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IMPACT OF MODULATED MAGNETIC FIELD ON THE ADVENT OF FERROCONVECTION IN A PERMEABLE MEDIUM

C. BALAJI[1*], C. RUDRESHA[2], V. V. SHREE[3], S. MARUTHAMANIKANDAN[4], §

ABSTRACT. The effect of time-periodic magnetic field modulation on the onset of ferroconvection in a magnetic fluid saturated densely packed porous medium is examined. In many systems, such as charges in electrostatic field and ferromagnetic resonance, modulation of a suitable parameter can have marked effects on the motion and can result in increased stability of the system. Making use of isothermal boundary conditions, the subsequent eigenvalue problem is attacked by means of the regular perturbation method with the assumption of small amplitude of modulation. The onset criteria are derived based on the assumption that the principle of exchange of stabilities holds good. The thermal Rayleigh number shift is determined as a function of magnetic force, porous parameters, and magnetic field modulation frequency. The influence of various physical factors is perceived to be significant at moderate values of magnetic field modulation frequency. It is found that subcritical instability is possible for low frequency magnetic field modulation. The effect of magnetic mechanism is shown to be attenuating the stabilizing influence of magnetic field modulation for moderate and large values of the frequency of modulation. However, the stabilizing effect of magnetic field modulation gets amplified due to an increase in the values of Vadasz number. Further, it is delineated that the normalized porosity and magnetic field modulation work in tandem in destabilizing the system for low frequency modulation. The study reveals that the effect of magnetic field modulation could be exploited to control ferroconvective instability in a porous medium saturated with magnetic fluids.

Keywords: Magnetic fluid, Magnetic field modulation, Perturbation method, Stability, Porous medium.

AMS Subject Classification: 35Q30, 00A73, 76E06, 76S99.

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1. Introduction

Ferromagnetic liquids are a type of smart liquids not available in nature freely, but they can be synthesized by various processes. These fluids are also called ferrofluids or magnetic fluids. They are magnetized by magnetic fields and are made by dissolving microscopic magnetic particles (iron-Fe, cobalt-Co, nickel-Ni, etc.) in a non-magnetic liquid carrier (ester, kerosene, hydrocarbons, etc.) and wrapping these particles in a surfactant-like organic solution to prevent particles aggregation in the presence of a magnetic field. Ferromagnetic fluids draw the attention of many researchers in view of their diverse uses in the domains such as magnetic imaging resonance, data storage devices, magnetic specific drug delivery, thermal engineering, aerospace and to mention a few (Popplewell [1], Berkovsky et al., [2], Horng et al., [3] and Odenbach [4]). The notion of ferroconvection to thermal expansion in a horizontal layer enclosing ferromagnetic fluid is similar to that of Rayleigh-Bénard convection and has sparked considerable interest in the literature in view of its potential values as heat exchanger. Finlayson [5] first described how an advection of magnetic liquid with variability in magnetic susceptibility yields a non-uniformity in magnetic body force resulting in thermomagnetic convection. Many researchers, drawing sufficient inspiration from the work of Finlayson, have examined the ferroconvective instability problem under a variety of handy constraints (Stiles et al.,[6], Maruthamanikandan [7], Sankar et al., [8], Soya Mathew et al., [9],Vidya shree et al., [10]). In recent times, using the higher order Galerkin technique, Vidya shree et al., [11] revealed that the effect of MFD viscosity on the onset of Brinkman-ferroconvection with second sound enhances the ferroconvective threshold.

Modulation of an appropriate parameter may have significant effects on the motion of various sectors such as charges in an electrode material and ferromagnetic resonant, and can result in greater system's stability. The alteration in the magnetic field with respect to time on the threshold of ferroconvection and the conflict between harmonic and subharmonic modes using the Floquet theory has been examined in some detail (Aniss et al., [12], Kaloni et al., [13], Matura and Lucke [14]). In an experimental study, Engler and Odenbach [15] showed that, depending on the frequency of the external magnetic field, the onset of thermomagnetic convection of a magnetic fluid is significantly affected by stationary and periodically modulated magnetic field. The rate of temperature distribution through an electrically charged couple stress liquid under the influence of a magnetic field fluctuation with internal heat source is discussed in detail by Keshri et al., [16]. Of late, Balaji et al., [17, 18] studied the effect of porous medium and couple stresses on the advent of ferroconvection subjected to a time-dependent magnetic field by means of the regular perturbation method with the assumption of minimum amplitude of modulation. It is found that subcritical instability manifests by virtue of modulated magnetic field for low frequency in the presence of porous media and magnetic field modulation has a stabilizing effect on the system in the presence of both magnetization and couple stresses.

Convective heat transfer through fluid saturated porous materials has elicited a lot of attention in view of its diverse utilization in science and technology including geothermal power resource usage, nuclear waste eradication, building thermal shielding, waste removal in aquifers, drying processes and so forth. The pioneering work on fluid saturated permeable structure located between two identically flat surfaces and heated directly beneath in the traditional composition was elucidated by Horton and Rogers [19], and Lapwood [20], and the overall problem is now known as the Horton-Rogers-Lapwood problem or Darcy-Bénard problem. However, several researchers have dealt with the topic in depth and the expanding body of research in the area is well documented (Nield and Bejan [21], and Vafai [22]).

Vadasz [23] investigated the effect of Darcy porous medium subjected to Coriolis effect using the linear and weakly nonlinear stability theory. Govender [24] described the impact of the Vadasz number on the resonance of a rotating porous medium located far from the rotational axes. It is demonstrated that a frozen period estimation is adequate for large Vadasz numbers if indeed the impact of resonance is signified by slight changes in the Rayleigh number. Thomas and Maruthamanikandan [25] examined the role of the Vadasz number on the threshold of ferroconvective instability problem under the impact of varying gravity field in respect to time and confirmed that the strength of the Vadasz number reinforces the destabilizing effect of gravity modulation for small and moderate frequency values. Further, many authors made a study on the porous medium instability problem under a variety of handy constraints (Saravanan and Sivakumar [26], Saravanan [27], Jagadeesha et al., [28, 29], Sankar et al., [30], Vasanth and Hanumagowda [31]). The effect of magnetic parameters, Vadasz number and temperature modulation in the cases of symmetric, asymmetric and bottom wall modulation, were discussed by Thomas et al., [32]. Of late, Rudresha et al., [33, 34] investigated the influence of electric field modulation on the onset of electroconvection in a fluid saturated compactly packed isotropic and anisotropic porous layer using the regular perturbation method.

Convection control is a phenomenon that is vital and intriguing in a wide range of magnetic fluid technologies and at the same time it is conceptually challenging. The problem of unmodulated Rayleigh-Bénard convection in ferromagnetic liquids has received ample attention. However, no significant attention has been devoted to studying the impact of modulated magnetic field on the advent of ferroconvection in a magnetic fluid saturated porous layer. In this paper, the analysis presented is based on the assumption that the amplitude of magnetic field modulation is very minimal and the convective currents are weak resulting in the avoidance of nonlinear effects. As a result, depending on the frequency of magnetic field modulation, the advent of ferroconvection can be advanced or delayed in the presence of a porous medium.

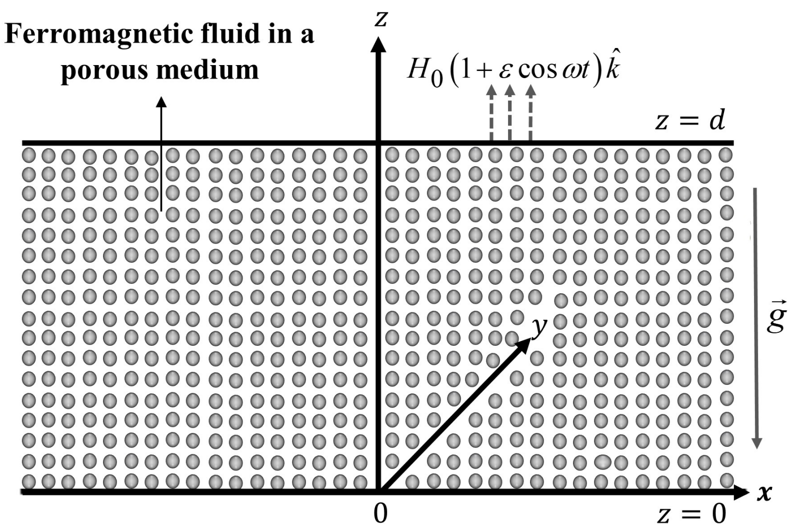

FIGURE 1. Schematic representation of the problem.

2. Mathematical formulation

We consider a Boussineq ferrofluid layer saturated densely packed porous medium placed in the middle of two horizontal infinite planes with d as the thickness of the fluid layer, gravity force acts on the fluid towards down. A time-periodic magnetic field is imposed in vertical direction $\vec{H}_0^{ext}(t) = H_0 (1 + \varepsilon \cos \omega t) \hat{k}$ as shown in Fig. 1. The origin of the Cartesian coordinate system $(x,\, y,\, z)$ is at the bottom of the fluid layer, and the z-axis is directed vertically upward. The upper and lower surfaces are retained at different uniform temperatures with a gradient ΔT.

The current study's governing equations thus take the form [12, 25]

$$\nabla \cdot \vec{q} = 0 \tag{1}$$

$$\rho_0 \left[\frac{1}{\varepsilon_p} \frac{\partial \vec{q}}{\partial t} + \frac{1}{\varepsilon_p^2} (\vec{q} \cdot \nabla) \vec{q} \right] = -\nabla p + \rho \vec{g} - \frac{\mu_f}{K} \vec{q} + \nabla \cdot \left(\vec{H} \vec{B} \right) \tag{2}$$

$$\varepsilon_p C_1 \frac{DT}{Dt} + (1 - \varepsilon_p) (\rho_0 C)_s \frac{\partial T}{\partial t} + \mu_0 T \left(\frac{\partial \vec{M}}{\partial T} \right)_{V,\, H} \cdot \frac{D\vec{H}}{Dt} = K_1 \nabla^2 T \tag{3}$$

$$\rho = \rho_0 \left[1 - \alpha (T - T_a) \right] \tag{4}$$

where $C_1 = \rho_0 C_{V,H} - \mu_0 \vec{H} \cdot \left(\frac{\partial \vec{M}}{\partial T} \right)_{V\,,H}$.

The simplified Maxwell's equations for a non-conducting fluid [5] without considering displacement current are as follows

$$\nabla \cdot \vec{B} = 0, \nabla \times \vec{H} = 0, \vec{B} = \mu_0 \left(\vec{H} + \vec{M} \right). \tag{5}$$

The linearized magnetic equation of state is

$$M = M_0 + \chi_m (H - H_0) - K_m (T - T_a) \tag{6}$$

The lower and upper surface temperatures respectively are $T = T_a + (1/2)\ \Delta T$ at $z = 0$ and $T = T_a - (1/2)\ \Delta T$ at $z = d$.

2.1. **Boundary conditions.** In a ferromagnetic fluid saturated porous layer of Darcy model, we use the impermeable condition for velocity and isothermal conditions for both temperature and magnetic potential. The boundary conditions at $z = 0$ and $z = 1$ are

$$w = T = \frac{\partial \phi}{\partial z} = 0 \tag{7}$$

2.2. **Basic state.** The solution pertaining to the basic quiescent basic state is given by

$$\begin{aligned} \vec{q} = \vec{q}_b = \vec{0}, p = p_b(z), \rho = \rho_b(z), T = T_b(z), \\ \vec{H} = \vec{H}_b = (0, 0, H_0(z,t)) = H_0^{ext}(t)\hat{k}, \vec{M} = \vec{M}_b = (0, 0, M_0(z)), \\ \vec{B} = \vec{B}_b = (0, 0, B_0(z)). \end{aligned} \tag{8}$$

In the basic state, the pressure, magnetic field, temperature, magnetic induction and magnetization equations are as follows

$$-\frac{\partial p_b}{\partial z}-\rho_b g+B_0\frac{\partial H_b}{\partial z}=0, T_b=T_a+\Delta T\left(\frac{1}{2}-\frac{z}{d}\right), B_b=\mu_0\left(M_0+H_0\right),$$
$$\rho_b=\rho_0\left(1-\alpha\,\Delta T\left(\frac{1}{2}-\frac{z}{d}\right)\right), H_b=\left[1+\frac{\gamma_a\,\Delta T}{(1+\chi_0)}\left(\frac{1}{2}-\frac{z}{d}\right)\right]\frac{H_0\left(1+\varepsilon G\right)}{(1+\chi_0)},\tag{9}$$
$$M_b=\left[M_0+\frac{H_0\gamma_a\,\Delta T}{(1+\chi_0)}\left(\frac{1}{2}-\frac{z}{d}\right)\right]\frac{(1+\varepsilon G)}{(1+\chi_0)}$$

where $\chi_0=\frac{M_0}{H_0},\ \gamma_a=\frac{\chi_0}{T_a}$ and $G=\mathrm{Re}\left(e^{-i\omega t}\right)=\cos\omega t$.

3. Linear stability analysis

The perturbed state is taken into account in order to look into the conditions under which the quiescent solution is balanced against small disturbances. We study the stability of the basic state using the method of small perturbations. On the basic state we superpose infinitesimal perturbations of the form

$$\vec{q}\ =\ \vec{q}_b+\vec{q}\,',\quad p\ =\ p_b+p',\quad \rho=\rho_b+\rho',\quad T=T_b+T',$$
$$\vec{H}=\vec{H}_b+\vec{H}',\quad \vec{M}=\vec{M}_b+\vec{M}',\quad \vec{B}=\vec{B}_b+\vec{B}'\tag{10}$$

where the primes represent infinitesimal perturbations. Substituting (10) into Eqs. (1) through (6) and using the basic state solution, we obtain the following equations

$$\nabla\cdot\vec{q}\,'=0\tag{11}$$

$$\frac{\rho_0}{\varepsilon_p}\left[\frac{\partial\vec{q}\,'}{\partial t}\right]=-\nabla p'+\alpha\rho_0 gT'\hat{k}-\frac{\mu_f}{K}\vec{q}\,'+\mu_0\left(M_0+H_0\right)\frac{\partial\vec{H}'}{\partial t}$$
$$-\left(\frac{\mu_0\chi_0 H_0\left(1+\varepsilon G\right)\Delta T}{T_a\left(1+\chi_0\right)d}\right)\frac{\partial\phi'}{\partial z}\hat{k}\tag{12}$$
$$+\left(\frac{\mu_0\chi_0^2H_0^2(1+\varepsilon G)^2\Delta T}{T_a^2(1+\chi_0)^3d}\right)T'\,\hat{k}$$

$$C_3\frac{\partial T'}{\partial t}-\varepsilon_pC_2\left(\frac{\Delta T}{d}\right)w'+\frac{\varepsilon_p\mu_0\chi_0H_0^2}{T_a(1+\chi_0)^2}\left(\frac{\partial T'}{\partial t}-w'\,\frac{\Delta T}{d}\right)(1+\varepsilon G)^2$$
$$-\frac{\mu_0\chi_0}{(1+\chi_0)}\left(\frac{\partial\phi'}{\partial z}\right)\frac{\partial}{\partial t}H_0\left(1+\varepsilon G\right)-\frac{\mu_0\chi_0}{(1+\chi_0)}H_0\left(1+\varepsilon G\right)\frac{\partial}{\partial t}\left(\frac{\partial\phi'}{\partial z}\right)$$
$$-\frac{\mu_0\chi_0\,H_0}{T_a(1+\chi_0)^2}\left(1+\varepsilon G\right)T'\frac{\partial}{\partial t}H_0\left(1+\varepsilon G\right)\tag{13}$$
$$+\frac{\mu_0\chi_0^2H_0^2\Delta T}{T_a(1+\chi_0)^3d}(1+\varepsilon G)^2w'\ =\ K_1\nabla^2T'$$

$$\left(1+\chi_0\right)\nabla^2\phi'-\left(\frac{H_0\left(1+\varepsilon G\right)\chi_0}{T_a\left(1+\chi_0\right)}\right)\frac{\partial T'}{\partial z}=0\tag{14}$$

where $C_3=\varepsilon_pC_2+\left(1-\varepsilon_p\right)\left(\rho_0C\right)_s$, $C_2=\rho_0\,C_{V,H}$, $\vec{q}\,'=(u',v',w')$ and $\vec{H}=\nabla\phi'$. We eliminate the pressure term in Eq. (12) and then render the resulting equation and Eqs. (13) and (14) dimensionless by means of the following transformations $(x^*,y^*,z^*)=\left(\frac{x}{d},\frac{y}{d},\frac{z}{d}\right)$,

$W^* = \frac{w'}{\left(\frac{K_1}{C_2\,d}\right)}$, $T^* = \frac{T'}{\Delta T}$, $t^* = \frac{t}{\left(\frac{C_2\,d^2}{K_1}\right)}$, $\omega^* = \frac{\omega}{\left(\frac{K_1}{C_2\,d^2}\right)}$ and $\phi^* = \frac{\phi'}{\left(\frac{k_m \Delta T d}{(1+\chi_0)^2}\right)}$ to obtain (after dropping the asterisks for simplicity)

$$\left(\frac{1}{Va}\frac{\partial}{\partial t}+1\right)\nabla^2 W = \left[R + RM_1(1+\varepsilon\cos\omega t)^2\right]\nabla_1^2 T \\ -RM_1(1+\varepsilon\cos\omega t)^2\frac{\partial}{\partial z}\left(\nabla_1^2\phi\right) \tag{15}$$

$$\lambda_p\frac{\partial T}{\partial t} - W + \frac{M_2}{\varepsilon_p}\left(\frac{(1+\varepsilon G)^2}{(1+\chi_0)^2}\right)W + M_2\left(\frac{(1+\varepsilon G)^2}{\chi_0\,(1+\chi_0)}\right)\left(\frac{\partial T}{\partial t} - W\right) \\ - M_2\frac{1}{H_0}\frac{\partial}{\partial t}\frac{H_0\,(1+\varepsilon G)}{(1+\chi_0)}\left(\frac{\partial\phi}{\partial z}\right) - M_2\left(\frac{(1+\varepsilon G)^2}{(1+\chi_0)}\right)\frac{\partial}{\partial t}\left(\frac{\partial\phi}{\partial z}\right) \\ - M_2\frac{1}{H_0}T\frac{\partial}{\partial t}H_0\,(1+\varepsilon G) = \nabla^2 T \tag{16}$$

$$\left(\frac{\partial^2}{\partial z^2}+\nabla_1^2\right)\phi = \frac{\partial T}{\partial z}. \tag{17}$$

The parameter M_2 is equivalent to the order of 10^{-6} . Hence M_2 can be omitted in further calculations (Finlayson [5]). It is suitable to state the whole problem in terms of the vertical component of the velocity W. Upon combining Eqs. (15), (16) and (17), we obtain the following equation

$$\left(\frac{1}{Va}\frac{\partial}{\partial t}+1\right)\left(\lambda_p\frac{\partial}{\partial t}-\nabla^2\right)\nabla^4 W = R\nabla^2\nabla_1^2 W \\ +RM_1(1+\varepsilon f)^2\nabla_1^4 W \tag{18}$$

The boundary conditions in Eq. (7) can be expressed in terms of W in the form [25]

$$W = \frac{\partial^2 W}{\partial z^2} = \frac{\partial^4 W}{\partial z^4} = 0 \text{ at } z=0 \text{ and } z=1. \tag{19}$$

4. Method of solution

We seek the eigenfunctions W and the eigenvalues R associated with the system of Eqs. (18)-(19) for a modulated magnetic field that is different from the constant magnetic field by a small amplitude ($\varepsilon < 1$). The eigenfunction W and eigenvalue R should be a function of ε and they should be obtained for a given buoyancy-magnetization parameter M_1, Vadasz number Va and frequency ω. Since ε is assumed to be very small, we expand these eigenfunctions and eigenvalues in a power series of ε, in accordance with the theory of small parameter perturbation, in the form (Vanezian [35])

$$(W,R) = (W_0,R_0) + \varepsilon(W,R_1) + \varepsilon^2(W_2,R_2) + ... \tag{20}$$

where R_0 is the Rayleigh number of Darcy ferroconvection without modulation. Substituting (20) into Eq. (18) and equating the coefficients of like powers of ε, we obtain the following system of equations up to $O\left(\varepsilon^2\right)$

$$LW_0 = 0 \tag{21}$$

$$LW_1 = R_1 \nabla^2 \nabla_1^2 W_0 + R_1 M_1 \nabla_1^4 W_0 + 2GR_0 M_1 \nabla_1^4 W_0 \tag{22}$$

$$\begin{aligned} LW_2 = R_1 \nabla^2 \nabla_1^2 W_1 + R_2 \nabla^2 \nabla_1^2 W_0 + R_1 M_1 \nabla_1^4 W_1 \\ + R_2 M_1 \nabla_1^4 W_0 + 2GR_0 M_1 \nabla_1^4 W_1 + 2GR_1 M_1 \nabla_1^4 W_0 \end{aligned} \tag{23}$$

where

$$L = \left(\frac{1}{Va}\frac{\partial}{\partial t} + 1\right)\left(\lambda_p \frac{\partial}{\partial t} - \nabla^2\right)\nabla^4 - R_0\left[\frac{\partial^2}{\partial z^2} + (1 + M_1)\nabla_1^2\right]\nabla_1^2. \tag{24}$$

In the absence of magnetic field modulation, the zeroth order solution is similar to that of the corresponding Rayleigh-Bénard ferroconvective problem in a permeable medium. Therefore, the problem's marginally stable solution in the absence of modulation is taken to be

$$W_0 = \left[e^{i(\alpha_x x + \alpha_y y)}\right]\sin \pi z \tag{25}$$

where α_x and α_y are wavenumbers in x and y directions respectively. Substituting (25) into Eq. (21), we obtain the following Rayleigh number equation

$$R_0 = \frac{\left(\pi^2 + \alpha^2\right)^3}{\alpha^2\left[\pi^2 + (1 + M_1)\alpha^2\right]}. \tag{26}$$

Equation (26) is the expression for the thermal Rayleigh number as a function of wavenumber and buoyancy-magnetization parameter for the unmodulated Rayleigh-Bénard ferroconvection problem. The variation of Rayleigh number R_0 versus wavenumber α for different values of buoyancy-magnetization parameter M_1 is displayed in Fig. 2. The destabilizing influence of buoyancy-magnetization mechanism is apparent from Fig. 2. Moreover, in the limiting case of $M_1 = 0$, one recovers the classical result concerning Darcy porous medium convection in a Newtonian fluid (Nield and Bejan [21]). Following the analysis of Venezian [35] and Maruthamanikandan et al., [36], we obtain the following expression for R_2 (the correction to the thermal Rayleigh number)

$$R_2 = -\ \frac{R_0^2 M_1^2 \alpha^6}{\left[\pi^2 + (1 + M_1)\alpha^2\right]}\sum_{n=1}^{\infty}\frac{C_n}{D_n}. \tag{27}$$

where

$$C_n = 2\left(-\omega^2\frac{\lambda_p}{Va}\left(n^2\pi^2 + \alpha^2\right)^2 + \left(n^2\pi^2 + \alpha^2\right)^3 - R_0\alpha^2\left[n^2\pi^2 + (1 + M_1)\alpha^2\right]\right)$$

$$\begin{aligned} D_n = \left(-\omega^2\frac{\lambda_p}{Va}\left(n^2\pi^2 + \alpha^2\right)^2 + \left(n^2\pi^2 + \alpha^2\right)^3 - R_0\alpha^2\left[n^2\pi^2 + (1 + M_1)\alpha^2\right]\right)^2 \\ + \omega^2\left(\frac{1}{Va}\left(n^2\pi^2 + \alpha^2\right)^3 + \lambda_p\left(n^2\pi^2 + \alpha^2\right)^2\right)^2 \end{aligned}$$

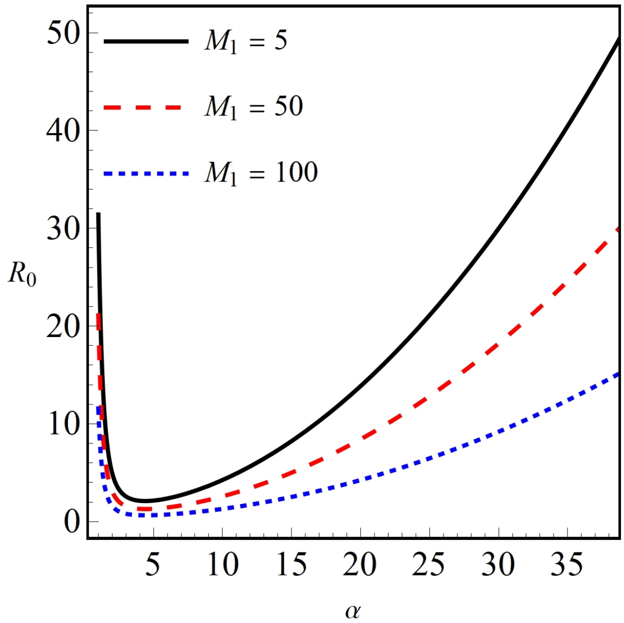

Figure 2. Variation of thermal Rayleigh number R_0 versus wavenumber α for different values of buoyancy-magnetization parameter M_1.

5. Results and discussion

The influence of time-periodically varying magnetic field on the commencement of thermal convection in a horizontal ferromagnetic smart fluid saturated densely packed porous structure is studied analytically using linear stability theory. The shift in the thermal Rayleigh number is computed using the regular perturbation technique, which is based on minimal magnetic field modulation amplitude. The expression for the correction Rayleigh number R_2 is obtained as a function of the modulation frequency ω, the buoyancy-magnetization parameter M_1, the Vadasz number Va and the normalized porosity λ_p. The influence of these parameters on the stability of the system is investigated meticulously. The sign of R_{2c} is responsible for the stabilizing and destabilizing effect of magnetic field modulation on the stability of the system. A positive R_{2c} implies that the magnetic field modulation effect is stabilizing, while a negative R_{2c} is indicative of the destabilizing effect of magnetic field modulation.

The variation of critical correction Rayleigh number R_{2c} with frequency ω for various values of different parameters arising in the study is shown in Figs. 3 through 8. We find from these figures that R_{2c} is negative over a small range of values of ω (when ω is below 5), indicating that the effect of magnetic field modulation is to destabilize the system, with ferroconvection commencing faster than that of the unmodulated system. However, there exists a moderate and large range of frequency (when ω is between 5 and 200) over which magnetic field modulation has the stabilizing effect on the system with ferroconvection occurring at a later point compared to the unmodulated system. As a result, subcritical instability arises over a small range of values of ω and supercritical instability is possible over a moderate and large range of values of frequency. It is also worth noting that when ω is large enough, the magnetic field modulation effect vanishes altogether.

Figs. 3 and 4 show the impact of the buoyancy-magnetization parameter M_1 on the stability of the system. The parameter M_1 is the ratio of magnetic force to gravitational force. It is noticed that R_{2c} increases with an increase in the parameter M_1 over a small

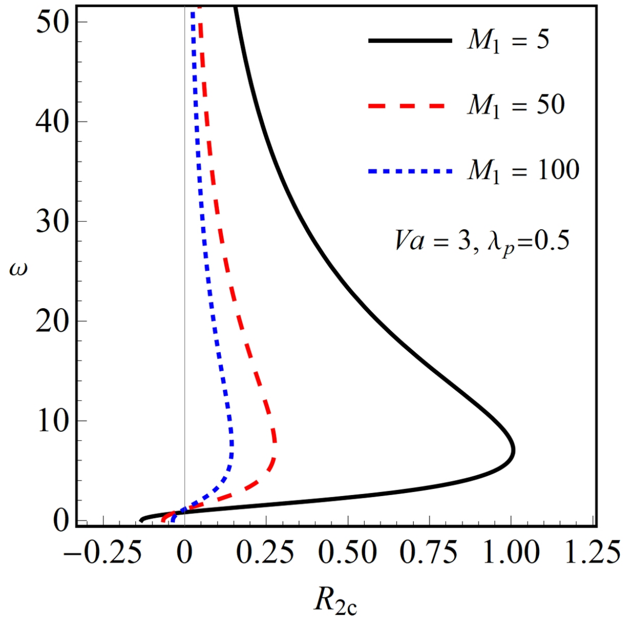

FIGURE 3. Variation of R_{2c} with ω for distinct values of M_1.

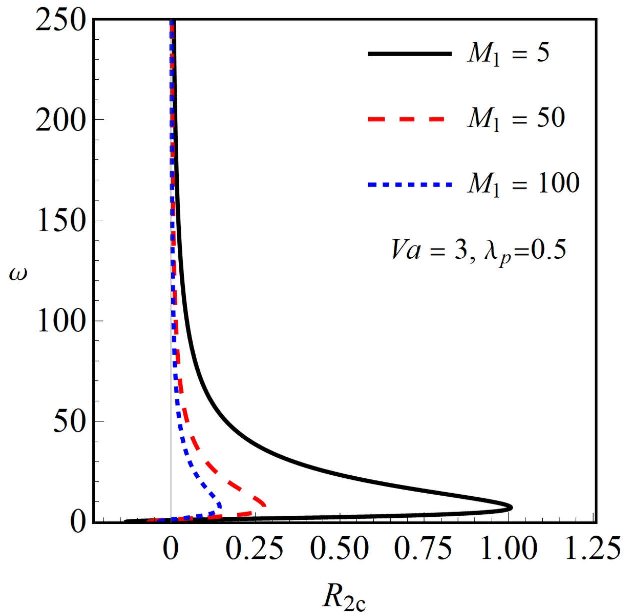

FIGURE 4. Variation of R_{2c} with ω for distinct values of M_1.

range of values of the frequency ω. However, the reverse pattern occurs over a moderate and large range of values of the frequency ω. Furthermore, when ω is small, the parameter M_1 minimizes the destabilizing influence of magnetic field modulation, whereas for moderate and large values of ω, the parameter M_1 reduces the stabilizing effect of magnetic field modulation. This is due to the fact that when M_1 increases, either magnetic force increases or gravitational force decreases, which shows that increasing M_1 increases the magnetic force and makes the system more unstable. It is also noted that R_{2c} increases

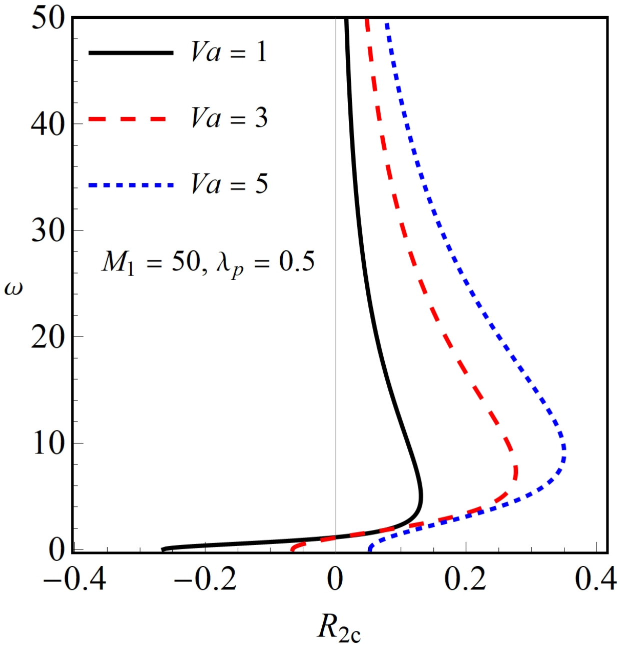

FIGURE 5. Variation of R_{2c} with ω for distinct values of Va.

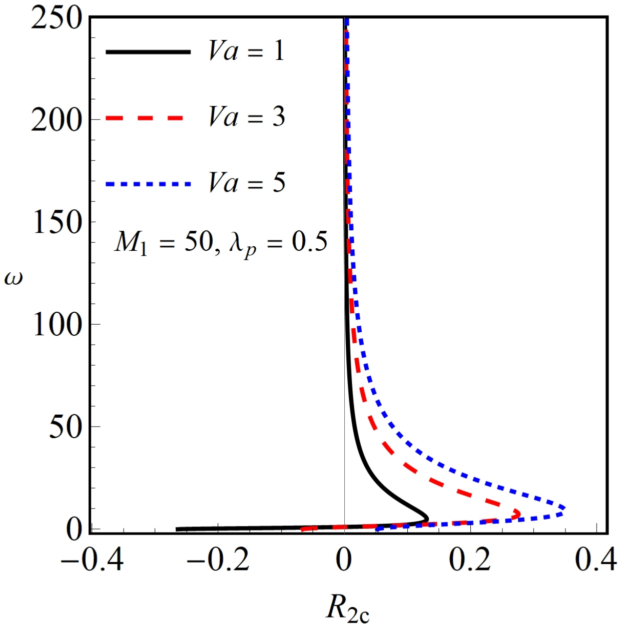

FIGURE 6. Variation of R_{2c} with ω for distinct values of Va.

with increasing the values of ω, reaches a peak, and then decreases with further increase of ω. The intensity of magnetic forces determines the frequency in which the peak value is reached.

The influence of Vadasz number Va on the stability of the system is shown in Figs. 5 and 6 with the parameters M_1 and λ_p are being fixed. The parameter Va includes the Prandtl and Darcy numbers as well as the porosity of the porous domain. It should

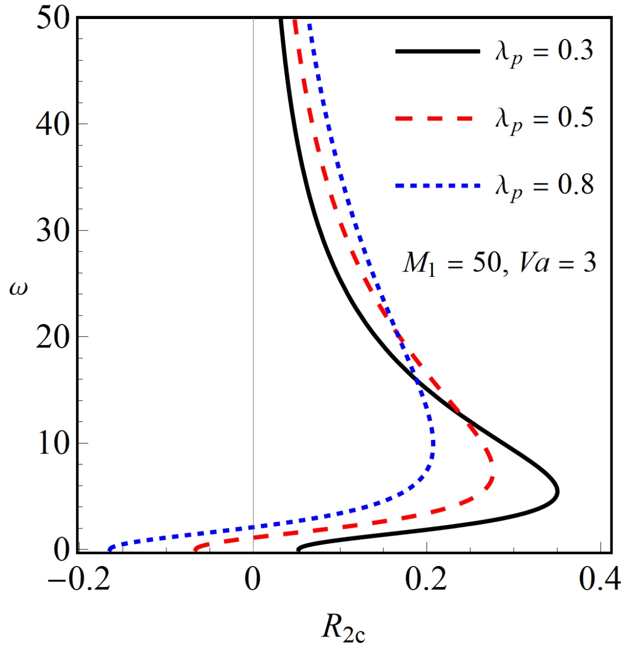

FIGURE 7. Variation of R_{2c} with ω for distinct values of λ_p.

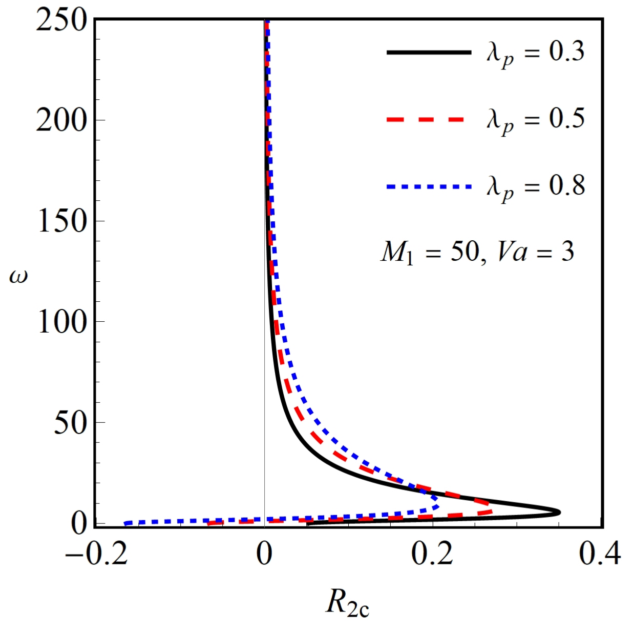

FIGURE 8. Variation of R_{2c} with ω for distinct values of λ_p.

be mentioned that when Va increases, either porosity and Prandtl number increases or Darcy number decreases. We find from these figures that critical correction Rayleigh number R_{2c} increases with increasing the values of Va over entire range of values of ω (when ω lies between 0 and 200) indicating that the effect of Vadasz number has the stabilizing influence on ferroconvection in a magnetic field modulated ferrofluid saturated porous medium. It can also be seen that increase in Va results in the delay of porous medium ferroconvection. This is due to the increase in pore space and the viscous effect, which is responsible for slowing down ferrconvective instability. Further, irrespective of the magnitude of the modulation frequency ω, the Vadasz number Va enhances the stabilizing effect of magnetic field modulation.

The variation of critical correction Rayleigh number R_{2c} versus frequency ω for different values of the normalized porosity λ_p is depicted in Figs. 7 and 8. The normalized porosity is the fraction of porosity to that of specific heat ratio. It is clear that when λ_p increases, either porosity increases or specific heat ratio decreases. We find from these figures that an increase in λ_p increases R_{2c}, provided ω is moderate and large, which indicates that the role of λ_p is to enhance the stability of the system. On the other hand, when ω is small, a strong destabilizing effect of λ_p can be seen from Fig. 7. Therefore, the normalized porosity exhibits a dual impact on the stability of the magnetic field modulated ferrofluid saturated porous layer. In addition, the effect of normalized porosity λ_p is to augment the destabilizing nature of magnetic field modulation when ω is small (when ω lies below 15) and to amplify the stabilizing influence of magnetic field modulation when ω is moderate and large (when ω lies between 15 and 200).

6. Conclusions

The combined influence of modulated magnetic field and the fluid saturated densely packed permeable structure on the threshold of ferroconvection is studied by means of the regular perturbation technique. The following conclusions are drawn:

1. Subcritical instability is noticeable for low frequency of the magnetic field modulation .
2. For moderate and large values of the frequency of magnetic field modulation, buoyancy-magnetization mechanism destabilizes the system and it reduces the stabilizing effect of magnetic field modulation.
3. The effect of the Vadasz number is to stabilize the system and the stabilizing effect of magnetic field modulation is enhanced due to increasing the values of the Vadasz number regardless of the magnitude of the frequency of magnetic field modulation.
4. Porous medium ferroconvection can be delayed by enhancing the normalized porosity provided the frequency of magnetic field modulation is moderate and large.
5. The effects of buoyancy-magnetization mechanism, porous medium and magnetic field modulation are nullified for large values of the frequency of magnetic field modulation.

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