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FAULT ESTIMATION BASED ON OBSERVER FOR FRACTIONAL-ORDER LU SYSTEM WITH BIFURCATION PROBLEM

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Abstract. Control systems have been susceptible to faults in recent years due to the complexities of their equipment and long-term function. These faults can reduce system performance, impose losses and casualties, and/or reduce final product quality. In order to detect such defects in a control system, fault estimation (FE) has been adopted in several studies. In this paper Estimating the sensor-based fault of a fractional-order Lu system with disturbances such as noise was investigated. This fault estimation problem is implemented using a bifurcation problem in the fractional-order Lu system. This study tries to synthesize the perception of bifurcation problem and fault estimation. Noise disturbance in nonlinear systems makes fault diagnosis a complex problem that should be managed over Takagi-Sugeno (TS) approximated fuzzy models. This approximation method can predict the faults and states of system. To model the observer, the TS fuzzy model was incorporated with a proportional plus integral. The results proved that the observer has a desirable performance to measure states and fault of sensor. Based on the estimations, the actual value of the parameters gives us the deviation value, which is equal to the low amount of deviation here, due to the bifurcation that happens in the system.

Keywords: Fault estimation. Noise disturbance. Fractional-order Lu system. Bifurcation problem.

AMS Subject Classification:

1. Introduction

Today modern societies are heavily depended on industrial systems and modern technology. These systems and technologies are prone to faults, which may cause loss of life and property. These faults in operation reduce the performance of systems and sometimes causes complete system failure. As the science of control systems progress and becomes more complicated, so the security and reliability of these systems have drawn a great deal of attention. Since no system can work flawlessly in practice, fault estimation is of particular importance [1]. Sensor-based fault make the process fail to show the

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actual value and the system moves away from its operating point. In this regard, the issue of safety, productivity, and economic use of industrial systems and equipment are of particular importance. Therefore, diagnosing the fault and isolating it in the systems is inevitable [2]. In many control systems, only a limited number of system modes that determine its output can be measured. In the control loop, a viewer is used to estimate the system's immeasurable states. In fact, the viewer uses estimates in the control loop to estimate unmeasurable states. Therefore, it can be said that the viewer is designed for a system in which the system specifications, including control inputs and outputs of the system are completely clear [3]. So far, many studies have been performed to illustrate the effectiveness of sensor-based fault estimation and diagnosis.

The method of fault diagnosis based on equality transactions is based on the production of residuals, so that by creating a comparative operation, a residue is created between the use of high-precision mathematical calculations and the values obtained from physical measurements. If no residue is generated, it indicates that there is no fault [4-5]. The state variables is estimated by state observer using the signal measurement of the output and control variables. While, the theory of state variables estimation is developed for feedback mode, this theory is used in several other applications like fault prediction and compensation. When it comes to the observer's discussion, we need an observable system. Without it, state variables design is not possible [6-8].

A neural observer uses the fault equations that is derived from the observer's output and performs the remaining production operation to estimate and identify of the fault. The main point to design and implement a neural observer is to use a neural network to identify and reconstruct the observer's state. One of the advantages of a neural network is that it is able to identify and model a nonlinear system on its own at any time [9-10].

In [11], a PI observer is employed for FE in a chaotic system. A two-stage Kalman filter and an adaptive Kalman filter are utilized for FE in [12, 13]. Although comprehensive methods are proposed, designing an observer using the L1 observer in these methods remains a challenge. Hence, [14] has introduced an FE technique based on the L1 observer through the nonlinear Takagi-Sugeno (TS) system. Despite an advanced model, this method's computational complexities remain a major challenge. A nonlinear time-invariant FE observer is proposed in [15]. Not only it neutralizes disturbances, but it also improves the transient FE function. Despite its excellent performance, this method would not be efficient for complex systems.

Artificial intelligence, fuzzy systems, and intelligent optimization algorithms have recently been of great interest to researchers in engineering fields. Numerous studies have been conducted on the FE of nonlinear systems [16]. Chaotic particle swarm optimization with dynamic self-optimization has been employed to estimate the parameters of a complex system in [17]. In [18], an event-triggered adaptive hybrid fuzzy dynamic surface control strategy has been proposed for a class of uncertain multi-input multi-output systems through a fuzzy-adaptive observer. Moreover, a fuzzy T-S system-based FE method is introduced for nonlinear systems susceptible to unknown faults and inputs [19, 20]. A type-2 fuzzy model is used in [21]. Despite the high performance of intelligent optimization algorithms and fuzzy systems, the different results of optimization techniques and the consideration of only one chaotic system in fuzzy models remain major challenges.

Conceptual fuzzy observers Takagi-Sugeno design is based on a concept called Parallel Distributed Compensation [22]. Ref [22] provides a way to identify a fault using a fuzzy observer in which all cases are estimated by one observer; this estimate indicates the fault event tree. Due to uncertainties, such as turbulence and unspecified inputs, it became clear that the system that was supposed to detect the fault based on the above method had to be robust in order to be able to show the actual fault. In light of this, the fuzzy observer combination with the sliding mode was performed in the robust operation [23]. The approach was to define two fuzzy sliding observers, one as an observer that tries to detect a fault by driving a switch term, and the other as an estimator.

By definition, bifurcation theory is to study changes in the qualitative or topological structure of a family in mathematical way. The family can be integral curves of a family of vector field and the solutions of a family with differential equations. Bifurcation is mostly used for mathematical study of dynamical systems and it happens when a small smooth change happens in the parameter values (the bifurcation parameters) of a system and triggers a sudden change in the behavior that can be qualitative or topological [24]. In a noisy environment, the fault of process and sensor is estimated at the same time. The process fault problem results in a bifurcation in the system. We want to mix the concept of fault and bifurcation. When we use fuzzy model, diagnosing fault of a nonlinear system is more feasible.

A review of the literature suggests that further FE research is required. Hence, this paper introduces a method Fault diagnosis and estimate of state of the fractional-order Lu system under fault and noise condition with bifurcation problem were investigated. A line of the design-based mechanism was applied in the Takagi-Sugeno observer that successfully estimated the faults similarly states. Based on the estimations, parameters deviation provided that make with the the process fault is determined.Fault diagnosis and estimate of state of the fractional-order Lu system under fault and noise condition with bifurcation problem were investigated. A line of the design-based mechanism was applied in the Takagi-Sugeno observer that successfully estimated the faults similarly states. Based on the estimations, parameters deviation provided that make with the the process fault is determined.

The contributions of the present work include the following:

1. Fault detection and state estimation of low-order fractional systems under fault and noise conditions with branch problems,

2. Utilization of a T-S observer for nonlinear systems,

3. Comparison to adaptive control systems to show the advantages of the proposed model, and

4. Provision of the results for both Lu and Genesio-Tesi chaotic systems.

In sector 2, the fractional-order Lu system and its Takagi-Sugeno fuzzy model is described. Deign of Takagi-Sugeno fuzzy is introduced in sector 3. In sector 4, the simulation in diagnosis and estimation of sensor fault are illustrated and finally, the outcome of this paper is provided in sector 5.

2. SYSTEM MODELLING

A. Fractional-Order Lu System. Lu system is a novel chaotic system in which the Lorenz system and Chen system are connected and represents the transition from one to the other [25]. Lu system is described by

$$\begin{cases} \dfrac{dx}{dt} = a(y-x), \\ \dfrac{dy}{dt} = -xz + cy, \\ \dfrac{dz}{dt} = xy - bz. \end{cases} \tag{1}$$

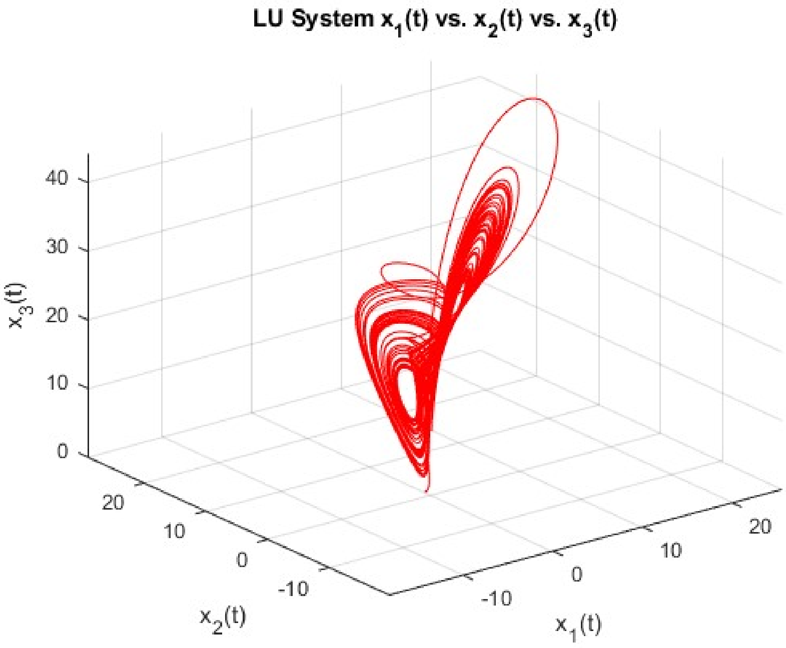

FIGURE 1. Chaotic attractor of the integer-order Lu system with $(a, b, c) = (36, 3, 20)$.

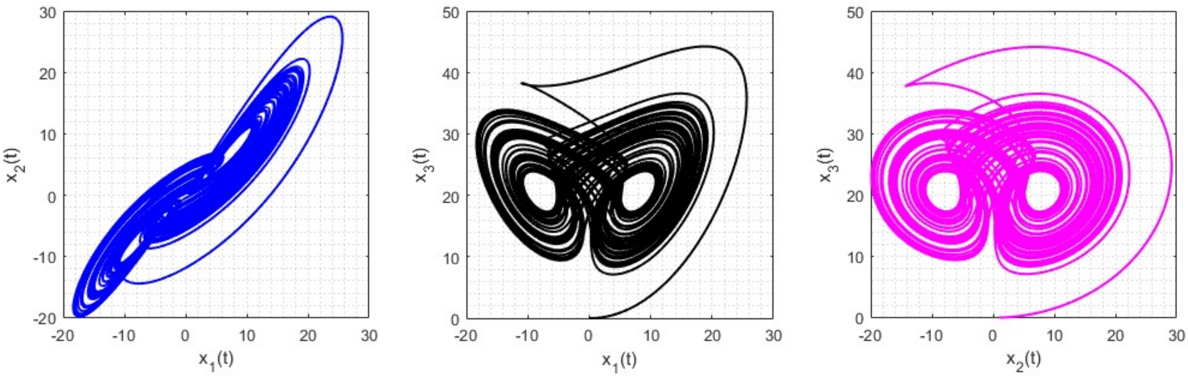

FIGURE 2. 2D Chaotic attractor of the integer-order Lu system

Where $(a, b, c) \in \mathbb{R}^3$ represents constant parameters. When $(a, b, c) = (36, 3, 20)$, system (1) is featured with a chaotic attractor as shown in Fig. 2 Therefore, the fractional-order Lu system is as follow:

$$\begin{cases} \dfrac{d^4x}{dt} = a(y - x), \\ \dfrac{d^4y}{dt} = -xz + cy, \\ \dfrac{dz}{dt} = xy - bz. \end{cases} \tag{2}$$

Where q represents the fractional order that meet $0 < q \leq 1$ condition. In what follows, only the derivative order q vary along with parameters a, c, and the other system parameters remain unchanged. Simulations are done for $q = [0.9.0.1]$ in step size 0.1.

B. Takagi-Sugeno Fuzzy Representation. Most of physical system are highly complicated in practice so that it is not easy to develop reliable mathematical model for them. Still, most of these systems can be represented in some mathematical model form locally or as an aggregation of a group of mathematical models. In this paper, a complex nonlinear system with unknown inputs was represented using the T-S fuzzy dynamic model [26]. In addition, the following T-S fuzzy model is adopted:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r_F} \mu_i \left(A_i x(t) + B_i u(t) + F_i f_p(t) + H_i w(t) \right) \\ y(t) = Cx(t) + F f_s(t) \end{cases} \tag{3}$$

With

$$\sum_{i=1}^{N} \mu_i \left(\xi \right) = 1. \qquad ; \qquad 0 < \mu_i \left(\xi \right) \leq 1, \;\; \forall i \in \mathbb{N} \tag{4}$$

Where $x \in \mathbb{R}^n$ represents the state vector, $u \in \mathbb{R}^m$ is the input vector, $\overline{u} \in \mathbb{R}^q$. $q < n$ has the unknown inputs and $y \in \mathbb{R}^p$ is the measured outputs. Matrices $A_i \in \mathbb{R}^{n \times n}$ and $B_i \in \mathbb{R}^{n \times m}$ represent the state matrix and the input matrix associated with the ith local model respectively. Matrices $R_i \in \mathbb{R}^{n \times q}$ stand for the distribution matrices of unknown inputs. $D_i \in \mathbb{R}^n$ is given to cover the operating point of the system. Finally, ξ stands for decision vector which, to define the operation regimes, uses some subset of the known inputs and/or measured variables.

Matrices A_i, B_i, C_i and C can be determined through direct linearization of an a priori nonlinear model near operating points, or using an identification procedure [27-30]. In the rest of paper, we assume that the vector ξ depends on measurable variables.

C. Takagi-Sugeno fuzzy observer model. To estimate sensor-based faults and also the states of (3), y is filtered using another state $q(t) \in R^q$ and a new matrix –T, described by $\dot{q}(t) = -T(q(t) - y(t))$. It is amplified with another state variable, $\xi(t)$ [31]:

$$\begin{cases} \dot{\xi}(t) = \sum_{i=1}^{r_F} \mu_i \left(A_{1i} \xi(t) + B_{1i} u(t) + F_{1i} f(t) + H_{1i} w(t) \right) \\ \qquad\qquad v(t) = C_1 \xi(t) \end{cases} \tag{5}$$

Where

$$\xi(t) = \begin{bmatrix} x(t) \\ q(t) \end{bmatrix}, f(t) = \begin{bmatrix} f_a(t) \\ f_s(t) \end{bmatrix}, A_{1i} = \begin{bmatrix} A_i & 0 \\ TC & -T \end{bmatrix},$$

$$F_{1i} = \begin{bmatrix} F_i & 0 \\ 0 & TF \end{bmatrix}, B_{1i} = \begin{bmatrix} B_i \\ 0 \end{bmatrix}, H_{1i} = \begin{bmatrix} H_i \\ 0 \end{bmatrix}, C_1 = \begin{bmatrix} 0 & I_q \end{bmatrix} \tag{6}$$

The matrices F_{1i} are full column rank. Signals that change with time represent the faults and their kth derivatives are limited. This supposition leads to the structure below:

$$\begin{cases} \dot{f}(t) &= f_1(t) \\ & \vdots \\ \dot{f}_{k-1}(t) &= f_k(t) \\ f_k(t) &\leq f_0 \end{cases} \tag{7}$$

The structure of the proportional integral fuzzy observer is as below:

$$\begin{cases} \dot{\hat{\xi}}(t) &= \sum_{i=1}^{r_F} \mu_i\left(\hat{\xi}\right)\left(A_{1i}\hat{\xi}(t) + B_{1i}u(t) + F_{1i}\hat{f}(t) + K_{pi}\left(v(t) - \hat{v}(t)\right)\right) + g_r(t) \\ \hat{v}(t) &= C_1\hat{\xi}(t) \\ \dot{\hat{f}}(t) &= \sum_{i=1}^{r_F} \mu_i\left(\hat{\xi}\right) K_{li}\left(v(t) - \hat{v}(t)\right) + \hat{f}_1(t) + g_f(t) \\ \dot{\hat{f}}_j(t) &= \sum_{i=1}^{r_F} \mu_i\left(\hat{\xi}\right) K_{li}^j\left(v(t) - \hat{v}(t)\right) + \hat{f}_{j+1}(t) + g_{f_j}(t) \text{ for } j: 1\cdots k-1 \end{cases} \tag{8}$$

That K_{pi}, K_{li} and K_{li}^j are the gains. $g_r(t)$, $g_f(t)$ and $g_{f_i}(t)$ are described to improve the effect of the immeasurable decision variables. The observer (8) and also the system (5) represented as amplified forms:

$$\begin{cases} \dot{\overline{\xi}}(t) = \sum_{i=1}^{r_F} \mu_i\left(A_{2i}\overline{\xi}(t) + B_{2i}u(t) + H_{2i}w(t) + J_{1i}f_k(t)\right) \\ \overline{v}(t) = C_2\overline{\xi}(t) \end{cases} \tag{9}$$

3. SIMULATION RESULTS

A.

$$\begin{cases} \dot{\hat{\overline{\xi}}}(t) = \sum_{i=1}^{r_F} \mu_i\left(A_{2i}\hat{\overline{\xi}}(t) + B_{2i}u(t) + K_i\left(\overline{v}(t) - \hat{\overline{v}}(t)\right)\right) + g(t) \\ \hat{\overline{v}}(t) = C_2\hat{\overline{\xi}}(t) \end{cases} \tag{10}$$

That

$$\overline{\xi}(t) = \begin{bmatrix} \xi(t) \\ f(t) \\ f_1(t) \\ \cdots \\ f_{k-1}(t) \end{bmatrix}, \hat{\overline{\xi}}(t) = \begin{bmatrix} \hat{\xi}(t) \\ \hat{f}(t) \\ \hat{f}_1(t) \\ \cdots \\ \hat{f}_{k-1}(t) \end{bmatrix}, g(t) = \begin{bmatrix} g_r(t) \\ g_f(t) \\ g_{f_1}(t) \\ \cdots \\ g_{f_{k-1}}(t) \end{bmatrix}$$

$$J = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ I_{n_f} \end{bmatrix}, H_{2i} = \begin{bmatrix} H_{1i} \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, A_{2i} = \begin{bmatrix} A_{1i} & F_{1i} & 0 & 0 & \cdots & 0 \\ 0 & 0 & I_{n_v} & 0 & \cdots & 0 \\ 0 & 0 & 0 & I_{n_v} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & I_{n_v} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$B_{2i} = \begin{bmatrix} B_{1i} \\ 0 \\ 0 \\ \cdots \\ 0 \end{bmatrix}, K_i = \begin{bmatrix} K_{pi} \\ K_{li} \\ K_{li}^1 \\ \cdots \\ K_{li}^{k-1} \end{bmatrix}, C_2 = \begin{bmatrix} C_1 & 0 & 0 & \cdots & 0 \end{bmatrix}$$

$$\overline{e}(t) = \overline{\xi}(t) - \hat{\overline{\xi}}(t) \tag{11}$$

$$\hat{e}_y(t) = \overline{v}(t) - \hat{\overline{v}}(t)$$

And

$$\hat{\bar{e}}(t) = \sum_{i=1}^{r_F} \mu_i \left(\hat{\bar{\xi}}\right) \overline{A}_i \bar{e}(t) + G\varepsilon(t) + \Delta.z(t) - g(t) \tag{12}$$

That

$$\overline{A}_i = A_{2i} - K_i C_2,\ \overline{\Delta}A = \sum_{i=1}^{r_F} \overline{\mu}_i A_{2i},\ \overline{\Delta}B = \sum_{i=1}^{r_F} \overline{\mu}_i B_{2i},\ \overline{\mu}_i = \mu_i(\xi) - \mu_i\left(\hat{\xi}\right),$$

$$G = \begin{bmatrix} J & H_{2i} \end{bmatrix},\ \Delta = \begin{bmatrix} \overline{\Delta}A & \overline{\Delta}B \end{bmatrix},\ \varepsilon(t) = \begin{bmatrix} f_k(t) \\ w(t) \end{bmatrix},\ z(t) = \begin{bmatrix} \overline{\xi}(t) \\ u(t) \end{bmatrix}.$$

And

$$g = \begin{cases} 0 & \text{if } |e_y| < \varepsilon \\ \sigma_1 \delta_1^2 \frac{\hat{\bar{\xi}}^T \hat{\bar{\xi}}}{2e_y^T e_y} P^{-1} C_2^T e_y + \sigma_2 \delta_2^2 \frac{u^T u}{2e_y^T e_y} P^{-1} C_2^T e_y & \text{if } |e_y| \geq \varepsilon \end{cases}$$

$$\sigma_1 = \frac{1}{\alpha\lambda_0}, \sigma_2 = \frac{\lambda}{\lambda\lambda_0(1+\alpha) - 1}$$

In this simulation, white noise is considered with a variance of 0.1. The initial state of the system is $x = [0.0.0.0.200.500. - 100]^T$ and the initial value of estimation is $x = [0.0.0.0.0.0.0]^T$. The value of the parameters is $(r = 12.\sigma = 8.b = 8.3)$.

Here, the process fault is applied at $t = 0.5$ sec to the system. The calculated observer gains are given in Table 1.

K_p	K_i
$\begin{bmatrix} 132000 & 73000 \\ 3912000 & 355000 \\ -640000 & -111000 \\ 2000 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 134000 & 73000 \\ 3911000 & 345000 \end{bmatrix}$

TABLE 1. Fuzzy observer gains in sensor fault occurrence.

Minimum of μ

$$\begin{bmatrix} \varphi_i & \text{PJ} & PH_{2i} & PH_{2i} & \delta_1 \cdot I & I \\ * & -\mu \cdot I & 0 & 0 & 0 & 0 \\ * & * & -\mu \cdot I & 0 & 0 & 0 \\ * & * & * & -\lambda \cdot I & 0 & 0 \\ * & * & * & * & -\lambda_0 \cdot I & 0 \\ * & * & * & * & * & -\mu I \end{bmatrix} < 0$$

And

$$\varphi_i = (PA_{2i} - N_i C_2) + (PA_{2i} - N_i C_2)^T\ .\ K_i = P^{-1} N_i \tag{13}$$

Figures 3 and 4 show normal condition and the estimations of states response, assuming that the reference value of states are zero, when a fault happens. Following the early transient, the state trajectories moves close to $x = [0. - 300.0.0.0.0.100]^T$ however, at

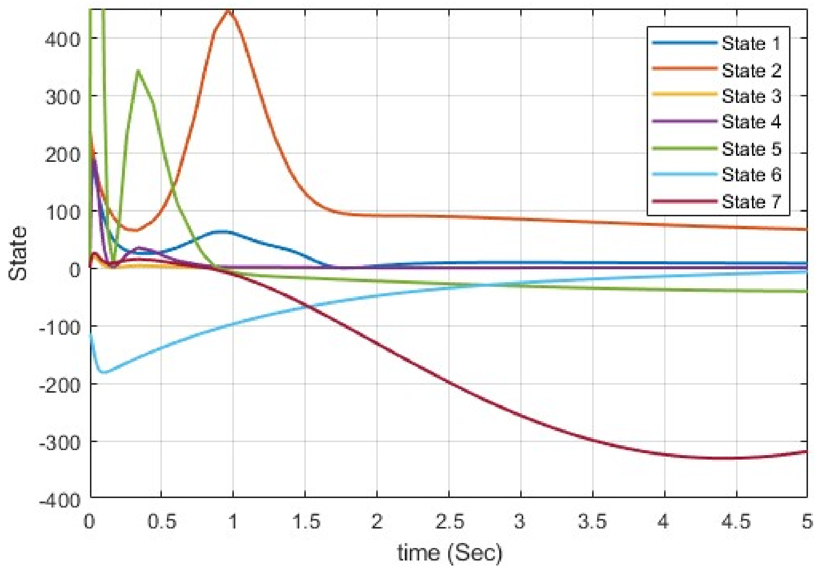

FIGURE 3. State response of state variables in normally condition.

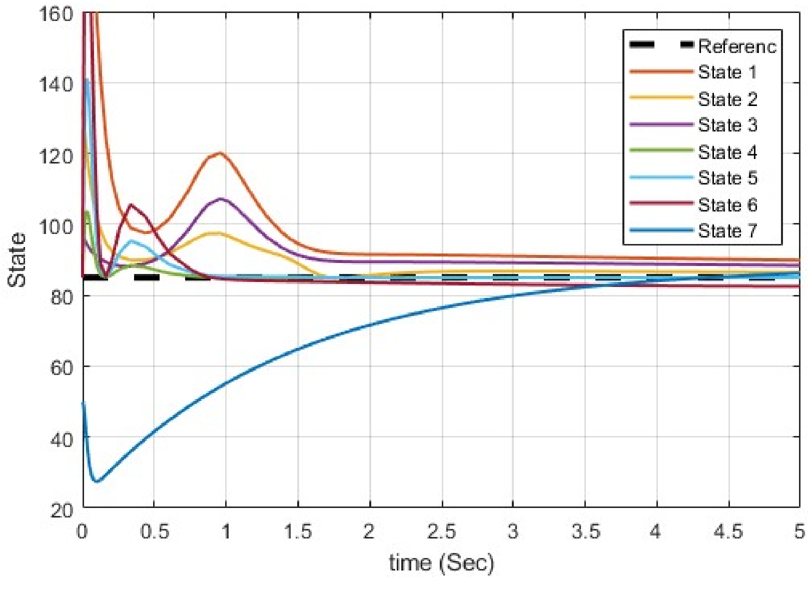

FIGURE 4. State response of state variables in sensor fault occurrence condition.

$t = 0.5$, which is the moment that the fault happens, the solution settles into an irregular oscillation. Therefore, the fault creates the bifurcation and alters the dynamics treatment. In chaotic conditions, estimation of the states is possible for the observer.

Fault of the actuator is used at 10^{th}s. The actuator fault signal contains a ramp and a sin. The obtained gains of the observer are listed in Table 2. Figure 5 illustrates the estimations, states and system respond. As mentioned, the time of using the control force in the system is at $t = 15$s and at $= 20$s when the system enters a steady-state. Figure 6

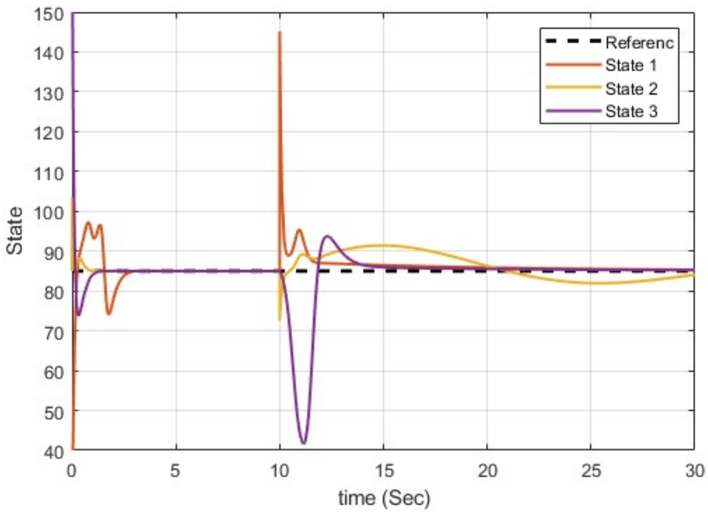

FIGURE 5. Estimations of the states in actuator fault occurrence at $t = 10$ sec.

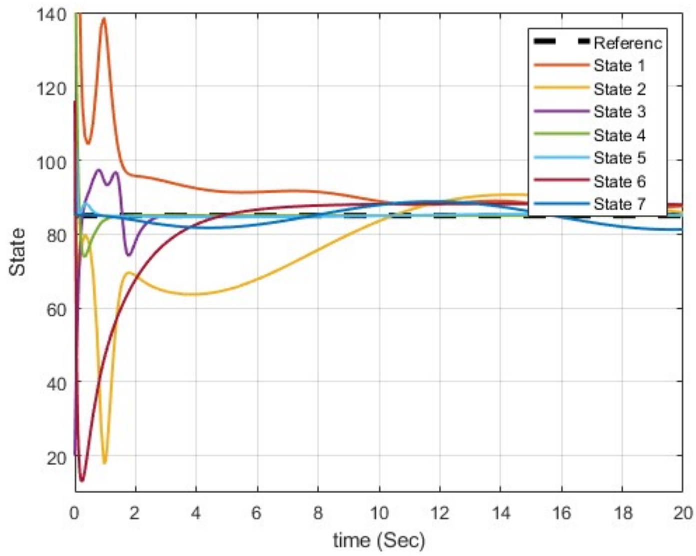

FIGURE 6. The estimation errors of variables in actuator fault occurrence.

illustrates the estimation errors. Clearly, the model gives a good estimate of the variables before and after using the control force.

4. Comparing the proposed algorithm with adaptive control model

Chaotic systems have two characteristics: 1. unpredictable behavior and 2. sensitivity to initial conditions. The Lu chaotic system was already used to investigate the proposed T-S fuzzy observer model. Although simulations demonstrated the efficiency of this observer, the present work compared the proposed model to an adaptive observer (designed based on adaptive control principles) for a comprehensive evaluation.

The Genesio-Tesi system is a multi-state chaotic system. It is formulated as:

K_p	K_i
$\begin{bmatrix} 168054 & 58440 \\ 4896100 & 351670 \\ -624720 & -141450 \\ 4210 & 174.32 \\ -345.1 & 312.9 \end{bmatrix}$	$\begin{bmatrix} 14981000 & 2963000 \\ 487000 & 394000 \end{bmatrix}$

Table 2. Fuzzy observer gains in actuator fault occurrence.

$$\begin{cases} \dfrac{dx}{dt} = y, \\ \dfrac{dy}{dt} = z, \\ \dfrac{dz}{dt} = -ax - by - cz + x^2. \end{cases} \tag{14}$$

where a=6, b=2.92, and c=1.2.
Figure 7 depicts the Genesio-Tesi chaotic system.

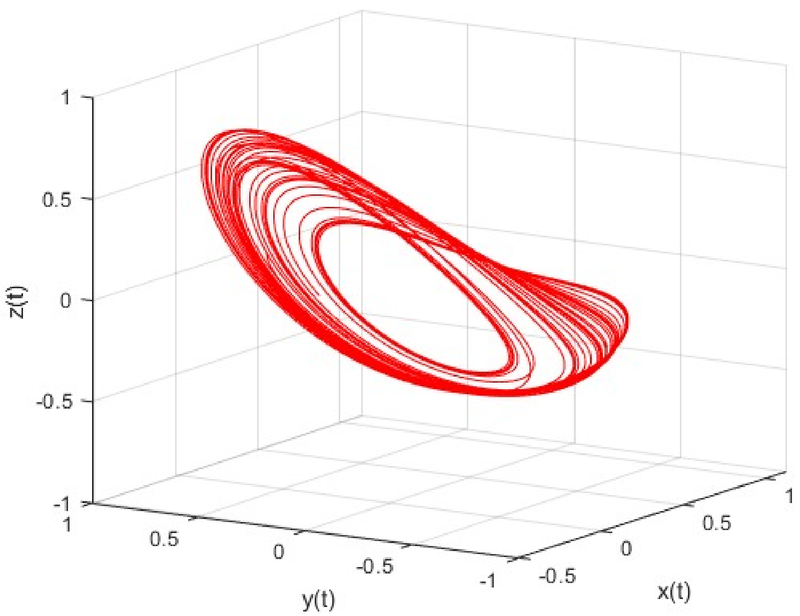

Figure 7. Genesio-Tesi chaotic system.

Figure 8 compares the proposed T-S fuzzy observer model and adaptive control model. As can be seen, the proposed model had a good speed and accuracy relative to the adaptive control model. Based on the simulations of the Lu and Genesio-Tesi chaotic systems, it can be said that the T-S fuzzy observer model had satisfactory performance and would remain satisfactory for other chaotic systems.

5. Conclusions

The actuator fault estimation of the fractional-order Lu system was examined with noise and fault of the sensor. To this end, the proportional-integral TS fuzzy observer was used.

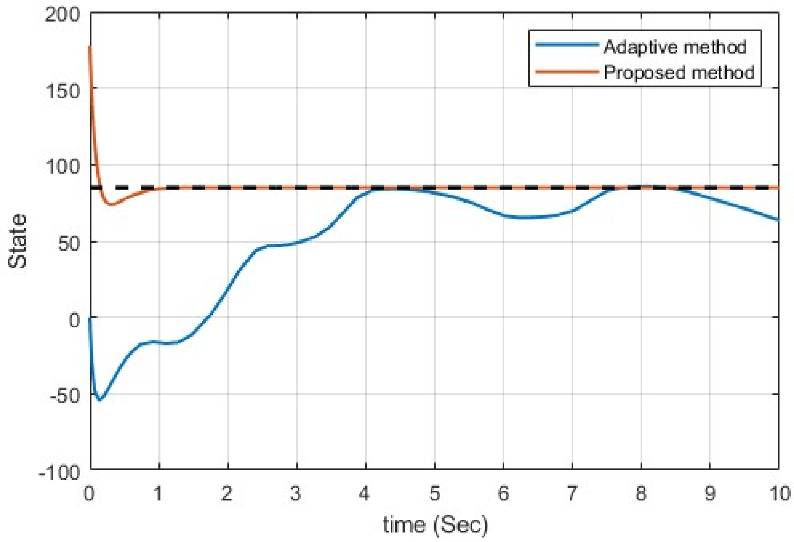

FIGURE 8. compares the proposed T-S fuzzy observer model and adaptive control model.

Only the numerical analysis was taken into account, which can describe a large portion of the system. In addition, a high frequency sinusoidal signal and actuator and sensor fault disturbed the system. The proposed model estimates the process, sensor, and actuator fault and the system in a desirable way. Practically speaking, these observers are able to estimate the value of bifurcation in other industrial system that have similar behavior with fractional-order Lu system. Since a precise examination can alter the behavior of the system and with fault-tolerant control, the system returns to its good performance and system failure is restored. In addition, the proposed system was compared to the Lu and Genesio-Tesi chaotic systems to measure its performance. Furthermore, the comparison of the proposed system to the adaptive control model demonstrated the efficiency of the proposed control model.

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