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INDEPENDENT DOMINATION NUMBER OF GRAPHS THROUGH VERTEX SWITCHING

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ABSTRACT. Let $G = (V, E)$ be a graph with vertex set V and edge set E. An independent dominating set S of G is a subset of V with the property that every vertex in $V - S$ is adjacent to some vertex in S and no two vertices within S are adjacent. The number of vertices in a minimum independent dominating set in the graph G is called the independent domination number $i(G)$ of G. In this article, the independent domination number of graphs obtained through vertex switching have been computed with appropriate illustration.

Keywords: Graphs, independent domination, independent dominating set, independent domination number, vertex switching.

AMS Subject Classification: 05C69, 05C76.

1. INTRODUCTION

Let $G = (V, E)$ be a simple graph with vertex set V and edge set E. The number of edges incident to the vertex v is called the degree of v, and it is denoted by $deg(v)$. The vertex of degree one is called a pendant vertex. The set of all vertices adjacent to a vertex v is called the open neighborhood $N(v)$ of v. Domination in graphs is a promising topic of study with a wide range of applications. Several significant models of domination theory are available in the literature. The abstract structure of these models are shown in the form of networks which are indeed graphs. The dominating set of a graph G is a subset S of V such that every vertex in $V - S$ has at least one neighbor in S. The minimum size of a dominating set is the domination number [9] and is denoted by $\gamma(G)$. The domination concepts are studied extensively by Hedetniemi and Laskar [8, 9, 10]. Different types of domination are available in the literature [11, 14, 16]. One such type of domination is an independent domination in graphs which was introduced by Berge [3] and Ore [15] in the year 1962. A set of pairwise non- neighborhood vertices of G is called the independent

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dominating set (abbreviated as IDS). The minimum size of an IDS in G is the independent domination number(abbreviated as IDN) and it is denoted by $i(G)$. Cockayne and Hedetniemi [4, 5] studied extensively the IDN, $i(G)$. The properties of IDS are referred to [2, 6, 12, 21]. It has various applications in topological networks, computer networks, biological networks and electrical power supply lines, radio stations and land surveying problems. The common choice for vertices used for the transmission of data in any form of network is an IDS. Allan and Laskar [1] have proved that claw-free graphs have equal domination and IDN. This result was further studied by Vaidya and Pandit [18] for graphs having induced subgraphs as claw-free graphs. Independent domination in regular graphs was studied by Goddard et. al [7].

The operation of switching of a Vertex v in G is done by deleting all incident edges of v in G and adding edges between vertices that are not adjacent to v in G. The resultant graph is denoted by $\widetilde{G}$. Vaidya and Pandit [19, 20] have found the independent domination number for path, wheel, cycle, flower, helm and girth graphs through the concept of vertex switching operations. Vertex switching in graphs has a numerous applications in topological networks, social networks, and electrical power supply lines. Some applications of vertex switching in graphs are stated as follows:

- In a confidential message transfer system, the data messenger should not be a single individual or a unique person whose identity is not known to anyone. The system of vertex switching can be utilized to prevent message leakage by third parties.

- In the case of an electric circuit, when there is a drop-down in voltage or failure of a node, the concept of switching a vertex comes to the rescue.

- In social networks, customers receive recommendations about products continuously, despite any breakup in the customer networks. The solution to all these problems can be solved by choosing a vertex within the same network and changing its neighbors without affecting the entire system.

This motivates to study the concept of vertex switching in graphs. In this article, IDN of graphs obtained through vertex switching of a vertex in the complete bipartite graph $K_{m,n}$, tadpole graph $T(m,n)$, lollipop graph $L(m,n)$ and sunlet graph S_n are computed.

2. Basic Notations

In this section, the basic definitions and concepts pertaining to this paper have been studied with appropriate examples. In this article, we consider an undirected graph $G = (V, E)$ having vertex set V and edge set E.

Definition 2.1. [9] *A dominating set of a graph G is a subset S of V such that every vertex in $V - S$ has atleast one neighbor in S. The minimum cardinality of the dominating set of G is the domination number $\gamma(G)$.*

Definition 2.2. [9] *An independent dominating set of a graph G is a subset S of V, that is dominating as well as independent. The minimum size of an independent dominating set of G is the independent domination number $i(G)$.*

Example 2.1. *The graph G with dominating set $\{v_1, v_2, v_3\}$ and independent dominating set $\{v_1, v_4, v_5, v_6, v_7\}$ are shown in Figure* 1.

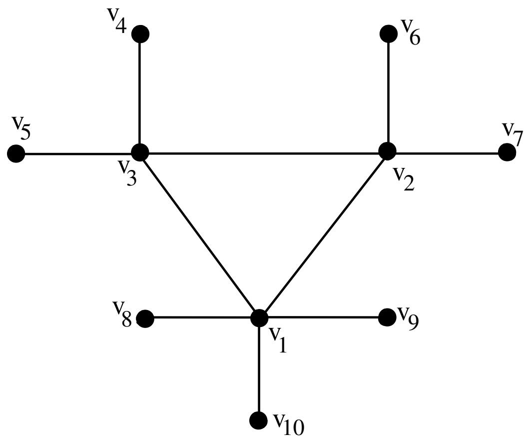


FIGURE 1. The graph G with $\gamma(G) = 3, i(G) = 5$

Definition 2.3. [19] *The vertex switching of v in G is the removal of all edges adjacent to v and adding edges between vertices which are not adjacent to v in G. The resultant graph is denoted by $\widetilde{G}$.*

Definition 2.4. *An edge in a graph G is said to be a bridge, if and only if the removal of an edge disconnects the graph.*

Definition 2.5. *The graph G is said to be bipartite if the vertex set V can be partioned into two disjoint subsets V_1 and V_2 such that each edge of G joins a vertex in V_1 to a vertex in V_2. A bipartite graph G is complete if every vertex in V_1 is connected to every vertex in V_2 and it is denoted by $K_{m,n}$.*

Definition 2.6. [13] *A tadpole graph $T(m, n)$ is a graph obtained by joining a cycle graph C_m to a path graph P_n using a bridge.*

In this paper, the vertex of the cycle C_m, which is connected to the initial vertex of the path P_n is called as bridge vertex.

Definition 2.7. [13] *The lollipop graph $L(m, n)$ is a graph obtained by joining a complete graph K_m with m vertices, to a path graph P_n with n vertices using a bridge.*

Definition 2.8. [17] *The sunlet graph S_n is a graph obtained by attaching n pendant edges to a cycle graph C_n. It has $2n$ pendant vertices.*

3. Independent Domination Number of Graphs through Vertex Switching

In this section, the IDN of graphs obtained through vertex switching operation of various graphs are computed.

Theorem 3.1. *Let $K_{m,n} (m \geq 2, n \geq 2)$ be a complete bipartite graph. Let $\widetilde{K_{m,n}}$ be a graph obtained from $K_{m,n} (m \geq 2, n \geq 2, (m < n))$ by switching a vertex v arbitrarily. Then the IDN of $\widetilde{K_{m,n}}$ is*

$$i(\widetilde{K_{m,n}}) = \begin{cases} m-1 & if \quad v \text{ is in the vertex set } V_1 \\ m+1 & if \quad v \text{ is in the vertex set } V_2 \text{ except for } n = m+1 \\ m & if \quad n = m+1 \text{ when } v \text{ is in the vertex set } V_2 \end{cases}$$

Proof. Let $G(V,E) = K_{m,n}(m \geq 2, n \geq 2, m < n)$ be a complete bipartite graph with $|V_1(G)| = m$, $|V_2(G)| = n$. The vertices and edges of $K_{m,n}$ are $m+n$ and mn respectively. Let the vertices of V_1 be $u_1, u_2, ..., u_m$ and V_2 be $v_1, v_2, ..., v_n$. Let $H(V,E) = \widetilde{K_{m,n}}(m < n)$ be the graph obtained by switching a vertex v of $K_{m,n}(m < n)$.

Case 1. Let us assume that the vertex $v = u_i \in V_1$, $1 \leq i \leq m$ be a switched vertex in $\widetilde{K_{m,n}}$ with $|V_1(H)| = m-1$ and $|V_2(H)| = n+1$. The switched vertex $v = u_i$ is adjacent to $u_1, u_2, u_3, ..., u_{i-1}, u_{i+1}, u_{i+2}, ..., u_m$ and not adjacent to the vertices in the vertex set V_2. Then the set of all vertices of $V_1(H)$ dominates all the vertices of $\widetilde{K_{m,n}}$ and is independent. Thus, the IDS contains all vertices of $V_1(H)$ and it is of size $m-1$. Therefore, the set of all vertices $\{u_1, u_2, ..., u_{i-1}, u_{i+1}, u_{i+2}, ..., u_m\}$ forms an IDS of $\widetilde{K_{m,n}}$. Hence, the IDN of $\widetilde{K_{m,n}}(m < n)$ is $m-1$, if switched vertex v is in the vertex set V_1 of $\widetilde{K_{m,n}}$.

Case 2. Let $v = v_j \in V_2, 1 \leq j \leq n$ be a switched vertex in $\widetilde{K_{m,n}}$. The switched vertex v_j and the vertices $u_1, u_2, ..., u_m$ dominate all the vertices of $\widetilde{K_{m,n}}$. Thus, the set $\{u_1, u_2, ..., u_m, v_j\}$ forms a dominating set of size $m+1$ except for $n = m+1$. Clearly, this set is independent. Therefore, the set $\{u_1, u_2, ..., u_m, v_j\}$ is an IDS of $\widetilde{k_{m,n}}$. Hence, the IDN of $\widetilde{K_{m,n}}(m < n)$ is $m+1$, if the switched vertex v is in the vertex set V_2 except for $n = m+1$. When $n = m+1$ the independent domination number of this set is m , if the switched vertex v is in the vertex set V_2.

□

Corollary 3.1. *If G is a complete bipartite graph $K_{n,n}$ with $|V_1| = |V_2| = n$, then the IDN of $\widetilde{K_{n,n}}$ is $n-1$.*

Proof. Let $G(V,E) = K_{n,n}$ be a complete bipartite graph with $|V_1(G)| = |V_2(G)| = n$. The number of vertices and edges of $K_{m,n}$ are $2n$ and n^2 respectively. Let the vertices in V_1 be $u_1, u_2, ..., u_n$ and V_2 be $v_1, v_2, ..., v_n$. Let $H(V,E) = \widetilde{K_{n,n}}$ be the graph obtained by switching a vertex v of the complete bipartite graph $K_{n,n}$. Then $|V_1(H)| = n-1$ and $|V_2(H)| = n+1$. Let $v \in V_1$ be an arbitrarily chosen switched vertex of $\widetilde{K_{n,n}}$ Then the vertex set $V_1(H)$ dominates all the vertices in $V_2(H)$ and it is of independent nature. Thus, the IDN of $\widetilde{K_{n,n}}$ is $n-1$. Similarly, we can show that, if $v \in v_2$ is a switched vertex in $\widetilde{K_{n,n}}$ then the IDN is also $n-1$. Therefore, the IDN of $\widetilde{K_{n,n}}$ is always $n-1$. □

Example 3.1. *The IDN of $i(\widetilde{K_{5,5}})$ is 4 and is shown in Figure 2.*

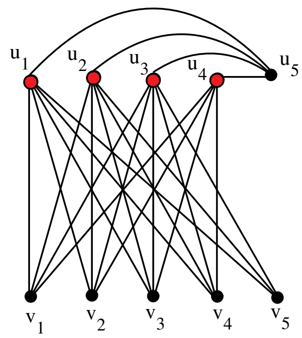


FIGURE 2. $i(\widetilde{K_{5,5}}) = 4$

Corollary 3.2. *If G is a complete bipartite graph $K_{m,n}$ with partite sets V_1 of size m and V_2 of size n then $\widetilde{K_{m,n}}$ is also a complete bipartite graph of size $m-1$ and $n+1$.*

Theorem 3.2. *Let $T(m,n)(m \geq 3, n \geq 1)$ be a tadpole graph. Let $\widetilde{T_1(m,n)}$ be a graph obtained by switching a pendant vertex of the path in $T(m,n)(m \geq 3, n \geq 1)$. Then the IDN of $\widetilde{T_1(m,n)}$ is*

$$i(\widetilde{T_1(m,n)}) = \begin{cases} 1 & if \quad m = 3 \text{ and } n = 1, 2 \\ 2 & \quad otherwise. \end{cases}$$

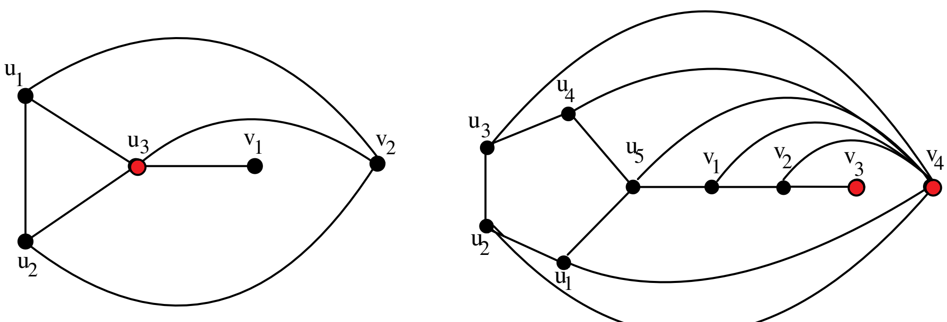
Proof. Let $\widetilde{T_1(m,n)}$ be the graph obtained by switching a pendant vertex v_n of the path in $T(m,n)(m \geq 3, n \geq 1)$. Let the vertices of cycle C_m and path P_n in the tadpole graph be $u_1, u_2, ..., u_m$ and $v_1, v_2, ..., v_n$ respectively. The pendent vertex v_n of $T(m,n)(m \geq 3, n \geq 1)$ is adjacent to the vertices in $\widetilde{T_1(m,n)}$ if and only if they are non − adjacent in $T(m,n)(m \geq 3, n \geq 1)$.

Case 1: $m \geq 3$ and $n \geq 1$ except for $m = 3$ when $n \leq 2$.
Since the pendant vertex v_n of $T(m,n)$ is the switched vertex in $\widetilde{T_1(m,n)}$ which is incident to all the other vertices except the neighborhood vertex v_{n-1} of $T(m,n)$. Hence the switched vertex v_n and non-neighborhood pendant vertex v_{n-1} forms the dominating set and it dominates all the vertices of the graph. The vertices v_{n-1}, v_n of $\widetilde{T_1(m,n)}$ is also independent. Therefore, the IDN of $\widetilde{T_1(m,n)}$ is 2 for $m \geq 3, n \geq 1$ except for $m = 3, n \leq 2$.

Case 2: $m = 3$ and $n \leq 2$.
In this case, the vertex u_i with maximum degree on the cycle C_m dominates all the other vertices of the graph and it is independent. Hence, the IDN of $\widetilde{T_1(m,n)} = 1$ only when $m = 3, n \leq 2$.

□

Example 3.2. *The IDN of $\widetilde{T_1(3,2)}$ is 1 and $\widetilde{T_1(5,4)}$ is 2 which are shown in Figure 3 of A and B respectively.*



(A) (B)

FIGURE 3. (A) $i(\widetilde{T_1(3,2)}) = 1$, (B) $i(\widetilde{T_1(5,4)}) = 2$

Theorem 3.3. *Let $\widetilde{T_2(m,n)}$ be a graph obtained by switching an internal path vertex of the tadpole graph $T(m,n)(m \geq 3, n \geq 1)$. Then, the IDN of $\widetilde{T_2(m,n)}$ is*

$$i(\widetilde{T_2(m,n)}) = \begin{cases} 2 & if \quad m > 3 \text{ and } n = 1 \\ 3 & if \quad m \geq 3 \text{ and } n \geq 2. \end{cases}$$

Proof. Let $\widetilde{T_2(m,n)}$ be the graph obtained by switching an internal path vertex of the tadpole graph of $T(m,n)(m \geq 3, n \geq 1)$. Let the vertices of cycle C_m and path P_n in the tadpole graph be $u_1, u_2, ..., u_m$ and $v_1, v_2, ..., v_n$ respectively. The internal path vertex v_i of $T(m,n)(m \geq 3, n \geq 1)$ is adjacent to the vertices in $\widetilde{T_2(m,n)}$ if and only if they are non–adjacent in $T(m,n)(m \geq 3, n \geq 1)$.

Case 1: $m > 3$ and $n = 1$.
Since, there is no vertex v_i or u_i which dominates all other vertices of the graph $\widetilde{T_2(m,n)}$ and the switched vertex is adjacent to all other vertices except the non-adjacent pendant vertex of the graph $\widetilde{T_2(m,n)}$. The pendant vertex along with the switched vertex forms the IDS of the graph. Hence, the IDN of the graph $\widetilde{T_2(m,n)}$ is 2.

Case 2: $m \geq 3$ and $n \geq 2$.
In this case, the graph $\widetilde{T_2(m,n)}$ has two non-adjacent vertices. Therefore, atleast three vertices are required to dominate the graph $\widetilde{T_2(m,n)}$. The switched vertex is adjacent to all other vertices of $\widetilde{T_2(m,n)}$ except the pendant vertices of $\widetilde{T_2(m,n)}$. Thus the two pendant vertices along with the switched vertex forms a dominating set of the graph, which is also independent. Hence, the IDN of $\widetilde{T_2(m,n)}$ is 3 when $m \geq 3$ and $n \geq 2$.

□

Example 3.3. *The IDN of $\widetilde{T_2(3,5)}$ is 3 and is shown in Figure 4.*

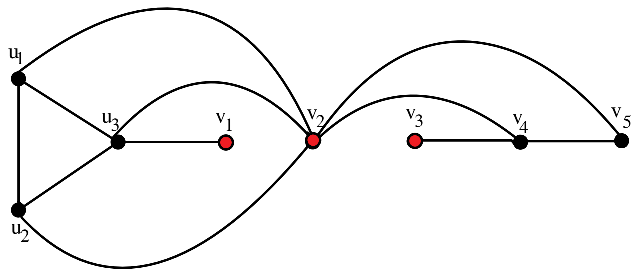


FIGURE 4. $i(\widetilde{T_2(3,5)}) = 3$

Theorem 3.4. *Let $\widetilde{L(m,n)}$ be a graph obtained by switching a pendant vertex of the lollipop graph $L(m,n)(m \geq 3, n \geq 1)$. Then the IDN of $\widetilde{L(m,n)}$ is*

$$i(\widetilde{L(m,n)}) = \begin{cases} 1 & if \quad m = 3 \text{ and } n = 1,2 \\ 2 & if \quad m \geq 3 \text{ and } n \geq 3. \end{cases}$$

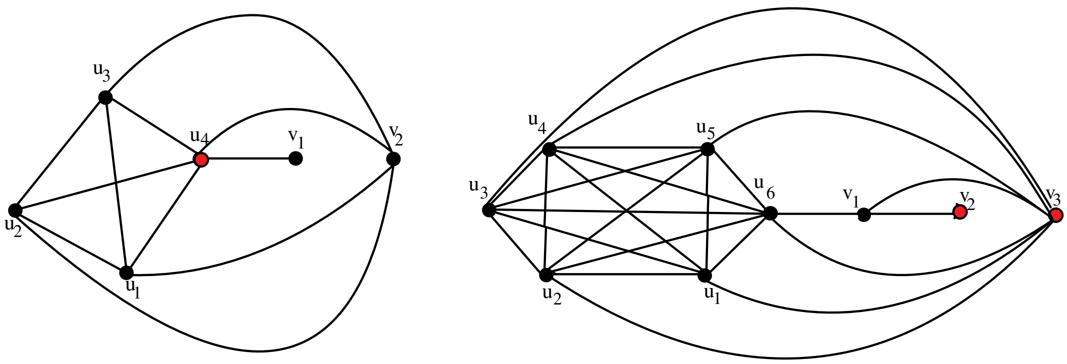
Proof. Let $\widetilde{L(m,n)}$ be the graph obtained by switching a pendant vertex of the lollipop graph $L(m,n)(m \geq 3, n \geq 1)$. Let the vertices of complete graph K_m and path P_n in the lollipop graph be $u_1, u_2, ..., u_m$ and $v_1, v_2..., v_n$ respectively. The IDN of $\widetilde{L(m,n)}$ is discussed in the following two cases.

Case 1: $m = 3$; $n = 1, 2$.
The vertex u_i which is adjacent to v_i dominates all the vertices of the graph $\widetilde{L(m,n)}$. The vertex u_i is dominating as well as independent. Hence, $i(\widetilde{L(m,n)}) = 1$ when $m = 3, n = 1, 2$.

Case 2: $m \geq 3, n \geq 2$.
The pendant vertex of $L(m,n)$ becomes the switch vertex of $\widetilde{L(m,n)}$. In $\widetilde{L(m,n)}$, the switch vertex is adjacent to all other vertices except its non-neighbourhood vertex. Therefore, the pendant vertex and the switch vertex forms an IDS for the graph $\widetilde{L(m,n)}$. Hence the IDN for the graph is two. That is, $i(\widetilde{L(m,n)}) = 2$ when $m \geq 3, n \geq 3$.

□

Example 3.4. *The IDN of $\widetilde{L(4,2)}$ is one and $\widetilde{L(6,3)}$ is two which are shown in Figure* 5 *of A and B respectively.*



(A) (B)

FIGURE 5. (A) $i(\widetilde{L(4,2)}) = 1$, (B) $i(\widetilde{L(6,3)}) = 2$

Theorem 3.5. *Let $\widetilde{S_n}$ be a graph obtained by switching an arbitrary pendant vertex of the sunlet graph S_n then the IDN of $\widetilde{S_n}$ is* 2.

Proof. Let the vertices of cycle C_n and the pendent vertices of n–sunlet graph be $u_1, u_2, ..., u_n$ and $v_1, v_2, ..., v_n$ respectively, where u_i is adjacent to v_i for $1 \leq i \leq n$. Let $\widetilde{S_n}$ be the graph obtained by switching an arbitrary pendant vertex of S_n. Let v_i be the arbitrarily chosen vertex. Then v_i becomes the switch vertex and is incident to all other vertices except non neighbourhood vertex u_i of $\widetilde{S_n}$. Therefore, every IDS must contain the vertex v_i and u_i. Hence, the IDN of the graph $\widetilde{S_n}$ is 2. □

Example 3.5. *The IDN of $\widetilde{S_7}$ is* 2 *and is shown in Figure* 6.

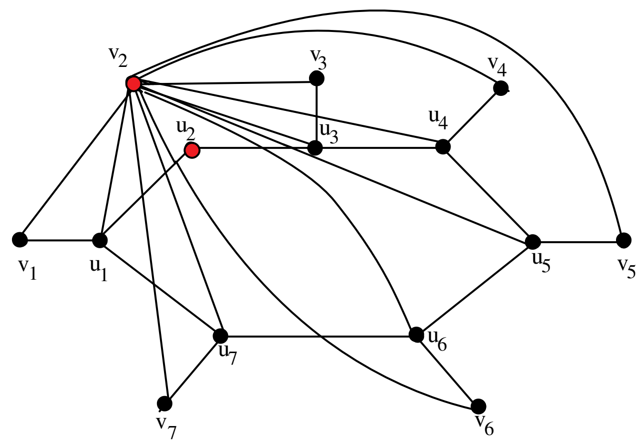


FIGURE 6. $i(\widetilde{S_7}) = 2$

Theorem 3.6. *Let $T(m,n)(m \geq 8, n \geq 1)$ be a tadpole graph. Let $\widetilde{T_0(m,n)}$ be a graph obtained by switching a vertex of the cycle in $T(m,n)(m \geq 8, n \geq 1)$. Then the IDN of $\widetilde{T_0(m,n)}$ is*

$$i(\widetilde{T_0(m,n)}) = \begin{cases} 4 & if \quad u_i \text{ is the bridge vertex of the cycle} \\ 3 & \text{otherwise.} \end{cases}$$

Proof. Let $\widetilde{T_0(m,n)}$ be the graph obtained by switching a vertex of the cycle in the tadpole graph of $T(m,n)(m \geq 8, n \geq 1)$. Let the vertices of cycle C_m and path P_n of the tadpole graph be $u_1, u_2, ..., u_m$ and $v_1, v_2, ..., v_n$ respectively. The vertex u_i of $T(m,n)(m \geq 8, n \geq 1)$ is adjacent to the vertices in $\widetilde{T_0(m,n)}$ if and only if they are non–adjacent in $T(m,n)(m \geq 8, n \geq 1)$.

Case 1: Let the switched vertex u_i be the bridge vertex of the cycle C_m in $\widetilde{T_0(m,n)}$.

The switched vertex u_i is adjacent to all the vertices in $\widetilde{T_0(m,n)}$ except the adjacent vertices of u_i in $T(m,n)$. The graph $\widetilde{T_0(m,n)}$ has three pendant vertices. The switched vertex and the three pendant vertices forms a dominating set for $\widetilde{T_0(m,n)}$. Clearly, these vertices are independent. Hence, the IDN of $\widetilde{T_0(m,n)}$ is 4, if the switched vertex u_i is the bridge vertex of C_m in $\widetilde{T_0(m,n)}$.

Case 2: Let the switched vertex u_i be any vertex other than the bridge vertex of the cycle C_m in $\widetilde{T_0(m,n)}$.

The vertex u_i is adjacent to all other vertices except the two pendant vertices u_{i-1} and u_{i+1}. Therefore, the set $\{u_{i-1}, u_i, u_{i+1}\}$ dominates all the vertices of the graph and it is independent. Hence, the IDN of $i(\widetilde{T_0(m,n)})$ is 3, if the switched vertex is the internal vertex (non-bridge vertex) of C_m in $\widetilde{T_0(m,n)}$. □

Example 3.6. *The IDN of $\widetilde{T_0(8,5)}$ is 4 and $\widetilde{T_0(9,4)}$ is 3 which are shown in Figure 7 and Figure 8 respectively.*

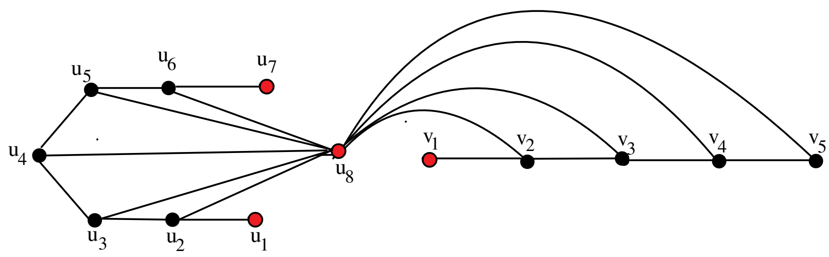


FIGURE 7. $i(\widetilde{T_0(8,5)}) = 4$

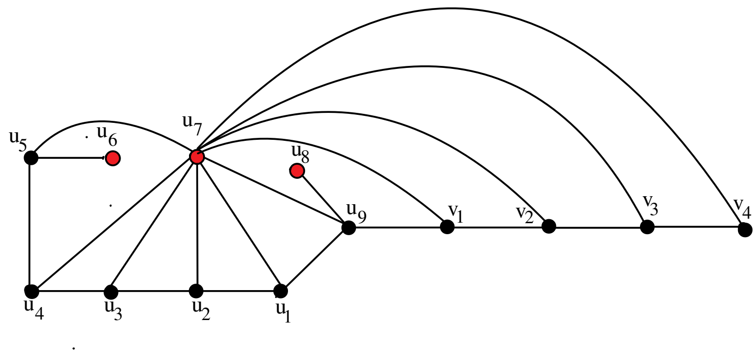


FIGURE 8. $i(\widetilde{T_0(9,4)}) = 3$

Theorem 3.7. *Let $\widetilde{T_0(m,n)}$ be a graph obtained by switching a vertex of the cycle in the tadpole graph $T(m,n)(m = 5,6,7, n \geq 1)$ then the IDN of $\widetilde{T_0(m,n)}$*

$$i(\widetilde{T_0(m,n)}) = \begin{cases} 4 & if \quad u_i \text{ is the bridge vertex of cycle and } m = 5,6,7; n \geq 4. \\ 3 & \quad \text{otherwise.} \end{cases}$$

Proof. Let $\widetilde{T_0(m,n)}$ be the graph obtained by switching a vertex of the cycle in the tadpole graph of $T(m,n)(m = 5,6,7, n \geq 1)$. Let the vertices of cycle C_m and path P_n in the tadpole graph be $u_1, u_2, ..., u_m$ and $v_1, v_2, ..., v_n$ respectively. The vertex u_i of $T(m,n)(m = 5,6,7, n \geq 1)$ is adjacent to the vertices in $\widetilde{T_0(m,n)}$ if and only if they are non–adjacent in $T(m,n)(m = 5,6,7, n \geq 1)$.

Case 1: Let the switched vertex u_i be the bridge vertex of the cycle C_m in $\widetilde{T_0(m,n)}$ and $n \geq 4$.

The graph $\widetilde{T_0(m,n)}$ has three pendant vertices. Therefore at least four vertices are required to dominate the graph $\widetilde{T_0(m,n)}$. From the definition of vertex switching, it is clear that the switched vertex is adjacent to all other vertices except the three pendant vertices. The switched vertex and the three non-adjacent pendant vertices form an IDS for the graph $\widetilde{T_0(m,n)}$. Hence, the IDN of the graph $\widetilde{T_0(m,n)}$ is 4 if the switched vertex u_i is the bridge vertex of the cycle C_m in $\widetilde{T_0(m,n)}$.

Case 2: Let the switched vertex u_i be any vertex other than the bridge vertex of the cycle C_m in $\widetilde{T_0(m,n)}$ and $n \geq 4$.

Let u_i be an internal(non-bridge) switched vertex of the cycle C_m in $\widetilde{T_0(m,n)}$. Then the switched vertex u_i is adjacent to all vertices except the two pendant vertices. Therefore the vertex u_i and the two pendant vertices dominates all the vertices of graph and it is independent. Hence, $i(\widetilde{T_0(m,n)}) = 3$ if the switched vertex is the internal(non-bridge) vertex of the cycle C_m in $\widetilde{T_0(m,n)}$.

Case 3: Let the switched vertex u_i be any vertex of the cycle C_m in $\widetilde{T_0(m,n)}$ and $m = 5, 6, 7; n < 4$.

In this case, the vertex u_i and the remaining two pendant vertices dominates all the vertices of the graph. Clearly, this set is independent. Hence, the IDN of $\widetilde{T_0(m,n)}$ is 3 when $m = 5, 6, 7, n < 4$. □

Example 3.7. *The IDN of* $\widetilde{T_0(6,3)}$ *is* 4 *and* $\widetilde{T_0(5,2)}$ *is* 3 *which are shown in Figure* 9 *and Figure* 10 *respectively.*

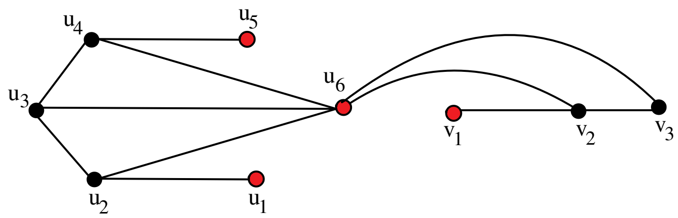


FIGURE 9. $i(\widetilde{T_0(6,3)}) = 4$

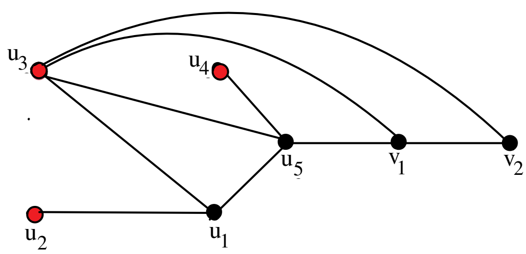


FIGURE 10. $i(\widetilde{T_0(5,2)}) = 3$

4. CONCLUSION

In this paper, we have computed the independent domination number of graphs obtained from complete bipartite graph, tadpole graph, lollipop graph and sunlet graph by switching a vertex. The independent domination number of graphs obtained in this article are tabulated in Table 1.

The computation of IDN of graphs through various graph operations is a potential area of research and it is interesting too. Investigating independent domination number for other family of graphs is an open area of research and future scope also.

Graphs	**Cases**	**IDN**
$K_{m,n}$	$m < n$	$m - 1$ if switched vertex is in V_1 $m + 1$ if switched vertex is in V_2
	$m = n$	$n - 1$ if switched vertex is in V_1 or V_2.
$T_1(m, n)$	$m = 3, n = 1, 2$	1
	otherwise	2
$T_2(m, n)$	$m > 3, n = 1$	2
	$m \geq 3, n \geq 2$	3
$T_0(m, n)$	u_i is the bridge vertex of cycle	4
	otherwise	3
$L(m, n)$	$m = 3, n = 1, 2$	1
	$m \geq 3, n \geq 2$	2
S_n	$\forall n$	2

Table 1. IDN of graphs obtained by switching a vertex.

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