparison of fractional and polynomial models: Modelling on number of subscribers in the Turkish mobile telecommunications market, International Journal on Applied Physics and Mathematics, 10(1), pp. 41-48.

[16] Önal, N. Ö., Karacuha, K., & Karacuha, E., Modelling on economic growth and telecommunication sector of Turkey using a fractional approach including error minimizing, In 3rd Asia-Pacific Conference on Applied Mathematics and Statistics, Feb 17-19,2020.

[17] Machado, J. T., Kiryakova, V., & Mainardi, F., 2011, Recent history of fractional calculus, Communications in nonlinear science and numerical simulation, 16(3), pp. 1140-1153.

[18] Samko, S. G., Kilbas A. A., & Marichev O. I., 1993, Fractional integrals and derivatives, theory and applications, Gordon and Breach Science Publishers, Switzerland; Philadelphia, Pa., USA.

[19] Axtell, M., & Bise, M. E., 1990, Fractional calculus application in control systems. In IEEE Conference on Aerospace and Electronics, pp. 563-566.

[20] Karaçuha, K., Tabatadze, V., & Veliev, E. I., 2020, Plane wave diffraction by strip with an integral boundary condition, Turkish Journal of Electrical Engineering and Computer Sciences, 28(3), pp. 1776-1790.

[21] E. Karaçuha, E. Ergün, N. Ö. Ö. Tuğrul, K. Karaçuha and V. Tabatadze, 2021, Analyzing Response Efficiency to COVID-19 and Underlying Factors of the Outbreak With Deep Assessment Methodology and Fractional Calculus, IEEE Access, 9, pp. 157812-157824.

[22] Alzabut, J., Tyagi, S., & Abbas, S., 2020, Discrete fractional-order BAM neural networks with leakage delay: existence and stability results, Asian Journal of Control, 22(1), pp. 143-155.

[23] Stamov, G. T., Alzabut, J. O., Atanasov, P., & Stamov, A. G., 2011, Almost periodic solutions for an impulsive delay model of price fluctuations in commodity markets, Nonlinear Analysis: Real World Applications, 12(6), pp. 3170-3176.

[24] Önal Tuğrul, N., Baser, C., Ergün, E., Karaçuha, K., Tabatadze, V., Eker, S., ... & Simsek, K., 2022, Modeling of mobile and fixed broadband subscriptions of countries with fractional calculus, Transport and Telecommunication Journal, 23(1), pp. 1-10.

[25] Conceição P. Human Development Report 2019, United Nations Development Programme, New York, NY, USA.
[26] Fantom, N., & Serajuddin, U. The World Bank's classification of countries by income. The World Bank, 2016.
[27] UNDP: Human Development Report 2005. New York , USA. Available Online: http://hdr.undp.org/sites/default/files/reports/266/hdr05_complete.pdf (accessed on May 5, 2020).
[28] UNDP: Human Development Report 2007. New York, USA. Available Online: http://hdr.undp.org/sites/default/files/reports/268/hdr_20072008_en_complete.pdf (accessed on May 5, 2020).
[29] UNDP: Human Development Report 2011. New York , USA. Available Online: http://hdr.undp.org/sites/default/files/hdr_2011_en_summary.pdf (accessed on May 5, 2020).
[30] UNDP: Human Development Report 2013. New York , USA. Available Online: http://hdr.undp.org/sites/default/files/hdr2013_en_summary.pdf (accessed on May 5, 2020).
[31] UNDP: Human Development Report 2015. New York , USA. Available Online: https://hdr.undp.org/content/human-development-report-2015 (accessed on May 5, 2020).
[32] UNDP: Human Development Report 2019. New York , USA. Available Online: http://hdr.undp.org/sites/default/files/hdr2019.pdf (accessed on May 5, 2020).
[33] UNDP: Human Development Index (HDI). Available Online: http://hdr.undp.org/en/content/human-development-index-hdi (accessed on May 5, 2020).

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