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ON THE INJECTIVE CHROMATIC NUMBER OF SPLITTING GRAPH AND SHADOW GRAPH OF CERTAIN REGULAR AND BIREGULAR GRAPHS

C. K. BHANUPRIYA[1*], M. S. SUNITHA[1], §

ABSTRACT. The injective chromatic number of a graph G, denoted by $\chi_i(G)$ is the minimum number of colors needed to color the vertices of G such that two vertices with a common neighbor are assigned distinct colors. The splitting graph and shadow graph are larger graphs obtained from a graph by a construction. In this article, $\chi_i(G)$ of splitting graph and shadow graph of certain classes of graphs are obtained in terms of number of vertices. Also obtained a lower and upper bound for the injective chromatic number of splitting graph and shadow graph of any graph.

Keywords: Injective coloring; injective chromatic number; splitting graph; shadow graph.

AMS Subject Classification: 05C15, 05C38, 05C76

1. INTRODUCTION

All graphs considered in this article are simple, finite and undirected. The sets $V(G)$ and $E(G)$ represent the vertex set and edge set of a graph G and the symbols $\Delta(G), \omega(G)$ and $N(u)$ denote the maximum degree, clique number of a graph and neighborhood set of a vertex $u \in V(G)$ respectively. For further graph-theoretic notations and terminologies refer [7] and [25].

In 2002, Hahn et al. [6] introduced the notion of injective coloring and injective chromatic number for a graph.

Definition 1.1. *An injective coloring of a graph G is a vertex coloring, that assigns different colors to pair of vertices that have a common neighbor.*

Definition 1.2. *The injective chromatic number of a graph G is the minimum number of colors required for attaining an injective coloring for a graph G and is denoted by $\chi_i(G)$.*

[1] Department of Mathematics, National Institute of Technology Calicut, Kozhikode, India, 673601.
e-mail: ckbhanupriya@gmail.com; ORCID: https://orcid.org/0000-0001-6273-6134.
e-mail: sunitha@nitc.ac.in.; ORCID:https://orcid.org/0000-0002-0090-9252.
[*] Corresponding author.
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In [6], the authors suggested the bounds of injective chromatic number in general and computed $\chi_i(Q_n)$, where Q_n represents the hypercubes. The authors routed, $\chi_i(Q_n)$ to the context of error correcting codes also.

Later, for a chordal graph G, Hell et al.[8], determined $\chi_i(G)$ at least as efficiently as one computes the chromatic number of $(G-B)^2$. Note that here, B represents the bridges of the graph G. The authors also designed a polynomial time algorithm which computes the injective coloring for a given chordal graph. Nevertheless, they extended their study about the injective coloring of split graphs also. Luzar et al., in [13], studied the injective coloring of different planar graphs having large girth using few colors. Kim et al. [10], in 2009 showed that, for a graph G, $\chi_i(G) \geq \frac{1}{2}\chi(G^2)$, where G^2 represents the square of G and $\chi(G)$ represents the chromatic number of G.

Further, in [11], A. Kishore and Sunitha introduced and studied about injective chromatic sum along with injective strength of graphs. The authors determined injective chromatic sum for certain graph classes. The authors also computed bounds of injective chromatic sum and determined injective chromatic sum for different product graphs and also introduced the notion of injective chromatic polynomial. Later in 2015, for join, union, direct product, Cartesian product, graph composition and disjuction of graphs, Song and Yue [21] computed sharp bounds (or the exact values) for the injective chromatic number. Further, in the same year, A. Kishore and Sunitha [12] studied about the coefficients of injective chromatic polynomials of different graph classes like complete graphs, cycles and wheel graphs. For different graph operations like join, union and corona of different graphs, the authors computed the injective chromatic polynomial.

In 1981, Sampathkumar and Walikar [18] introduced the concept of splitting graph of a graph and it is constructed as follows.

Definition 1.3. *Let G be a graph. For each vertex v of G, take new vertex v'. Join v' to all vertices of G adjacent to v. The graph $S(G)$ thus obtained is called the splitting graph of G.*

Definition 1.4. *The shadow graph $D_2(G)$ of a connected graph G is constructed by taking two copies of G say G' and G'' join each vertex v' in G' to the neighbors of the corresponding vertex v'' in G''.*

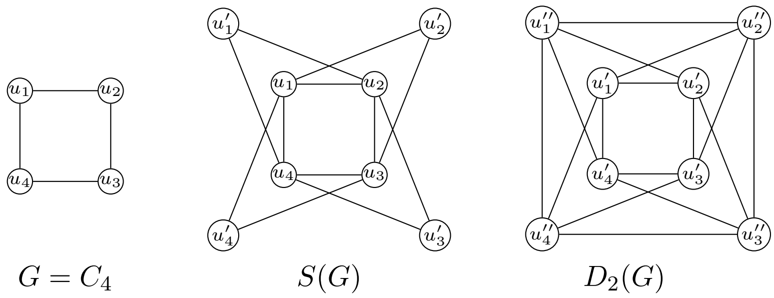

Figure 1: Example of splitting graph and shadow graph of C_4

The domination parameters of splitting graph and shadow graph is well studied in [3, 9, 20, 15]. The Wiener and Harary Index of Splitting Graphs are obtained in [5]. Further in 2017 Ponraj et al. obtained the energy of shadow graph and splitting graph in [24] and in 2022 determined the pair difference cordiality of shadow graph and splitting graph of certain graphs in [17]. Also other graph theoretic parameters for splitting graph and shadow graph are addressed by different authors [14, 19, 1, 23]. Also different coloring parameters of splitting graph and shadow graph are well studied by different authors in [16, 2, 22, 4].

Motivated from this, the topic of this work is the injective coloring of the splitting graph and the shadow graph of various kinds of graphs.
The following result is useful for our main results.

Proposition 1.1. [6]

(1) *Let G be an arbitrary graph of order at least four. Then $\chi_i(G) = |V(G)|$ if and only if either G is a complete graph, or G has diameter 2 and every edge of G is contained in a triangle.*

(2) *Let G be a graph with maximum degree Δ. Then $\chi_i(G) \geq \Delta$.*

(3) *Let P_n be a path of length n, then* $\chi_i(P_n) = \begin{cases} 1, & n = 1,2 \\ 2, & n > 2 \end{cases}$.

(4) *Let C_n be a cycle of length n, then* $\chi_i(C_n) = \begin{cases} 2, & n \equiv 0 \mod 4 \\ 3, & n \not\equiv 4 \mod 4 \end{cases}$.

2. Injective chromatic number of splitting graph of classes of graphs

The injective chromatic number of splitting graphs for several classes of graphs are shown in this section, along with bounds for the injective chromatic number of splitting graph of any given graph G. The injective chromatic number of a splitting graph of any path of length n is found in Theorem 2.1.

Theorem 2.1. *The injective chromatic number of splitting graph of P_n is*
$\chi_i(S(P_n)) = \begin{cases} n, & n = 1,2 \\ 4, & n > 2 \end{cases}$.

Proof. Let $v_1, v_2, \cdots, v_n$ be the vertices of P_n and $u_1, u_2, \cdots, u_n$ be the new set of of vertices for the construction of $S(P_n)$.
Case 1: $n = 1$.
$S(P_1)$ is a graph with two isolated vertices. It is injectively colored with a single color.
Case 2: $n = 2$.
By the construction, $S(P_2)$ is P_4 and P_4 is colored injectivey with 2 colors.
Case 3: $n > 2$.
Let $v_1 - v_2 - v_3 - \cdots - v_n$ be a path of length n. Now by the construction of splitting graph the new vertices u_1 and u_n are adjacent to v_2 and v_{n-1} respectively. Also u_i, $1 < i < n$ is adjacent to v_{i-1} and v_{i+1}. Thus the vertices v_1 and v_n are of degree 2 each and $v_i, 1 < i < n$ are of degree 4 each. Also u_1 and u_n are of degree 1 each and $u_i, 1 < i < n$ are of degree 2 each. Thus $\Delta(S(P_n)) = 4$. Since $\chi_i(G) \geq \Delta(G)$, we have, $\chi_i(S(P_n)) \geq \Delta(S(P_n)) = 4$. Now it is enough to provide an injective coloring of $S(P_n)$ with 4 colors. First color the vertices $v_1, v_2, \cdots, v_n$ sequentially as 1,1,4,4,1,1,4,4,$\cdots$. Now color the vertices $u_1, u_2, \cdots, u_n$ sequentially as 2,2,3,3,2,2,3,3,$\cdots$, which gives an injective coloring of $S(P_n)$ with 4 colors. Thus $\chi_i(S(P_n)) = 4$. □

The following theorem gives the injective chromatic number of $S(K_n)$.

Theorem 2.2. *The injective chromatic number of splitting graph of K_n is*
$\chi_i(S(K_n)) = \begin{cases} n, & n = 1,2 \\ 2n, & \textit{Otherwise.} \end{cases}$

Proof. Let $v_1, v_2, v_3, \cdots, v_n$ be the vertices of K_n and $u_1, u_2, u_3, \cdots, u_n$ be the new set of vertices for the construction of $S(K_n)$. For $n = 1$ or 2, the result follows from Theorem 2.1. For $n \geq 3$, the diameter of $S(K_n)$ is 2 and every edge of $S(K_n)$ lies on a triangle. Then by Proposition 1.1(1) $\chi_i(S(K_n)) = |V(S(K_n))| = 2n$. □

Let $C : V(G) \to \{1, 2, \cdots, n\}$ be an injective coloring of a graph G, $C(u)$ represents the color of the vertex u. Also define $N_2(u) = \{v \in V : \exists\ w \in V,\ u - w - v \text{ is a path in } G\}$. Note that the colors of the vertices in $N_2(u)$ cannot be assigned to the vertex u.

In the next theorem the injective chromatic number of splitting graph of cycles on n vertices is obtained.

Theorem 2.3. *The injective chromatic number of splitting graph of C_n is*

$$\chi_i(S(C_n)) = \begin{cases} 4, & n \equiv 0 \mod 4 \\ 6, & n = 3, 6 \\ 5, & Otherwise. \end{cases}$$

Proof. Let $v_1, v_2, v_3, \cdots, v_n$ be the vertices of C_n and $u_1, u_2, u_3, \cdots, u_n$ be the new set of vertices for the construction of $S(C_n)$.

Case 1: $n \equiv 0 \mod 4$.

Since $\Delta(S(C_n)) = 4$, $\chi_i(S(C_n)) \geq \Delta(S(C_n)) = 4$. Now it is enough to provide an injective coloring of $S(C_n)$ with 4 colors. First color the vertices $v_1, v_2, \cdots, v_n$ sequentially as $1, 1, 2, 2, 1, 1, 2, 2, \cdots$. Now color the vertices $u_1, u_2, \cdots, u_n$ sequentially as $3, 3, 4, 4, 3, 3, 4, 4, \cdots$, which gives an injective coloring of $S(C_n)$ with 4 colors.

Case 2: $n = 3, 6$.

Subcase 1: $n = 3$.

By Proposition 1.1(1), the result is obvious.

Subcase 2: $n = 6$.

In $S(C_6)$, note that, for any two vertices $u, v \in V(S(C_6))$, $d(u, v) \leq 3$ and for $w \in V(S(C_6)) - \{u, v\}$, either $d(u, w) = 2$ or $d(v, w) = 2$. Therefore not more than two vertices can have the same colors. Hence $\chi_i(S(C_6)) \geq \frac{|V(S(C_6))|}{2} = 6$. Also Figure 2 gives an injective coloring of $S(C_6)$ with 6 colors. Thus $\chi_i(S(C_6)) = 6$.

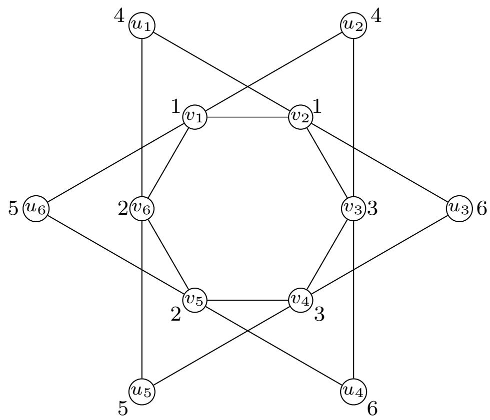

Figure 2: Injective coloring of $S(C_6)$

Case 3: $n \not\equiv 0 \mod 4$ and $n \neq 3, 6$.

For $n \not\equiv 0 \mod 4$, $\chi_i(C_n) = 3$, therefore three colors are needed to color the vertices of C_n. Also for $n \equiv 0 \mod 4$, a total $\chi_i(C_n) + 2$ colors are used to color the vertices of $S(C_n)$. Thus here $\chi_i(C_n) + 2 = 5$ colors are needed to color the vertices of $S(C_n)$.

Subcase 1: $n \equiv 1 \mod 4$.

First color the vertices $v_1, v_2, \cdots, v_{n-1}$ of C_n sequentially as 1,1,2,2,1,1,2,2,$\cdots$ and the vertex v_n with color 3. Now color the vertices $u_1, u_2, \cdots, u_n$ as follows.

- $N_2(u_1) = \{v_1, u_3, v_3, u_{n-1}, v_{n-1}\}$. Colors of the vertices in $N_2(u_1)$ are 1 and 2. Thus $C(u_1) = 3$.

- $N_2(u_2) = \{v_2, v_n, u_n, v_4, u_4\}$. Colors of the vertices in $N_2(u_2)$ are $1, 2$ and 3. Thus $C(u_2) = 4$.
- For $i = 3, 4, \cdots, n-2$, $N_2(u_i) = \{v_i, v_{i-2}, u_{i-2}, v_{i+2}, u_{i+2}\}$. Colors of the vertices in $N_2(u_i)$ are $1, 2, 3$ for i such that $i \equiv 3, 2 \mod 4$ and $1, 2, 4$ for i such that $i \equiv 0, 1 \mod 4$). Thus the vertices $u_3, u_4, \cdots, u_{n-2}$ are colored sequentially as 4,3,3,4,4,3,3$\cdots$.
- $N_2(u_{n-1}) = \{v_{n-1}, v_1, u_1, v_{n-3}, u_{n-3}\}$. Colors of the vertices in $N_2(u_{n-1})$ are $1, 2, 3, 4$. Thus $C(u_{n-1}) = 5$.
- $N_2(u_n) = \{u_n, u_2, v_2, u_{n-2}, v_{n-2}\}$. Colors of the vertices in $N_2(u_n)$ are $1, 2, 3, 4$. Thus $C(u_n) = 5$.

Which gives an injective coloring of $S(C_n)$ for $n \equiv 1 \mod 4$ with 5 colors. Similarly, we can injectively color for $S(C_n)$ where $n \equiv 2 \mod 4$ and $n \equiv 3 \mod 4$ with 5 colors. □

A sharp bound for the injective chromatic number of a splitting graph of any graph with maximum degree $\Delta(G)$ and number of vertices n is given as follows.

Theorem 2.4. *Let G be a graph with n vertices. Then $2\Delta(G) \leq \chi_i(S(G)) \leq 2n$.*

Proof. We have $\chi_i(G) \geq \Delta(G)$ and $\Delta(S(G)) = 2\Delta(G)$. Thus $\chi_i(S(G)) \geq 2\Delta(G)$. Also we have $\chi_i(G) \leq n$, where n is the number of vertices. Clearly the number of vertices of $S(G)$ is $2n$. Also the bound is sharp since the lower bound is attained for $S(P_n)$ for any $n > 2$ and the upper bound is attained for $S(K_n)$ for any n. □

In Theorem 2.5, the injective chromatic number of splitting graph of $K_{m,n}$ is obtained.

Theorem 2.5. *The injective chromatic number of splitting graph of $K_{m,n}$ is $\chi_i(S(K_{m,n})) = 2m$, $m \geq n$.*

Proof. Let $m \geq n$, $u_1, u_2, \cdots, u_m, v_1, v_2, \cdots, v_n$ be the vertices of $K_{m,n}$ and let $u'_1, u'_2, \cdots, u'_m$, v'_1, v'_2, $\cdots$, v'_n be the new set of vertices for the construction of $S(K_{m,n})$. By the construction of splitting graph of $K_{m,n}$, a vertex v_i is adjacent to all u_i and u'_i, $1 \leq i \leq m$. Hence the vertices u_i and u'_i, $1 \leq i \leq m$ are colored with distinct $2m$ colors. Let color i be the color of the vertex u_i, $1 \leq i \leq m$ and color $m+i$ be the color of the vertex u'_i, $1 \leq i \leq m$. Also no colored vertex is a common neighbor of the vertices $v_1, v_2, \cdots, v_n, v'_1, v'_2, \cdots, v'_n$. So from the $2m$ colors, $2n$ colors are used to color the vertices v_i and v'_i, $1 \leq i \leq n$. Let color i be the color of the vertex v_i, $1 \leq i \leq n$ and color $n+i$ be the color of the vertex v'_i, $1 \leq i \leq n$. Hence $\chi_i(S(K_{m,n})) = 2m$. □

Corollary 2.1. *The injective chromatic number of splitting graph of star graph S_{n+1} is $\chi_i(S(S_{n+1})) = 2n$.*

In the following section, the injective chromatic number of shadow graph of different classes of graphs and a bound for the injective chromatic number of shadow graph of any arbitrary graph are discussed.

3. Injective chromatic number of shadow graph of classes of graphs

In this section, we obtained the injective chromatic number of shadow graph of different classes of graphs and a sharp bound for the injective chromatic number of shadow graph of any graphs are obtained. Assume for the following theorems that, if G is a graph with n vertices, then the vertices of $D_2(G)$ be $u_1, u_2, \cdots, u_n$, $v_1, v_2, \cdots, v_n$ where $u_1, u_2, \cdots, u_n$ be the vertices of first copy G' of G and $v_1, v_2, \cdots, v_n$ be the vertices of second copy G'' of G. A result on the injective chromatic number of shadow graph of complete graph is obtained as a Corollary to Proposition 1.1(1).

Corollary 3.1. *The injective chromatic number of shadow graph of K_n is $\chi_i(D_2(K_n)) = 2n$.*

In Theorem 3.1, the injective chromatic number of $D_2(P_n)$ is obtained.

Theorem 3.1. *The injective chromatic number of shadow graph of P_n is $\chi_i(D_2(P_n)) = \Delta(D_2(P_n))$.*

Proof. The injective chromatic number of $D_2(P_2)$, $\chi_i(D_2(P_2)) \geq \Delta(D_2(P_2)) = 2$ by Proposition 1.1(2). Now Figure 4 provides an injective coloring of $D_2(P_2)$ using 2 colors. Thus $\chi_i(D_2(P_2)) = 2$. Now for $n > 2$, maximum degree of $D_2(P_n)$ is 4, then by Proposition 1.1(2), $\chi_i(G) \geq \Delta(G)$ for any graph G. Thus $\chi_i(D_2(P_n)) \geq 4$. Now providing an injective coloring of $D_2(P_n)$ with 4 colors shows that $\chi_i(D_2(P_n)) = 4$. Figure 3 gives the injective coloring of $D_2(P_n)$ with 4 color.

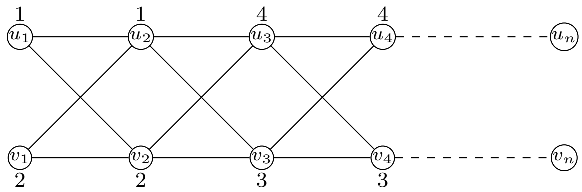

Figure 3: Injective coloring of $D_2(P_n)$

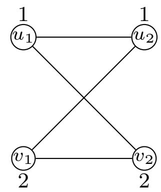

Figure 4: Injective coloring of $D_2(P_2)$

□

A sharp bound for the injective chromatic number of shadow graph of any graph G with maximum degree $\Delta(G)$ and number of vertices n is given in Theorem 3.2.

Theorem 3.2. *Let G be a graph with n vertices. Then $2\Delta(G) \leq \chi_i(D_2(G)) \leq 2n$.*

Proof. We have $\chi_i(G) \geq \Delta(G)$ and $\Delta(D_2(G)) = 2\Delta(G)$. Thus $\chi_i(D_2(G)) \geq 2\Delta(G)$. Also we have $\chi_i(G) \leq n$, where n is the number of vertices in G, number of vertices in $D_2(G)$ is $2n$. Also the bound is sharp since the lower bound is attained for $D_2(P_n)$ for any n and the upper bound is attained for $D_2(K_n)$ for any n. □

The injective chromatic number of $D_2(C_n)$ is obtained as follows.

Theorem 3.3. *The injective chromatic number of shadow graph of C_n is*

$$\chi_i(D_2(C_n)) = \begin{cases} 6, & n = 3, 6 \\ 4, & n \equiv 0 \mod 4 \\ 5, & n \neq 3, 6 \text{ and } n \not\equiv 0 \mod 4 \end{cases}.$$

Proof. **Case 1**: $n = 3, 6$.
For $n = 3$, the result follows from Corollary 2. Also in $D_2(C_6)$, note that, for any two vertices $u, v \in V(D_2(C_6))$, $d(u, v) \leq 3$ and for $w \in V(S(C_6)) - \{u, v\}$, either $d(u, w) = 2$ or $d(v, w) = 2$. Therefore at most two vertices can have the same colors. Hence $\chi_i(D_2(C_6)) \geq \frac{|V(D_2(C_6))|}{2} = 6$. Also Figure 5 gives an injective coloring of $D_2(C_6)$ with 6 colors. Thus $\chi_i(D_2(C_6)) = 6$.

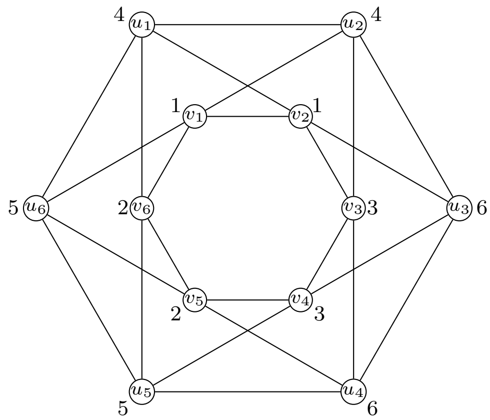

Figure 5: Injective coloring of $D_2(C_6)$

Case 2: $n \equiv 0 \mod 4$.
Since $\Delta(D_2(C_n)) = 4$, $\chi_i(D_2(C_n)) \geq \Delta = 4$. Now it is enough to provide an injective coloring with 4 colors. First color the vertices $v_1, v_2, \cdots, v_n$ sequentially as $1, 1, 2, 2, 1, 1, 2, 2, \cdots$. Next color the vertices $u_1, u_2, \cdots, u_n$ sequentially as $3, 3, 4, 4, 3, 3, 4, 4, \cdots$, which gives an injective coloring of $D_2(C_n)$ with 4 colors.
Case 3: $n \neq 3, 6$ and $n \not\equiv 0 \mod 4$.
For $n \not\equiv 0 \mod 4$, $\chi_i(C_n) = 3$, therefore three colors are needed to color the vertices of copies of C_n. Also for $n \equiv 0 \mod 4$, totally $\chi_i(C_n) + 2$ colors are used to color the vertices of $S(C_n)$. Thus $\chi_i(C_n) + 2 = 5$ colors are needed to color the vertices of $S(C_n)$.
Subcase 1: $n \equiv 1 \mod 4$.
First color the vertices $v_1, v_2, \cdots, v_{n-1}$ of C_n sequentially as 1,1,2,2,1,1,2,2$\cdots$ and the vertex v_n with color 3. Now color the vertices $u_1, u_2, \cdots, u_n$ as follows.

- $N_2(u_1) = \{v_1, u_3, v_3, u_{n-1}, v_{n-1}\}$. Colors of the vertices in $N_2(u_1)$ are $1, 2$. Thus $C(u_1) = 3$.
- $N_2(u_2) = \{v_2, v_n, u_n, v_4, u_4\}$. Colors of the vertices in $N_2(u_2)$ are $1, 2, 3$. Thus $C(u_2) = 4$.
- For $i = 3, 4, \cdots, n-2$, $N_2(u_i) = \{v_i, v_{i-2}, u_{i-2}, v_{i+2}, u_{i+2}\}$. colors of the vertices in $N_2(u_i)$ are $1, 2, 3$ for i such that $i \equiv 3, 2 \mod 4$ and $1, 2, 4$ for i such that $i \equiv 0, 1 \mod 4$. Thus the vertices $u_3, u_4, \cdots, u_{n-2}$ are colored sequentially as 4,3,3,4,4,3,3$\cdots$.
- $N_2(u_{n-1}) = \{v_{n-1}, v_1, u_1, v_{n-3}, u_{n-3}\}$. Colors of the vertices in $N_2(u_{n-1})$ are $1, 2, 3, 4$. Thus $C(u_{n-1}) = 5$.
- $N_2(u_n) = \{u_n, u_2, v_2, u_{n-2}, v_{n-2}\}$. Colors of the vertices in $N_2(u_n)$ are $1, 2, 3, 4$. Thus $C(u_n) = 5$.

This gives an injective coloring of $D_2(C_n)$ for $n \equiv 1 \mod 4$ with 5 colors. Similarly, we can injectively color $D_2(C_n)$ for $n \equiv 2 \mod 4$ and $n \equiv 3 \mod 4$ with 5 colors. □

In Theorem 3.4 the injective chromatic number of shadow graph of $K_{m,n}$ is obtained.

Theorem 3.4. *The injective chromatic number of shadow graph of $K_{m,n}$ is $\chi_i(D_2(K_{m,n})) = 2\ max\{m, n\}$.*

Proof. Without loss of generality, assume that $n \geq m$. Let $v_1, v_2, \cdots, v_n, u_1, u_2, \cdots, u_m$ be the vertices of the first copy of $K_{m,n}$ and $v'_1, v'_2, \cdots, v'_n, u'_1, u'_2, \cdots, u'_m$ be the vertices of the second copy of $K_{m,n}$ for the construction of $D_2(K_{m,n})$.
From the Figure 6, it can be seen that no vertices v_i's or v'_i's, $1 \leq i \leq n$ can have the same

colors, since any vertex u_j, $1 \leq j \leq m$ is a common vertex for the vertices v_i's and v_i''s. Thus $2n$ distinct colors are needed to color the vertices v_i and v_i', $1 \leq i \leq n$. The same set of n colors are enough to color the vertices u_j's and u_j''s for $1 \leq j \leq m$, since these vertices have no common vertices with the vertices v_i and v_i', $1 \leq i \leq n$.

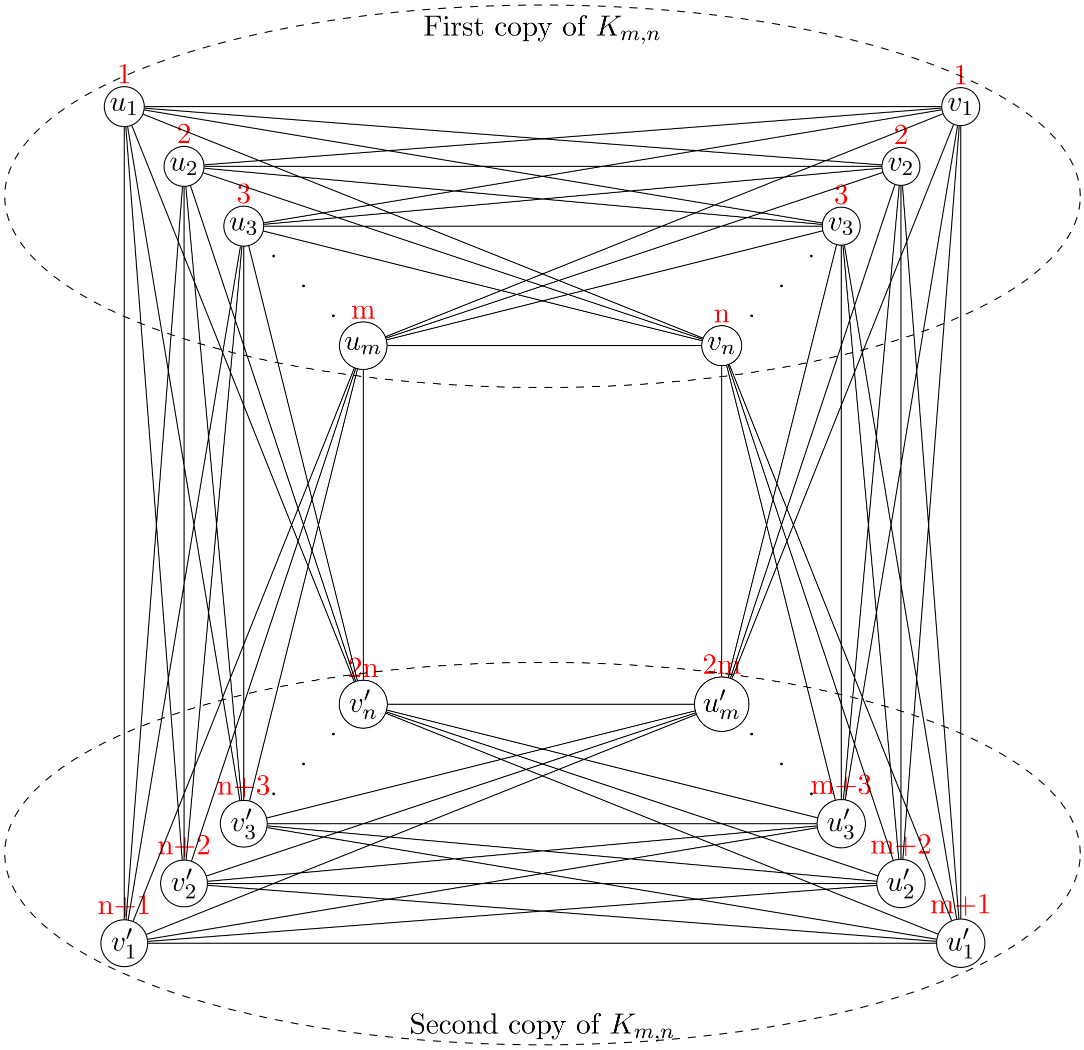

Figure 6: Injective coloring of $D_2(K_{m,n})$

□

Corollary 3.2. *The injective chromatic number of shadow graph of S_{n+1}, star graph with $n+1$ vertices is $\chi_i(D_2(S_{n+1})) = 2n$.*

4. Conclusions

Splitting graph, $S(G)$ and shadow graph, $D_2(G)$ are larger graphs obtained from G by means of construction. In this article the injective chromatic number of splitting graph and shadow graph of different classes of graphs are expressed in terms of number of vertices. Also a sharp lower and upper bound for the injective chromatic number of splitting graph and shadow graph of any graph is suggested. It is open to compute the injective chromatic number of splitting graph and shadow graph of any arbitrary graphs.

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Bhanupriya C K received her BSc degree in Mathematics from WMO Arts and Science College, Muttil, Wayanad, Kerala, India in 2014 and MSc degree in Mathematics from National Institute of Technology Calicut, Kerala, India in 2016. Currently she is doing research at the same institute. Her research interest includes Discrete Mathematics and Graph Theory.

Sunitha M S is currently working as a Professor of Mathematics in National Institute of Technology Calicut, Kerala, India. She is a Life Member of Academy of Discrete Mathematics and Applications (ADMA), India. Her research interest include Fuzzy Graph Theory and Algebraic Graph Theory.