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NULLITY AND ENERGY OF THE CUBIC POWER GRAPH

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ABSTRACT. Let G be a finite abelian group with identity 0, $G_1 = \{3t \mid t \in G\} \subseteq G$ and $G_2 = G \setminus G_1$. The cubic power graph Γ_{cpg} of G is an undirected simple graph with vertex set G, such that two distinct vertices x and y are adjacent in Γ_{cpg} if and only if $x + y \in G_1 \setminus \{0\}$. In this paper, we first prove certain properties of the cubic power graph Γ_{cpg} of finite abelian groups. Finally, we observe some properties on the nullity of the cubic power graph Γ_{cpg} and also show that the energy of the cubic power graph of a finite abelian group of order n is bounded below by $\sqrt{\frac{(n-3)(n-1)}{3}}$ if n is divisible by 3 and $n-1$ if n otherwise.

Keywords: Nullity of a graph; Energy of a graph, Cubic power graph; Power graph; Finite groups; Graphs from finite groups.

AMS Subject Classification: 83-02, 05C25.

1. Introduction

A graph $\Gamma = (V, E)$ is a combinatorial structure with vertex set V and edge set E. Suppose the cardinalities of V and E are n and m respectively and $\lambda_1, \lambda_2, \ldots, \lambda_n$ are eigenvalues of the adjacency matrix of Γ. The energy $\bar{E}(\Gamma)$ of Γ, according to Gutman [10] is the summation of the absolute values of these eigenvalues. i.e.,

$$\bar{E}(\Gamma) = \sum_{i=1}^{n} |\lambda_i|.$$

The eigenvalues along with their multiplicities is called the spectrum of a graph. The multiplicity of the number zero in the spectrum of Γ is the nullity of Γ[7], usually it is denoted by $\eta(\Gamma)$. A graph Γ is said to be non-singular if it has no zero eigenvalues. i.e., the nullity $\eta(\Gamma) = 0$. For two graphs Γ and Γ', if $\eta(\Gamma) \leq \eta(\Gamma')$, then the energy

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$\bar{E}(\Gamma) \geq \bar{E}(\Gamma')$. These concepts were introduced by Cvetkovic and Gutman [7] and Gutman [10] and about five decades ago and since then numerous researchers have employed them both to study different graphs. Sharma and Naresh [20] studied the nullity and the bounds for the energy of the central graph of Smith graphs. Specifically, they used the Huckel molecular orbital theory to study the spectral properties of Smith graphs and the applicability of the graph energy in determining the stability of unsaturated conjugate hydrocarbons which are isomorphic to central graph of Smith graphs. In the same vein, Ejima, Aremu and Audu [9] studied the energy of inverse graphs of dihedral and symmetric groups. They established that if $G = D_{2n}$ is the dihedral group of order $2n$, where $n \geq 3$ and $S = \{x \in G | x \neq x^{-1}\}$ is a subset of D_{2n}, then the energy of the inverse graph $\Gamma(G)$ satisfies $3 + 2\sqrt{2} \leq \bar{E}(\Gamma(G)) < 2m$ where m is the number of edges in $\Gamma(G)$.

In order to generalize the notions of nullity and multiliplicity of graphs, Aouchiche and Hansen [2] studied the bounds of the nullity number of graphs by providing the necessary condition on the extremal graphs corresponding to the bound using the maximum clique number of the graph. Their results improved the results of Cheng and Liu [6] who showed that $\eta(\Gamma) \leq n - D$, where η, n and D denote the nullity number, the order and the diameter of a connected graph Γ respectively. Also, Vatandoost and Pour [21] used the spectrum of a Cayley graph in the form of irreducible characters of the algebraic structure, and using representation and character of the structure, gave a lower bound for the maximum nullity of Cayley graph. Meanwhile, a decade long open problem of graph nullity bound was attempted by Zhou, Wong and Sun [17] and they proved that the nullity of an arbitrary graph of order n and maximum degree $\Delta \geq 1$ has an upper bound of $\frac{\Delta-1}{\Delta}n$. They further proposed that this upper bound of graphs nullity η can be improved to $\frac{(\Delta-2)n+2}{\Delta-1}$.

Very recently, Raveendra Prathap and Tamizh Chelvam [19] studied the cubic power graph of finite abelian groups. They obtained the diameter, the girth, the independence number, the clique number and the chromatic number of the cubic power graph of a finite group and that of its complement. Motivated by the work of Raveendra Prathap and Tamizh Chelvam [19], we propose to study the nullity and energy of the cubic power graph of a finite abelian group. In fact, we highlighted some new properties of the cubic power graphs of finite abelian groups and also prove that the nullity of Γ_{cpg} to be greater than zero. Furthermore, we studied the energy of certain class of cubic power graphs and their upper and lower bounds.

1.1. **Preliminaries.** In this section, we state some known results which will be useful in the proofs of our main results and for all the undefined terms of graph theory, we refer [1, 3, 4, 5, 12, 13, 19]. Throughout this paper, we let G be a finite abelian group with identity $\{0\}$, $G_1 = \{3t \mid t \in G\} \subseteq G$ and $G_2 = G \setminus G_1$. The cubic power graph Γ_{cpg} of G is an undirected simple graph with vertex set G, such that two distinct vertices x and y are adjacent in Γ_{cpg} if and only if $x + y \in G_1 \setminus \{0\}$. The following are some of the known results on cubic power graph of finite abelian groups.

Theorem 1.1. [19, Theorem 3.2] *Let G be a finite abelian group and $|G|$ is not divisible by 3. Then*

(1) $deg_{\Gamma_{cpg}}(x) = \begin{cases} |G| - 1 & \textit{if } x = -x \textit{ for } x \in G; \\ |G| - 2 & \textit{otherwise.} \end{cases}$

(2) $\Gamma_{cpg}(G)$ *is connected.*

(3) $diam(\Gamma_{cpg}(G)) = \begin{cases} 1 & \text{if } G \cong \mathbb{Z}_2^k \text{ for some integer } k \geq 1; \\ 2 & \text{otherwise.} \end{cases}$

(4) $gr(\Gamma_{cpg}(G)) = \begin{cases} \infty & \text{if } G \cong \mathbb{Z}_2; \\ 3 & \text{otherwise.} \end{cases}$

(5) $\Gamma_{cpg}(G)$ *is a refinement of the star graph* $K_{1,|G|-1}$.

Theorem 1.2. [3, Theorem 1] *Let* Γ *be a graph of order* n *and size* m. *If* $m < \frac{n}{2}, m = \lceil \frac{n}{2} \rceil$ *(for odd* n*) or* $m = \binom{n}{2} - 1$, *the* Γ *has non-zero nullity.*

For a finite abelian group G, let $G_1 = \{3t \mid t \in G\} \subseteq G$ and $G_2 = G \setminus G_1$. As observed in [19, Remark 2.2(1)], for a finite abelian group G such that $|G|$ is not divisible by 3, we have $G_1 = G$ and $G_2 = \emptyset$

Theorem 1.3. [19, Theorem 3.3(1)] *Let* G *be a finite abelian group. Assume that* $|G| = 3^\alpha.t$, $\alpha \geq 1, t$ *is relatively prime to 3 and* $\Sigma_{i=1}^k m_i = \alpha$. *Then*

$$deg_{\Gamma_{cpg(G)}}(x) = \begin{cases} \dfrac{|G|}{3^k} - 1 & \text{if } x = -x \text{ and } x \in G_1 = \{3t : t \in G\}; \\ \dfrac{|G|}{3^k} - 2 & \text{if } x \neq -x \text{ and } x \in G_1; \\ \dfrac{|G|}{3^k} - 1 & \text{if } x \in G_2 = G \setminus G_1. \end{cases}$$

Theorem 1.4. [19, Theorem 3.3(4)] *Let* G *be a finite abelian group. Assume that* $|G| = 3^\alpha.t$, $\alpha \geq 1, t$ *is relatively prime to 3 and* $\Sigma_{i=1}^k m_i = \alpha$. *Then the cubic power graph of* $\Gamma_{cpg}(G)$ *is isomorphic to disjoint union of induced subgraphs* $\langle\Gamma_{cpg}(G_1)\rangle$ *and* $\langle\Gamma_{cpg}(G_2)\rangle$.

Lemma 1.5. [8, Proposition 2.8] *Let* Γ *be a simple graph on* n *vertices and let the path* P_k *be an induced subgraph of* Γ, *where* $2 \leq k \leq n$. *Then*

$$\eta(\Gamma) = \begin{cases} n - k + 1 & \text{if } k \text{ is odd;} \\ n - k & \text{otherwise.} \end{cases}$$

Lemma 1.6. [8, Proposition 2.4] *Let* Γ *be a simple graph on* n *vertices and let the path* P_k *be a subgraph of* Γ, *where* $2 \leq k \leq n$. *Then* $\eta(\Gamma) \leq n - k$.

Lemma 1.7. [4, Lemma 1] *Let* Γ *be a graph of* n *vertices. Then* $\eta(\Gamma) = n$ *if and only if* Γ *is a graph without edges.*

Corollary 1.8. [4, Corollary 2] If Γ is a simple graph on n vertices, and Γ has at least one cycle, then

$$\eta(\Gamma) \leq \begin{cases} n - gr(\Gamma) + 2 & \text{if } gr(\Gamma) \equiv 0 \pmod 4; \\ n - gr(\Gamma) & \text{otherwise.} \end{cases}$$

Theorem 1.9. [16, Theorem 1.1] *The sum of of the degrees of the vertices in a graph equals twice the number of edges.*

2. Some Properties of Γ_{cpg}

This section outline some properties of the cubic power graph of finite abelian group G, $\Gamma_{cpg}(G)$.

Remark 2.1. Let G be a finite abelian group of order $n \leq 3$. The cubic power graph $\Gamma_{cpg}(G)$ of G is an empty graph and hence from Lemma 1.7, the nullity $\eta(\Gamma_{cpg}) = n$.

Theorem 2.2. *Let m and n be two coprime positive integers. Then $\Gamma(\mathbb{Z}_{mn})$ is not isomorphic to the complete product of cubic power graphs of $\mathbb{Z}_m$ and $\mathbb{Z}_n$.*

Proof. Note that $\mathbb{Z}_{mn}$ is isomorphic to the direct sum of two cyclic groups of order $\mathbb{Z}_m$ and $\mathbb{Z}_n$. Suppose $\Gamma_{cpg}(\mathbb{Z}_{mn})$ is isomorphic to the complete product of cubic power graphs $\Gamma_{cpg}(\mathbb{Z}_m)$ and $\Gamma_{cpg}(\mathbb{Z}_n)$. Then $V(\Gamma_{cpg}(\mathbb{Z}_{mn})) = V(\Gamma_{cpg}(\mathbb{Z}_m)) \cup V(\Gamma_{cpg}(\mathbb{Z}_n))$. Clearly, the order of $\Gamma_{cpg}(\mathbb{Z}_{mn}) = mn$ while the order of $\Gamma_m \nabla \Gamma_n$ is $m + n$ which is a contradiction to the definition of graph isomorphism. □

Lemma 2.3. *Let G be a finite abelian group of order $|G| = 3.t$ where t is an odd integer and $t \geq 5$. Let $\Gamma_{cpg}(G)$ be the cubic power graph of G. Then $|E(\Gamma_{cpg}(G))|$ is $\dfrac{|G|^2 - 4|G| + 3}{6}$.*

Proof. By assumption $|G| = 3.t, t$ is relatively prime to 3. In the notation of Theorem 1.3, $\alpha = 1$, $t \geq 5$ and $k = 1$. Hence $\mid G_1 \mid = \frac{|G|}{3}$ and $\mid G_2 \mid = G - \frac{|G|}{3}$. By Theorem 1.3, the sum of the degrees of the vertices of $\Gamma_{cpg}(G)$ will be as follows:

$$\sum_{x \in G} deg_{\Gamma_{cpg}}(x)) = \left(\frac{|G|}{3} - 1\right) + \left(\frac{|G|}{3} - 1\right)\left(\frac{|G|}{3} - 2\right) + \left(|G| - \frac{|G|}{3}\right)\left(\frac{|G|}{3} - 1\right) = \frac{|G|^2 - 4|G| + 3}{3}.$$

Using the handshaking Theorem 1.9, the sum of of the degrees of the vertices in a graph equals twice the number of edges. Hence

$$|E(\Gamma_{cpg}(G))| = \frac{|G|^2 - 4|G| + 3}{6}.$$

□

Lemma 2.4. *Let G be a finite abelian group of order not divisible by 3. Then the size of the cubic power graph $\Gamma_{cpg}(G)$ is $\dfrac{|G|^2 - 2|G| + 1}{2}$.*

Proof. Let G be a finite abelian group of odd order and not divisible by 3. By Theorem 1.1, we have

$$\sum_{x \in G} deg_{\Gamma_{cpg}}(x)) = (|G| - 1)(|G| - 2) + (|G| - 1) = |G|^2 - 2|G| + 1$$

and so by Theorem 1.9, we get that

$$|E(\Gamma_{cpg}(G))| = \frac{|G|^2 - 2|G| + 1}{2}.$$

□

3. Nullity and energy of Γ_{cpg}

One can check that the adjacency matrix of $\Gamma_{cpg}(\mathbb{Z}_5)$ is

$$A = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \end{pmatrix}.$$

The eigenvalues of A are the roots of the characteristics polynomial of the adjacency matrix A, which is nothing but $\lambda^5 - 8\lambda^3 - 8\lambda^2$. The roots $\lambda_i, 1 \leq i \leq 5$ of the polynomial are $0, 0, -2, 1+\sqrt{5}$ and $1-\sqrt{5}$. Therefore, the nullity $\eta(\Gamma_{cpg}(\mathbb{Z}_5)) = 2$ and the energy $\bar{E}(\Gamma_{cpg}(\mathbb{Z}_5)) = 2 + (1+\sqrt{5}) + (\sqrt{5}-1) = 2 + 2\sqrt{5}$.

Now, we observe below in Theorem 3.1 that, the cubic power graph of a finite abelian group is singular.

Theorem 3.1. *Let G be a finite abelian group of order at least 5. Then, the nullity of the cubic power graph $\eta(\Gamma_{cpg}(G))$ is not zero. Further $\eta(\Gamma_{cpg}(G)) \leq |G| - 3$ when $|G|$ is not divisible by 3.*

Proof. Consider the cubic power graph $\eta(\Gamma_{cpg}(G))$ of G with $|G| = n \geq 5$.

Case 1. Assume that $|G|$ is divisible by 3. By Lemma 2.3, the number of edges in $\Gamma_{cpg}(G)$ is $m = \dfrac{|G|^2 - 4|G| + 3}{6}$. Thus $m = \dfrac{(n-3)(n-1)}{6} < \binom{n}{2}$. By Theorem 1.2, $\eta(\Gamma_{cpg}(G))$ is not zero.

Case 2. Assume that $|G|$ is not divisible by 3. By Lemma 2.4, the number of edges in $\Gamma_{cpg}(G)$ is $m = \dfrac{|G|^2 - 2|G| + 1}{2}$. Thus $m = \dfrac{(n-1)^2}{2} < \binom{n}{2}$. By Theorem 1.2, $\eta(\Gamma_{cpg}(G))$ is not zero.

Later part of the statement follows Corollary 1.8. □

From Theorem 1.4, we have the following

Lemma 3.2. *Let $\Gamma_{cpg}(G)$ be a cubic power graph of finite abelian group G such that $|G|$ is divisible by 3. Then the nullity of $\eta(\Gamma_{cpg}(G)) = \eta(\langle\Gamma_{cpg}(G_1)\rangle) + \eta(\langle\Gamma_{cpg}(G_2)\rangle)$.*

From Lemma 1.5, we have the following.

Lemma 3.3. *Let G be a finite abelian group of order n and $\Gamma_{cpg}(G)$ be the cubic power graph of G. Assume that the P_k is an induced subgraph of $\Gamma_{cpg}(G)$ for $3 \leq k \leq n$. Then*

$$\eta(\Gamma_{cpg}(G) = \begin{cases} n-k+1 & \textit{if } k \textit{ is odd;} \\ n-k & \textit{otherwise.} \end{cases}$$

Theorem 3.4. *Let G be a finite abelian group of order $n \geq 5$ and $\Gamma_{cpg}(G)$ be the cubic power graph of G. Assume that n is divisible by 3. Then the energy of the cubic power graph $\bar{E}(\Gamma_{cpg}(G)) \geq \sqrt{\dfrac{(n-3)(n-1)}{3}}$.*

Proof. Suppose G is a finite abelian group n and n is divisible by 3. Let A be the adjacency matrix of $\Gamma_{cpg}(G)$ with $\lambda_1, \cdots, \lambda_n$ as its eigenvalues. Adopting the general form of energy of graphs as given by Brualdi [5]

$$\bar{E}(\Gamma(G)) \geq \sqrt{2m + n(n-1) \mid detA \mid^{2/n}}. \tag{3.1}$$

Recall that from Lemma 2.3, $m = \dfrac{(|G|-3)(|G|-1)}{6} = \dfrac{(n-3)(n-1)}{6}$. Also, by Theorem 3.1, $\det(A) = 0$. Substituting these in Equation 3.1, we have the following

$$\bar{E}(\Gamma(G)) \geq \sqrt{\frac{(n-3)(n-1)}{3}}. \tag{3.2}$$

□

Theorem 3.5. *Let G be a finite abelian group of order $n \geq 5$ and $\Gamma_{cpg}(G)$ be the cubic power graph of G. Assume that n is not divisible by 3. Then the energy of the cubic power graph $\bar{E}(\Gamma_{cpg}(G)) \geq (n-1)$.*

Proof. Assume that $|G|$ is not divisible by 3. Let A be the adjacency matrix of $\Gamma_{cpg}(G)$ with $\lambda_1, \cdots, \lambda_n$ as its eigenvalues. Adopting the general form of energy of graphs as given by Brualdi [5]

$$\bar{E}(\Gamma(G)) \geq \sqrt{2m + n(n-1) \mid detA \mid^{2/n}}. \tag{3.3}$$

Using Lemma 2.4, $m = \dfrac{(n-1)^2}{2}$. By Theorem 3.1 $\det(A) = 0$. Substituting these in Equation 3.3, we get that

$$\bar{E}(\Gamma(G)) \geq (n-1). \tag{3.4}$$

□

4. Conclusion

This study has highlighted the nullity and energy of cubic power graphs of finite abelian groups. We show that the nullity is greater than zero, this means, the cubic power graph can be associated to an alternant unsaturated conjugated hydrocarbon which is highly reactive with an unstable and open-shell electron configuration. We also show that the energy of the cubic power graph is bounded below by $\sqrt{\dfrac{(n-3)(n-1)}{3}}$ if n is divisible by 3 and $n-1$ if n is not divisible by 3.

Declarations

4.1. **Availability of data and materials.** Data sharing not applicable to this article as no data sets were generated or analysed during the current study.

4.2. **Conflict of Interests.** The authors declare that they have no competing interests.

4.3. **Authors' Contribution.** OE conceived the study, wrote the results and proofs, TTC validated the results and proofread the manuscript and KOA helped with the literature review. All authors read and approved the final manuscript.

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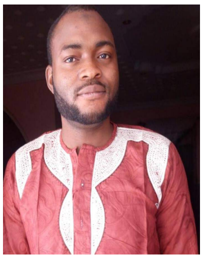

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